Extreme Mass Ratio Inspirals and Black Hole Spectroscopy

Marc Casals
University College Dublin

In collaboration with S.Dolan, A.Ottewill, B.Wardell

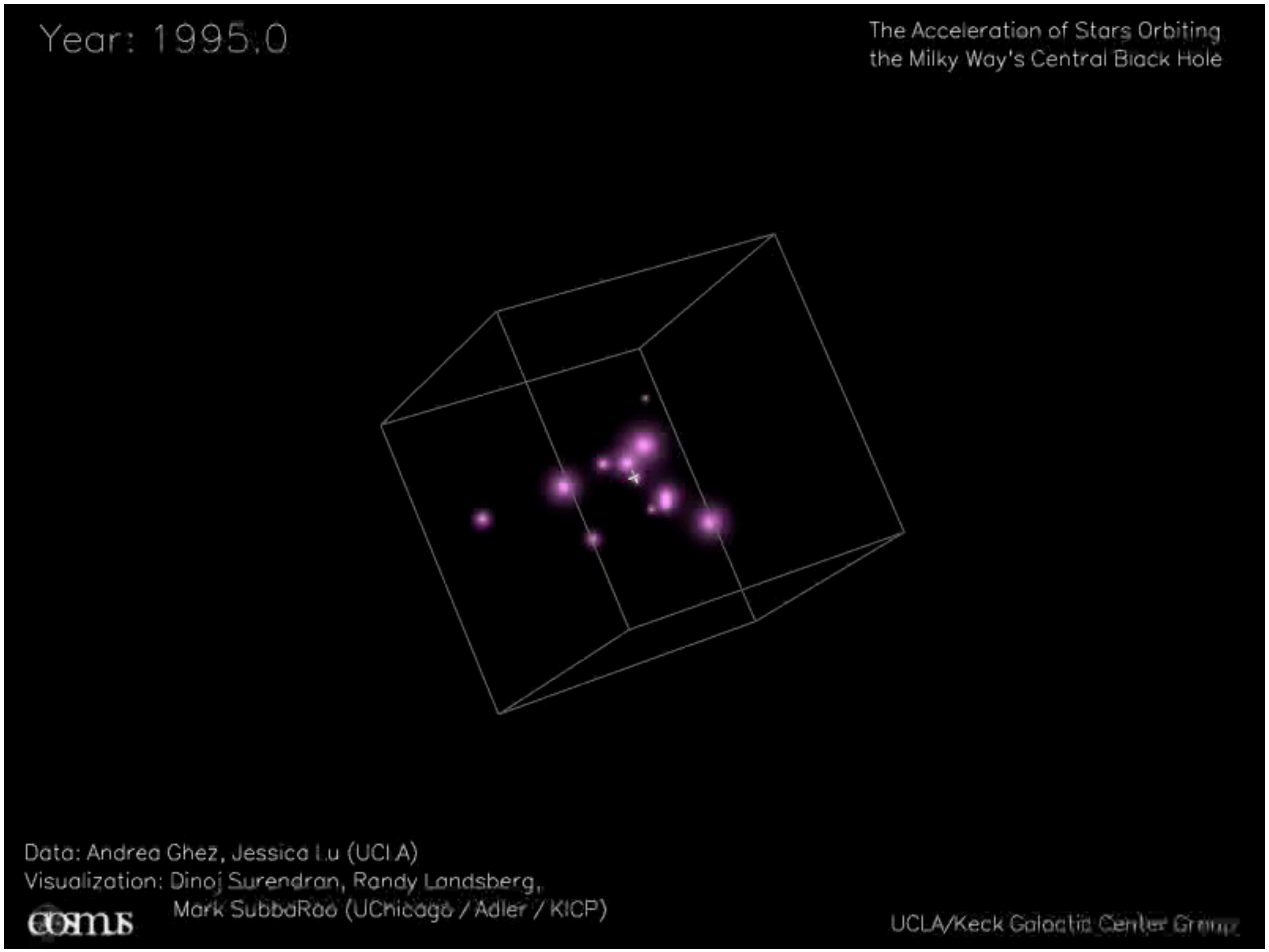
CENTRA-Instituto Superior Técnico, 13 December 2012

Outline

1. Self-Force for EMRIs
2. Green Function
3. Numerical Method
4. Analytical Method - Spectroscopy
5. Conclusions

Motivation - Black Hole Inspirals

Supermassive ($\approx 4 \cdot 10^6 M_\odot$) black hole at the centre of the Milky Way



Credit: UCLA

Motivation - Gravitational Waves

- Gravitational waves (ripples in spacetime) emitted during inspiral
- Evidence of their existence from binary pulsar (Nobel prize, 1993)
- Interferometers (LIGO, LISA) expected to detect GWs
- GWs are important for:
  - Mapping spacetime near black holes
  - Testing General Relativity
  - (Observing early Universe)

Linearized Einstein Equations

- **Einstein eqs.** of GR: 10 coupled, highly nonlinear 2nd order PDEs

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}$$

- Methods for solving the eqs. in the case of binary inspirals:

 - **Post-Newtonian approx.:** expansion in v/c. Valid at early stages of inspiral

 - **Numerical Relativity**: b-h masses $\frac{M}{m} \sim 1 - 100?$

 - **Linearize** eqs. for **Extreme Mass Ratio Inspiral**: $\frac{M}{m} \sim 10^4 - 10^8$

 Total metric $= g_{\mu\nu} + h_{\mu\nu} + O\left(\frac{m}{M}\right)^2$

 $g_{\mu\nu}$: due to M

 $h_{\mu\nu}$: perturbation (**gravitational waves**) due to m

Self-Force

- Inspiral of small mass ($\sim 10M_\odot$) around super-massive Black Hole ($\sim 10^5 - 10^9 M_\odot$) **deviates from geodesic** due to the action of its own 'regularized' field: **Self-Force** (DeWitt&Brehme'60 and Mino, Sasaki, Tanaka, Quinn, Wald'97)

- Self-field is singular at the location of particle -> regularization obeying **covariance** and **causality** $h_{\alpha\beta} \rightarrow h^R_{\alpha\beta}$

- Alternative viewpoint: motion is **geodesic** in spacetime with metric of super-massive black-hole plus 'regularized' metric of small mass (Detweiler&Whiting'03) $g_{\alpha\beta} + h^R_{\alpha\beta}$

- Motion of small bodies is open fundamental problem in GR

- S-F upholds **Cosmic Censorship** (Cardoso&al'11, Poisson&al'12,...)

S-F results in Schwarzschild with other methods

- 'Redshift' parameter $dt/d\tau$ as a function of orbital frequency is gauge invariant. Agreement between S-F and post-Newtonian (Detweiler'08)

- Correction to orbital radius&frequency of ISCO using mode-sum reg. (Barack,Sago'09)

- Correction to precession effect (rate of periastron advance) using mode-sum reg. (Barack,Damour,Sago'10)

- 'Geodesic' S-F orbit for gravitational case using mode-sum reg. (Warburton,Ackay,Barack,Gair,Sago'11)

- Self-consistent orbit and waveform for scalar charge using 'effective source' (Diener,Vega,Wardell,Detweiler'11)

Wave Eq.

- **Gravitational wave** (spin=2) eq.

$$\text{``}\Box h_{\mu\nu}\text{''} = T_{\mu\nu} \qquad \Box \equiv g_{\mu\nu}\nabla^{\mu}\nabla^{\nu}$$

Similar wave eq. for emag (spin=1) and scalar (spin=0) fields

- **Retarded Green function** defined by

$$\Box G_R(x, x') = \delta_4(x, x') \quad \text{with causality b.c.}$$

- **S-F** for scalar charge:

$$f_\mu = q^2(\delta^\nu_\mu + v^\nu v_\mu)\nabla_\nu \int_{-\infty}^{\tau^-} G_{ret}(z(\tau), z(\tau'))d\tau' + \text{local}$$

Global structude of G_{ret} is crucial!

Green Function

- Green function is important for:

  - Calculation of S-F

  - Study of classical stability of b-h's

  - Evolution of initial data in b-h spacetime

  - Quantum properties, eg, quantization of black hole area, 'gauge-gravity' duality,...

New Numerical method

- **Kirchhoff integral: evolution of initial data in space-time of b-h**

$$u(x) = \int_{t=0} \left[G_{ret}(x,x')\dot{u}^{ic}(\vec{x}') - u^{ic}(\vec{x}')\partial_t G_{ret}(x,x') \right] g^{tt}(x') d^3\vec{x}'$$

$$\Box u = 0$$

New Numerical method

- Kirchhoff integral: evolution of initial data in space-time of b-h

$$u(x) = \int_{t=0} \left[G_{ret}(x,x')\dot{u}^{ic}(\vec{x}') - u^{ic}(\vec{x}')\partial_t G_{ret}(x,x') \right] g^{tt}(x') d^3\vec{x}'$$

$$\Box u = 0$$

- New method: numerical evolution of a 'Peaked Gaussian':

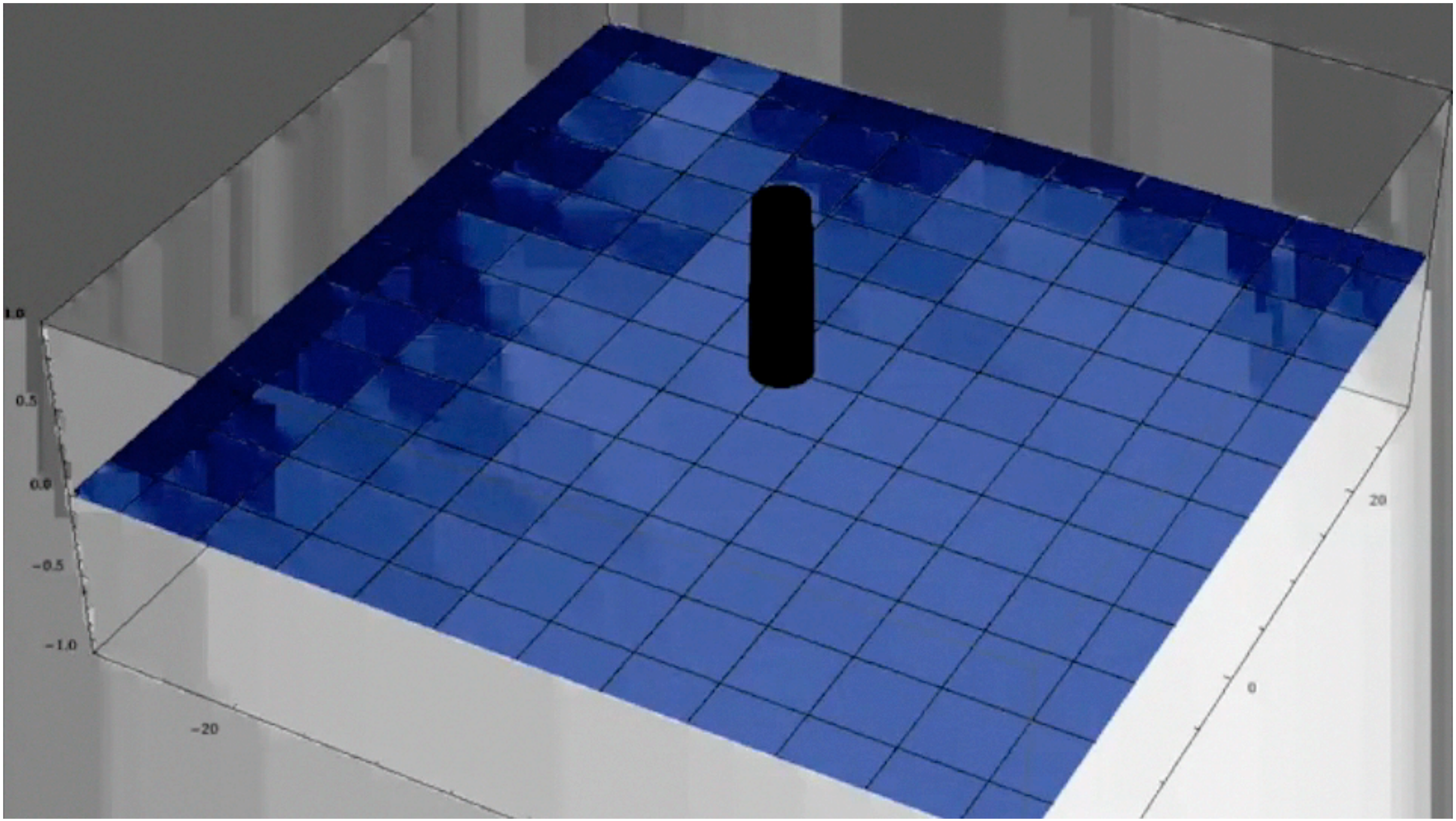
zero

$$G_{ret}(x,x'')$$

$$\frac{1}{(2\pi w^2)^{3/2}} e^{-|\vec{x}'-\vec{x}''|^2/(2w^2)} \approx \delta_3(x'-x'')$$

$$w \ll M$$

Numerical evolution of peaked Gaussian around equator of b-h



Method of Matched Expansions

- **Non-local part of S-F:** $\int_{-\infty}^{\tau^-} G_{ret}\, d\tau'$

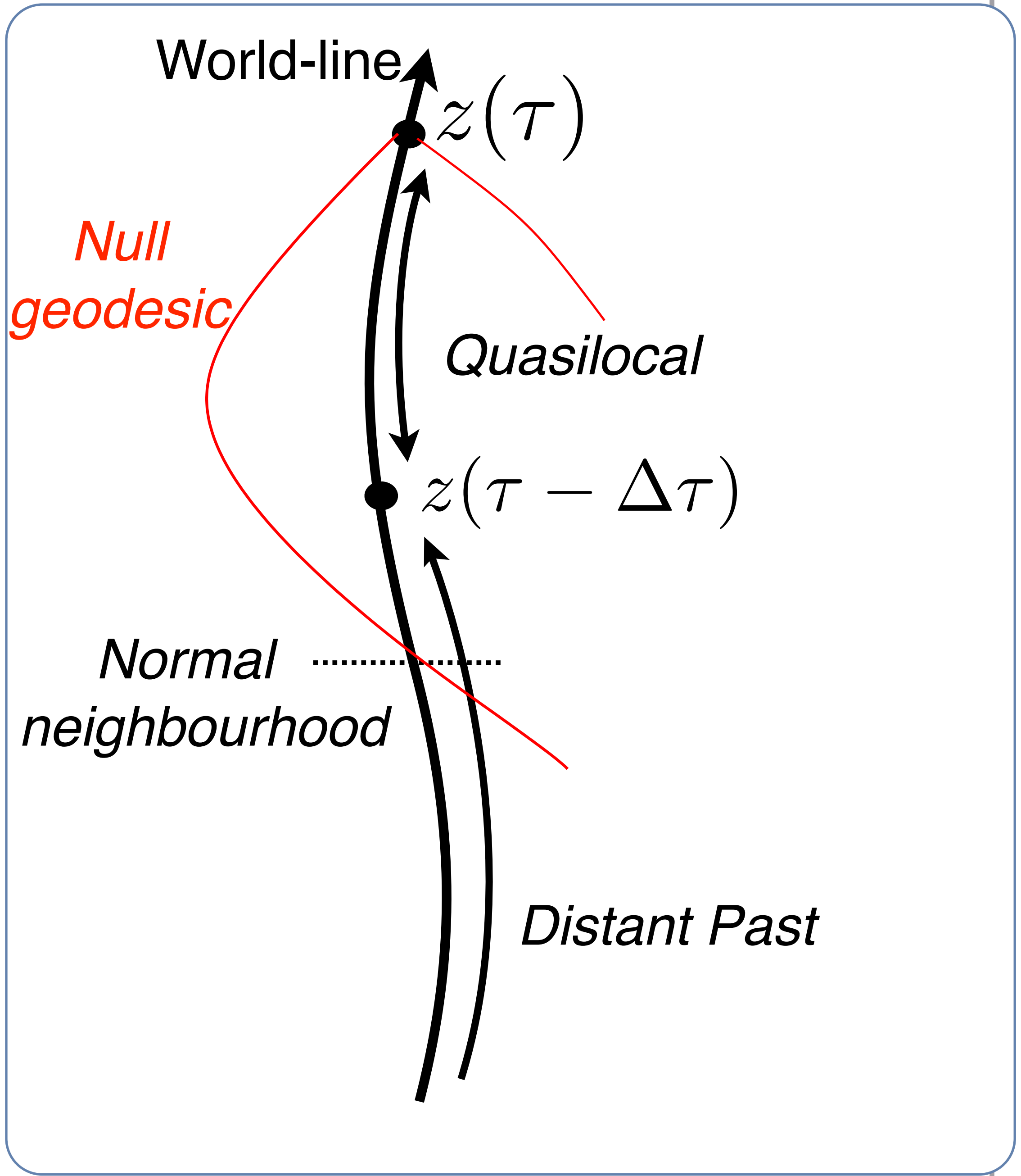
- **Matched expansions:** **choose** $\Delta\tau$ **:**

- before that point ('Quasilocal' region)

$$\int_{\tau-\Delta\tau}^{\tau^-} G_{ret}\, d\tau'$$

- after that point ('Distant Past')

$$\int_{-\infty}^{\tau-\Delta\tau} G_{ret}\, d\tau'$$



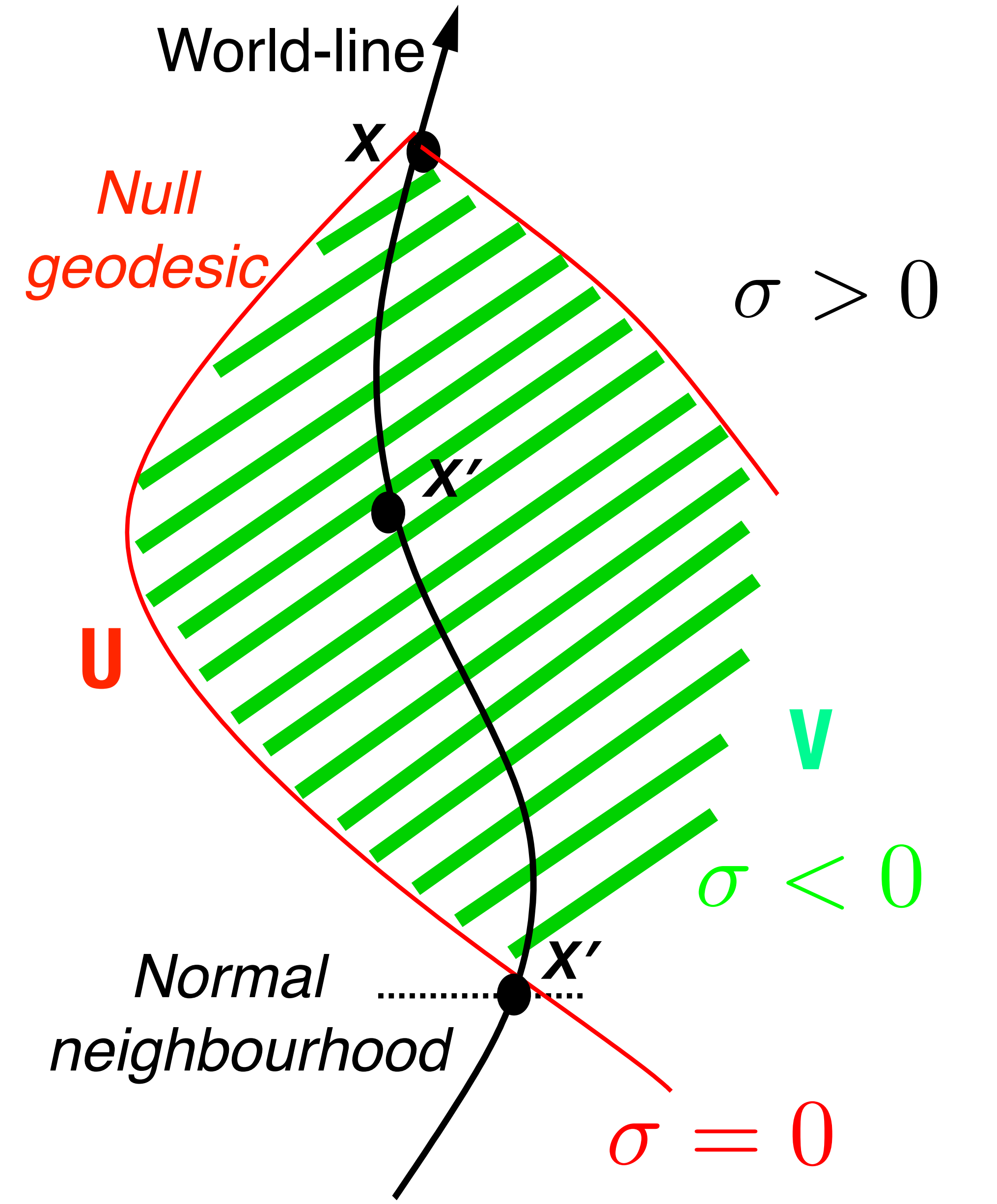
Quasilocal - Hadamard form

$$G_{ret}(x,x') = \underbrace{\theta(\Delta t)}_{\neq 0 \text{ in the past}} \left\{ \underbrace{U(x,x')\,\delta(\sigma)}_{\neq 0 \text{ on light cone}} + \underbrace{V(x,x')\,\theta(-\sigma)}_{\neq 0 \text{ inside light cone}} \right\}$$

- σ : **geodesic distance between x & x’**
- **U & V regular**
- **Valid in normal neighbourhood**

$$\int_{\tau-\Delta\tau}^{\tau^-} G_{ret}\; d\tau' = \int_{\tau-\Delta\tau}^{\tau} V\; d\tau'$$

$$V = \sum_{k=0}^{\infty} V_k \sigma^k$$



- **It renders regularization trivial**

Distant Past: Black Hole Spectroscopy

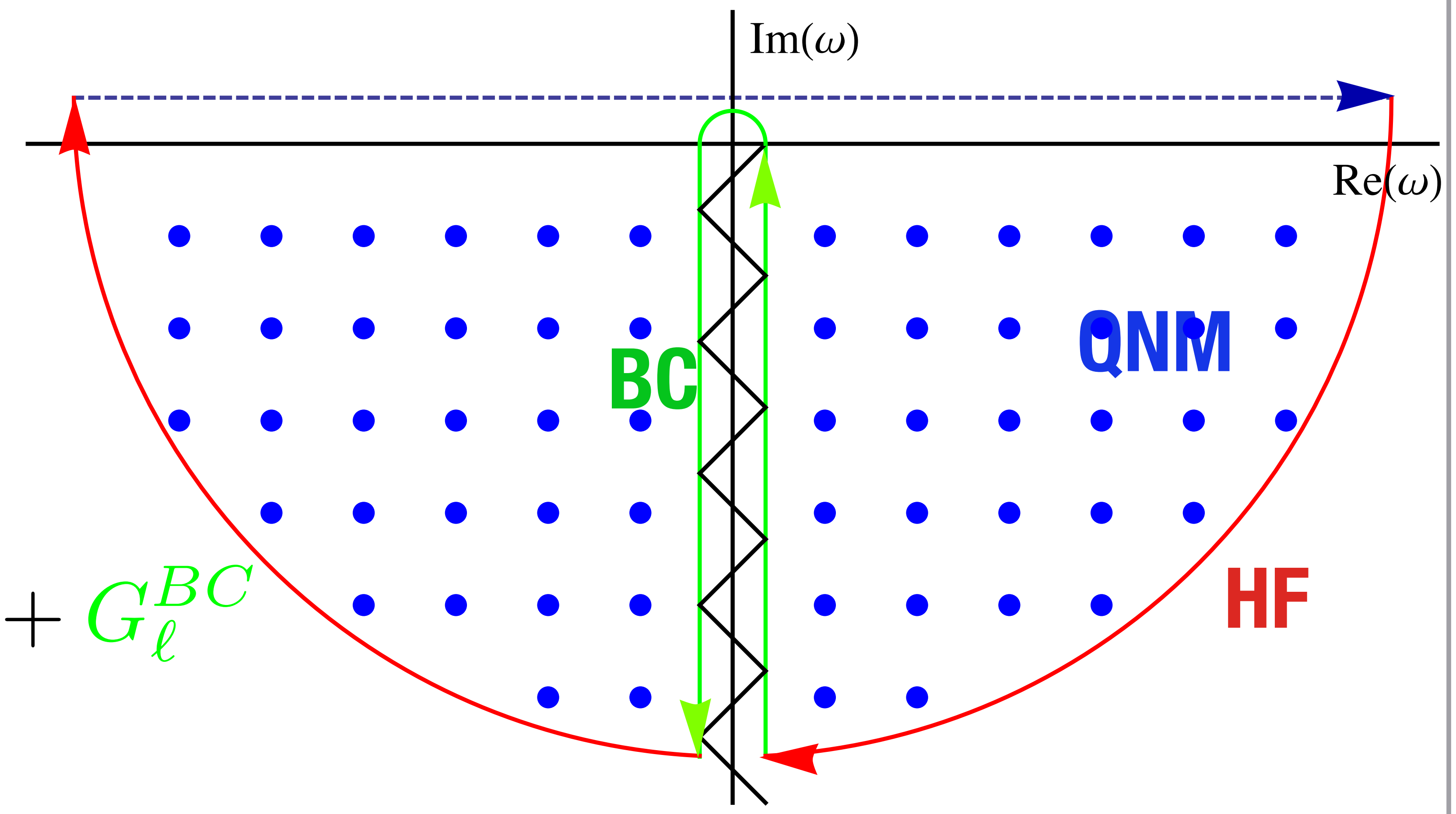
- **Multipolar decomposition:**

$$G_{ret}(x,x') = \frac{1}{rr'}\sum_{\ell=0}^{\infty}(2\ell+1)P_\ell(\cos\gamma)G_\ell^{ret}(r,r';t)$$

- **Fourier transform:**

$$G_\ell^{ret}(r,r';t) \equiv \int_{-\infty+ic}^{\infty+ic} d\omega\ G_\ell(r,r';\omega)e^{-i\omega t}$$

Complex-Frequency Plane



- **Residue theorem:**

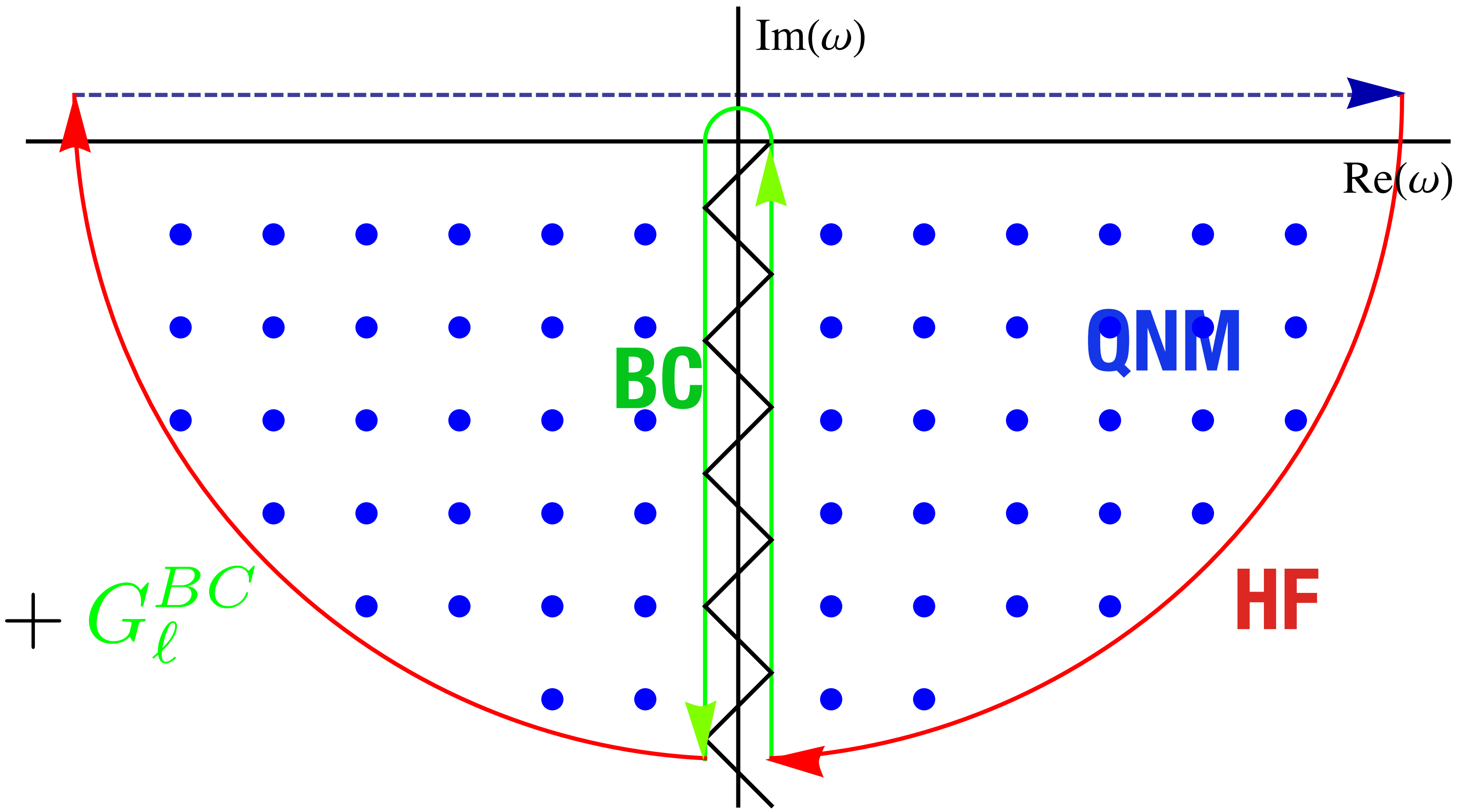
$$G_\ell^{ret} = G_\ell^{HF} + G_\ell^{QNM} + G_\ell^{BC}$$

G_ℓ^{HF} **Integral along high-frequency arc. Zero in Distant Past.**

G_ℓ^{QNM} **Sum over residues of poles (quasinormal modes)**

G_ℓ^{BC} **Integral around branch cut**

Complex-Frequency Plane



- **Residue theorem:**

$$G_\ell^{ret} = \cancel{G_\ell^{HF}} + G_\ell^{QNM} + G_\ell^{BC}$$

G_ℓ^{HF} **Integral along high-frequency arc. Zero in Distant Past.**

G_ℓ^{QNM} **Sum over residues of poles (quasinormal modes)**

G_ℓ^{BC} **Integral around branch cut**

Radial Equation

- **Green function modes:** $G_\ell(r,r';\omega) = \dfrac{R_\ell^{in}(r_<,\omega)\, R_\ell^{up}(r_>,\omega)}{W(\omega)}$

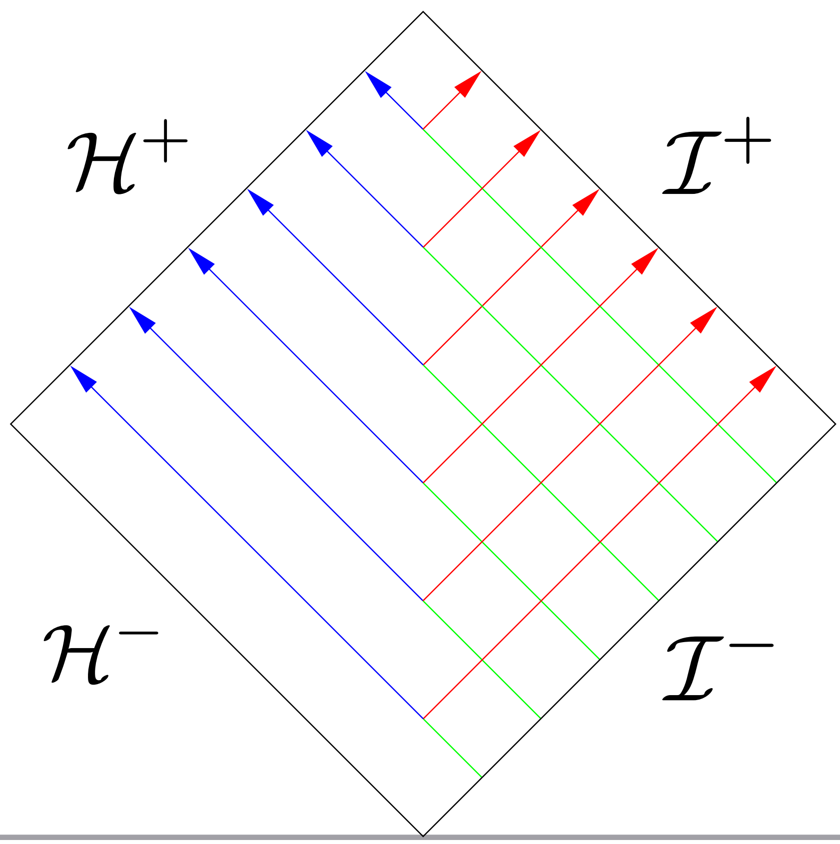
- **Radial ODE ('Regge-Wheeler eq.') for the perturbation:**

$$\left[\frac{d^2}{dr_*^2} + \omega^2 - V(r)\right] R_\ell(r,\omega) = 0 \qquad V(r) = \left(1 - \frac{1}{r}\right)\left[\frac{\ell(\ell+1)}{r^2} + \frac{(1-s^2)}{r^3}\right]$$

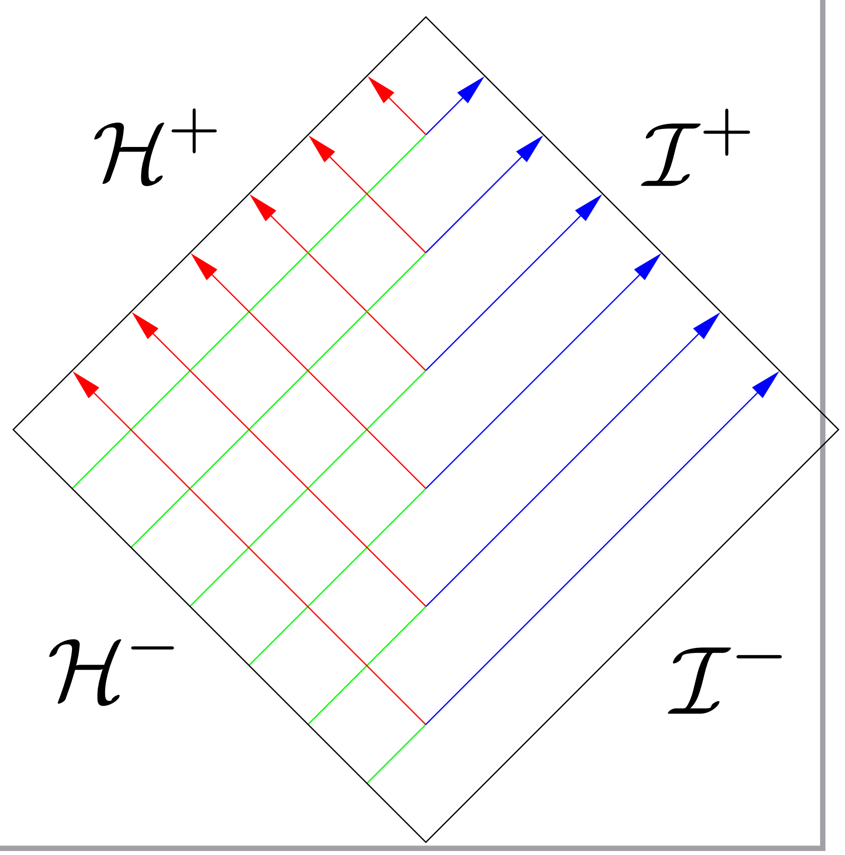
$$r_* = r_*(r) \in (-\infty,\infty) \qquad s = 0,1,2$$

- **Two lin. indep. slns.:**

'in'



'up'



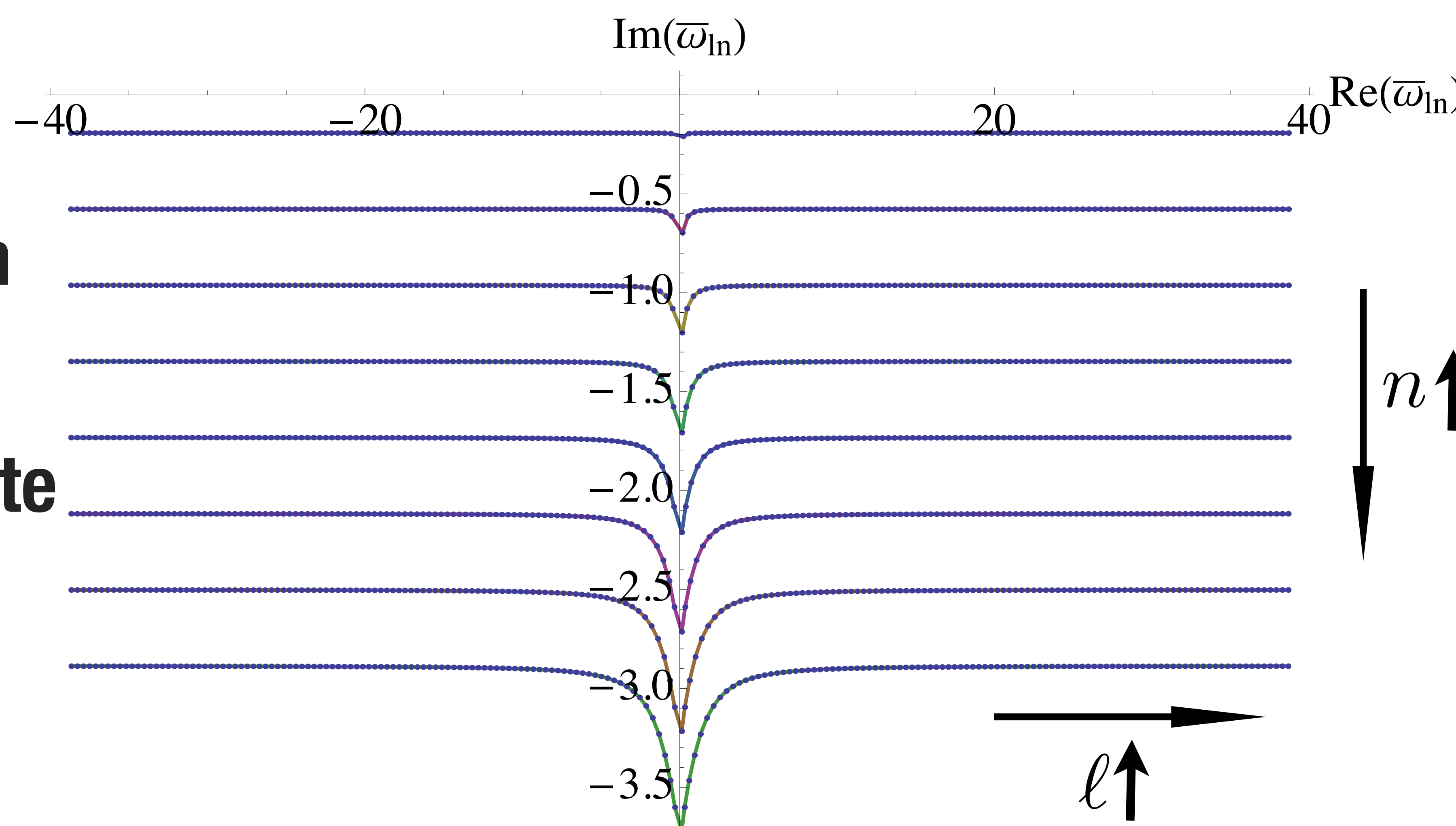
Quasinormal Modes

- **QNM frequencies: simple poles of** $G_\ell = \dfrac{R_\ell^{in}(r_<,\omega)\,R_\ell^{up}(r_>,\omega)}{W(\omega)}$ **in the complex-ω plane, ie,** $W(\omega_{ln}) = 0$

- **Boundary conditions:** $\underset{r_*\to-\infty}{e^{-i\omega_{\ell n} r_*} \sim R_\ell^{in}} \propto \underset{r_*\to\infty}{R_\ell^{up} \sim e^{+i\omega_{\ell n} r_*}}$

$\mathrm{Re}(\omega_{\ell n})$ **: freq. of oscillation**

$\mathrm{Im}(\omega_{\ell n})$ **: decay rate**



Branch Cut

Branch Cut

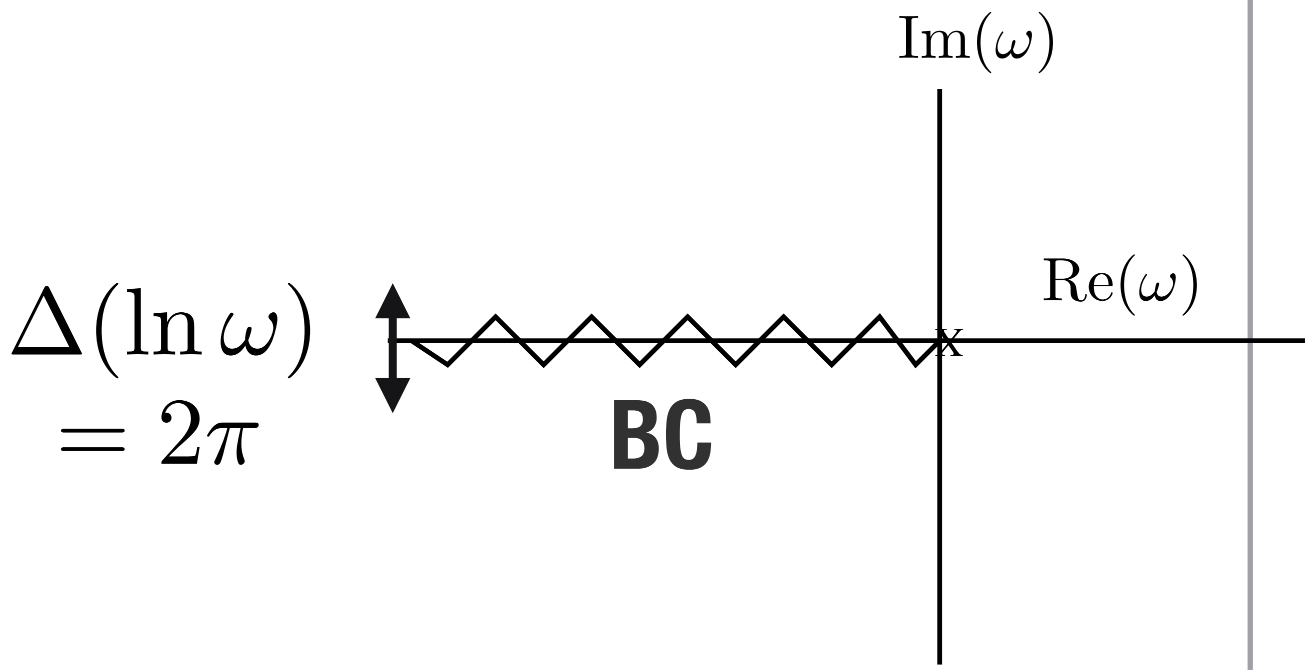
- Ahem...what is a BC??

Branch Cut

- **Ahem...what is a BC??**

Ex: $\ln \omega = \ln |\omega| + i \arg(\omega)$

$\arg(\omega) \in (-\pi, \pi]$



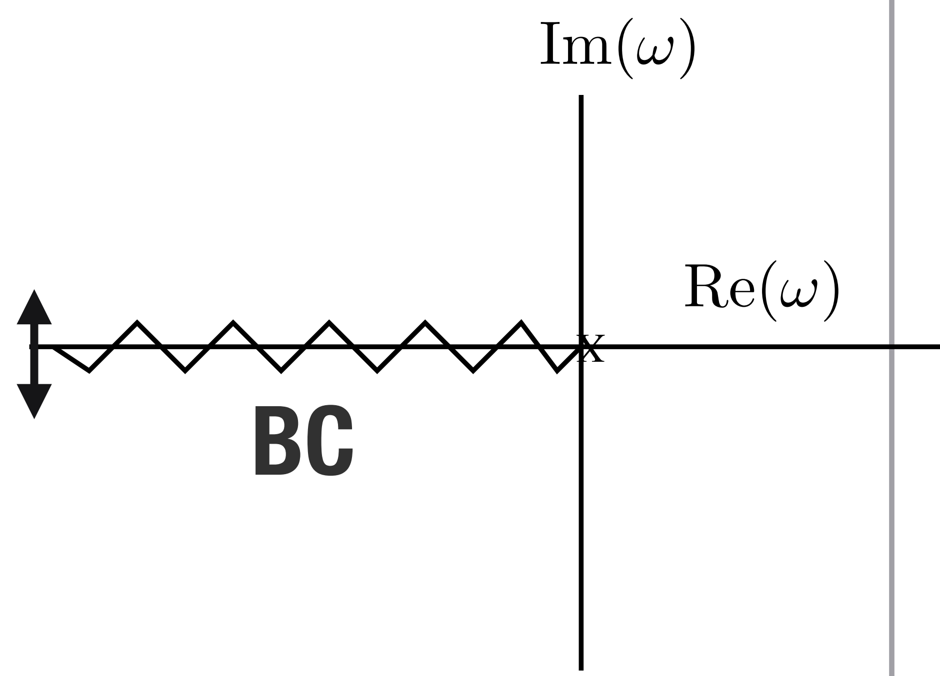
Branch Cut

- **Ahem...what is a BC??**

  Ex: $\ln\omega = \ln|\omega| + i\arg(\omega)$

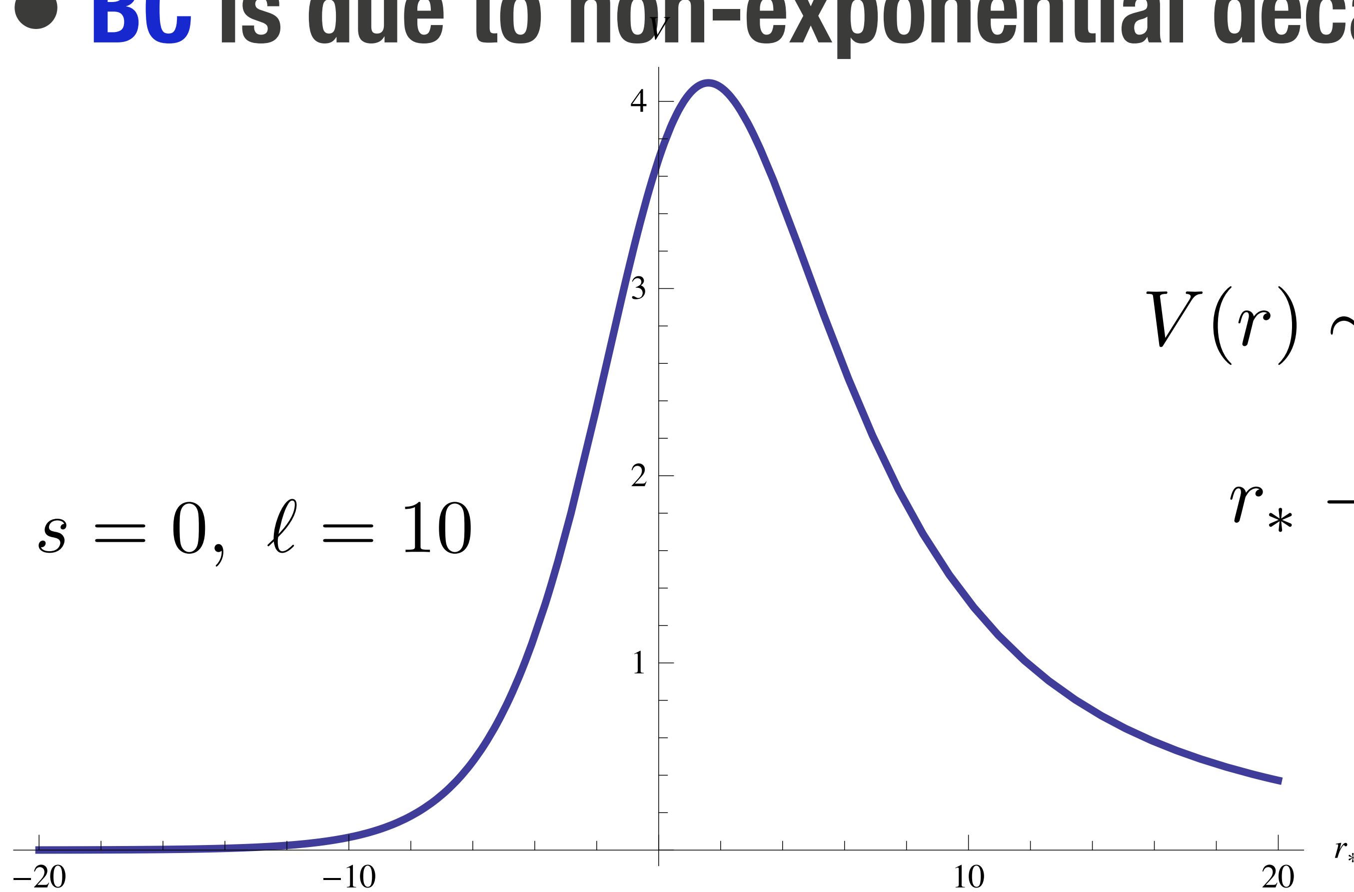
  $\arg(\omega) \in (-\pi, \pi]$

  $\Delta(\ln\omega) = 2\pi$



- **BC integral** $$G_\ell^{BC} = \int_0^{-i\infty} d\omega\ e^{-i\omega t}\Delta_\ell G$$

- **BC is due to non-exponential decay of potential at radial infinity:**



$s = 0,\ \ell = 10$

$$V(r) \sim \frac{\ell(\ell+1)}{r_*^2} + \frac{2\ell(\ell+1)r_h\ln(r_*/r_h)}{r_*^3}$$

$$r_* \to \infty$$

Methods for QNMs and BC

- **Large-$|\omega|$ asymptotics by analytic continuation to complex-r plane**
- **Small-$|\omega|$ asymptotics by 'MST method'**
- **Mid-$|\omega|$ by using series of confluent hypergeometric functions**

$$R_\ell^{up} \propto \sum_{n=0}^{\infty} a_n (1-2\nu)_n \, U(s+1-2\nu+n, 2s+1, -2\nu r)$$

New series on BC:

$$\Delta R_\ell^{up} \propto \sum_{n=0}^{\infty} a_n \frac{(-1)^n \Gamma(1+n-2\nu) U(s-n+2\nu, 2s+1, 2\nu r)}{\Gamma(1+s+n-2\nu)\Gamma(1-s+n-2\nu)}$$

this can be evaluated *on* the NIA!

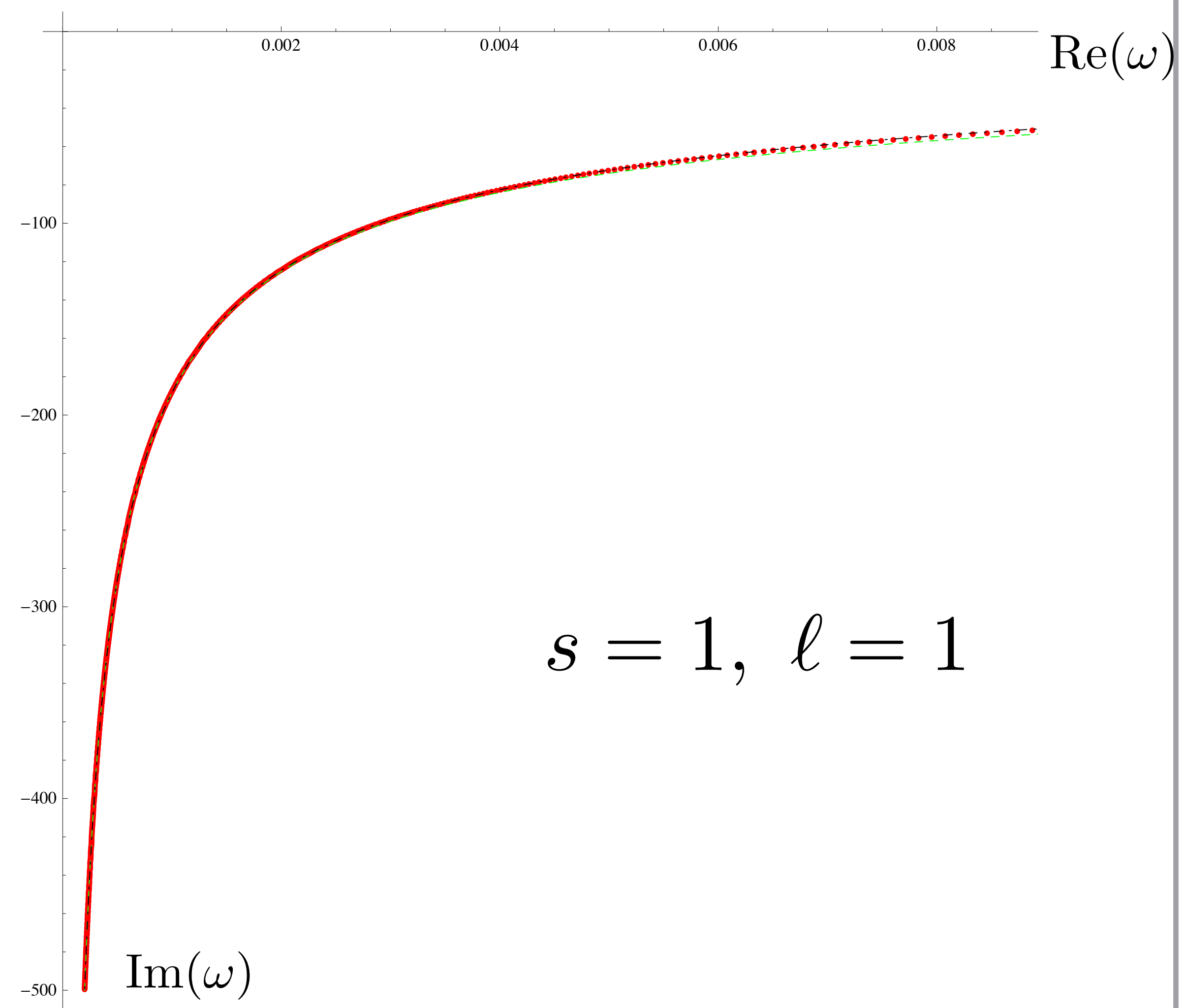
Results: QNMs

- **Large-n asymptotics:**

$$s = 0, 2: \quad \omega_{ln} \sim \frac{\ln 3}{4\pi} - \left(\frac{n}{2} + \frac{1}{4}\right) i + e^{\pi i/4} \frac{\Gamma^4(1/4)\left[(3-s)\ell(\ell+1) + (-1)^{s/2}\right]}{72\pi^{5/2}\sqrt{n}} + O\left(n^{-1}\right)$$

$s = 1:$

$$\omega_{\ell n} = -\frac{in}{2} \underbrace{- \frac{i\lambda^2}{2n} + \frac{e^{-i\pi/4}\pi^{1/2}\lambda^3}{2n^{3/2}} + \frac{3\pi\lambda^4}{4n^2} + \frac{e^{i\pi/4}\sqrt{\pi}\lambda^2\left[72\lambda^3(\pi + \ln 4) - 52\lambda^2 + 41\lambda + 12\right]}{96n^{5/2}}}_{\text{new}} + O\left(\frac{1}{n^3}\right)$$

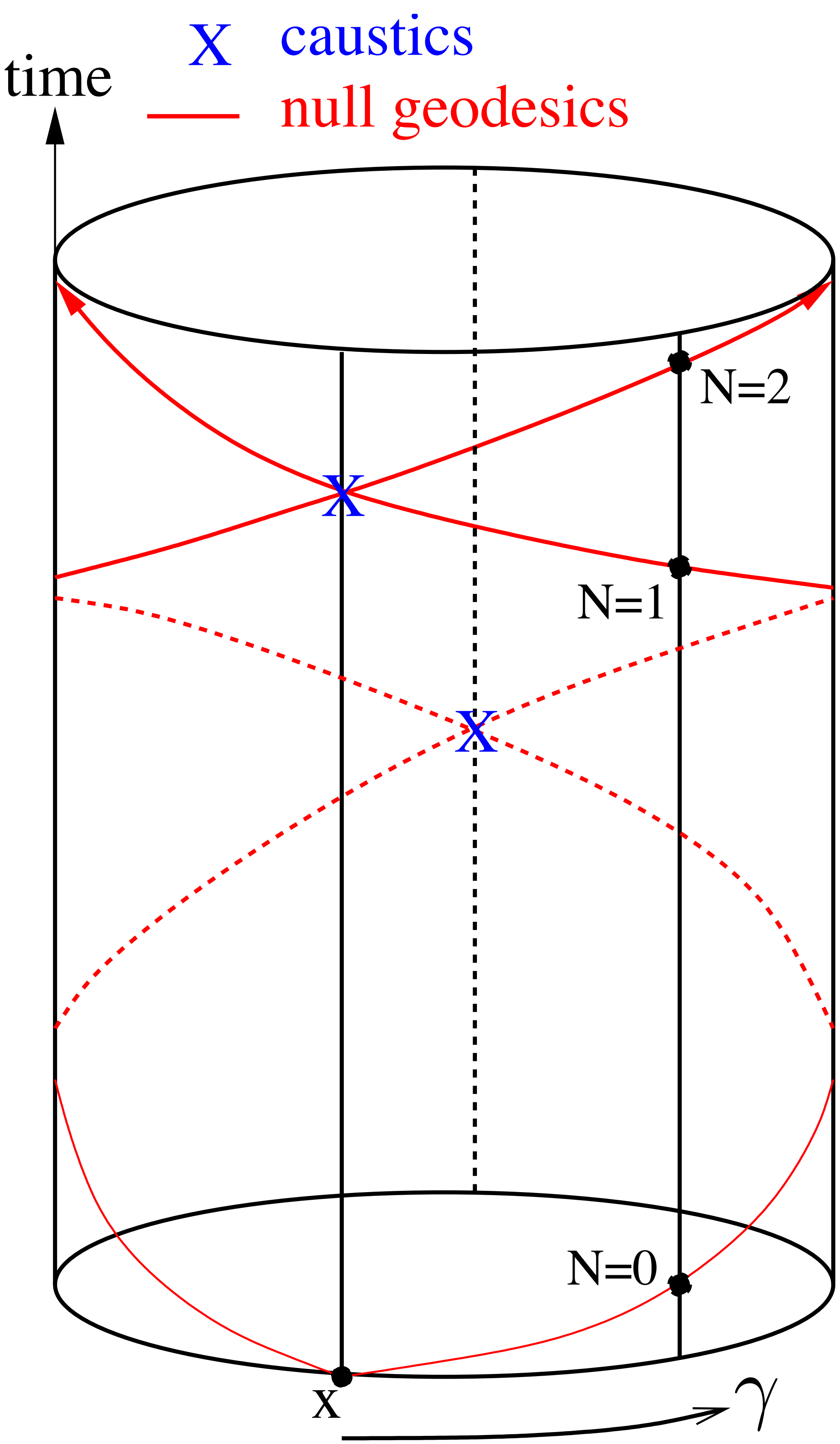


- **Note: highly-damped QNMs have been related to quantization of b-h area, Hawking radiation, small scales of space-time, etc**

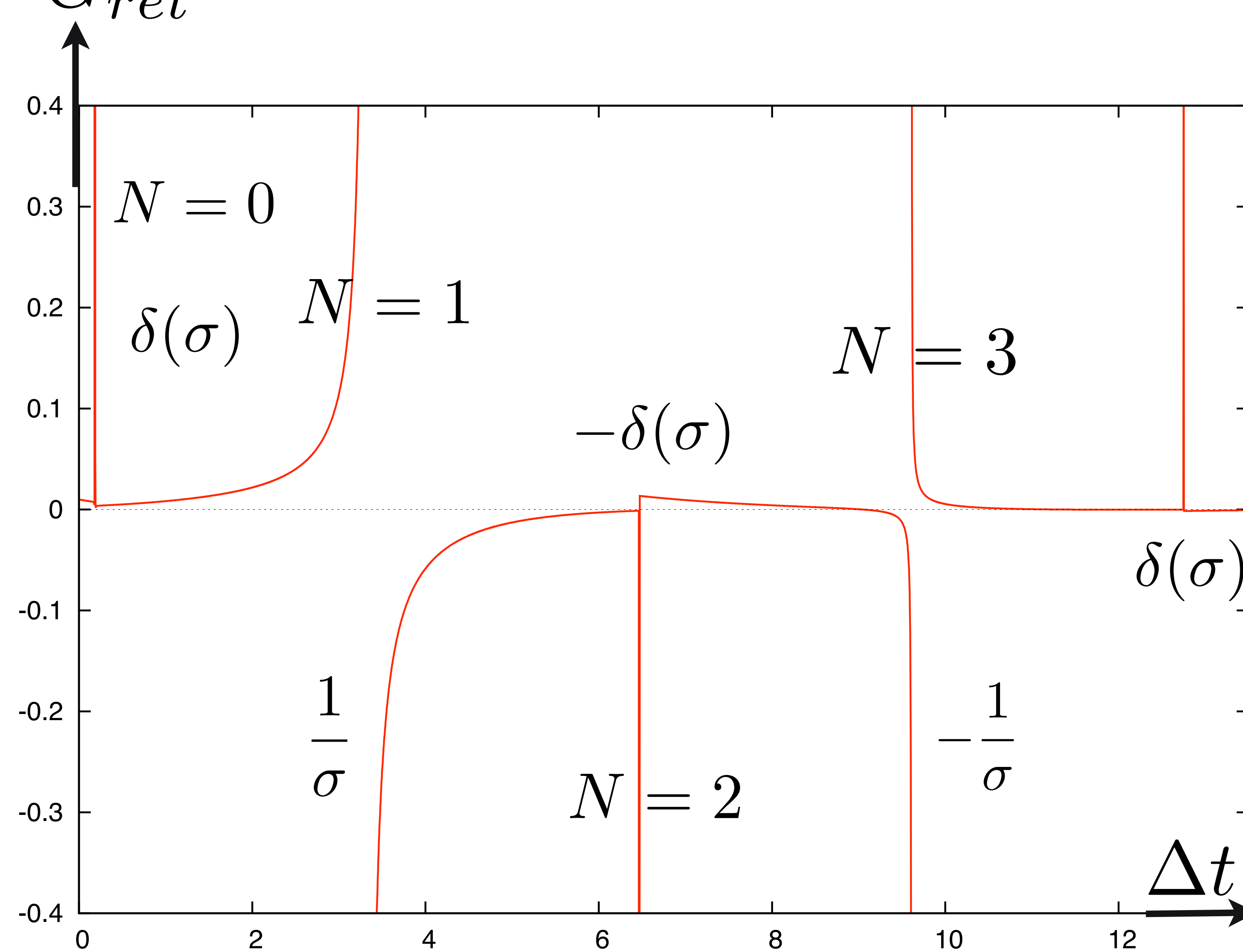
Results: QNMs

- **QNM yield singularity structure:** $G_{ret} \sim \delta(\sigma),\ \frac{1}{\sigma},\ -\delta(\sigma),\ -\frac{1}{\sigma}$

due to caustic-crossing in $\mathbb{S}^2$-topology



Seen in GR for first time!



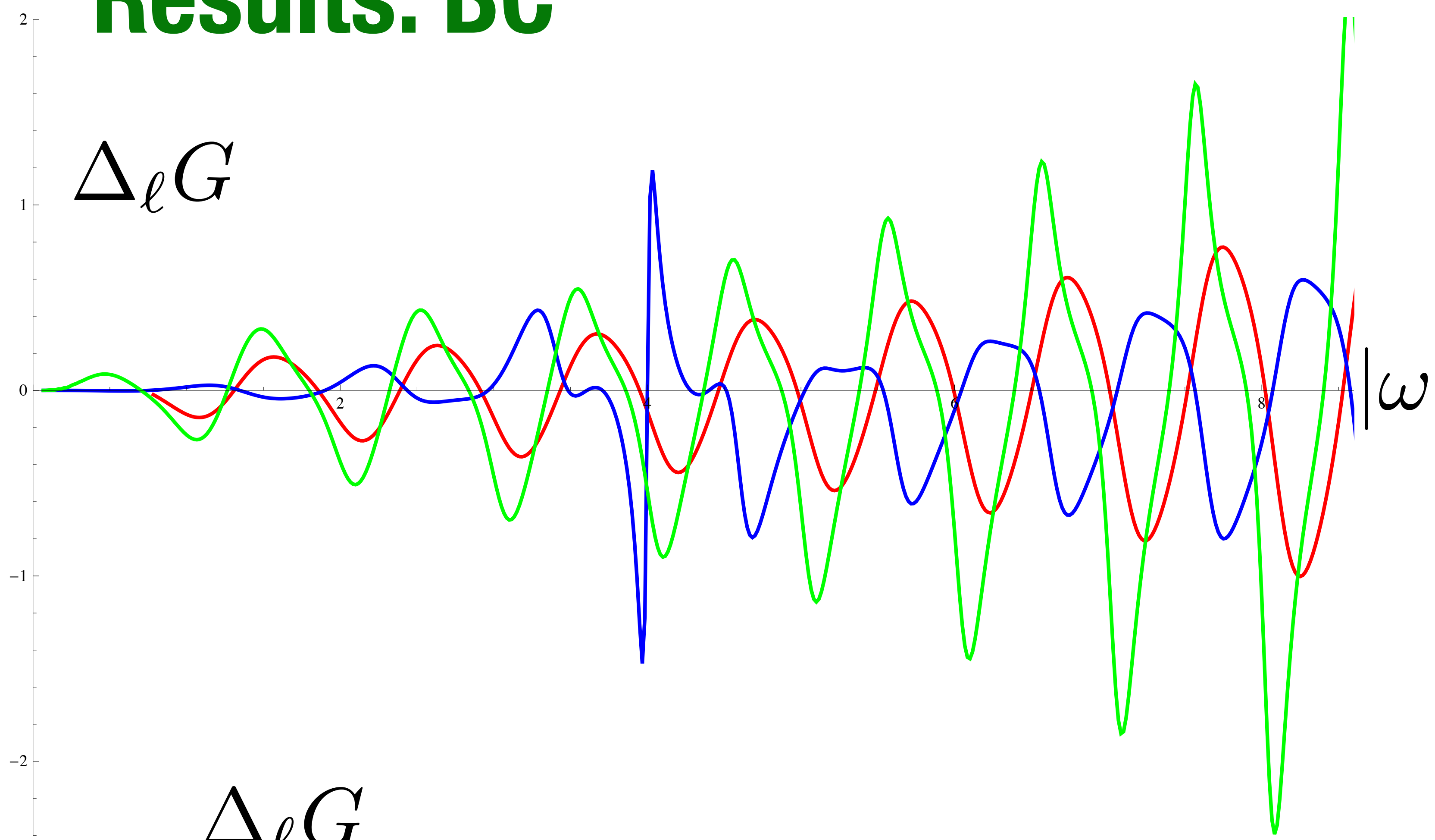
Results: BC

- **First ever analytic calculation**

$s = 2,\ \ell = 2$

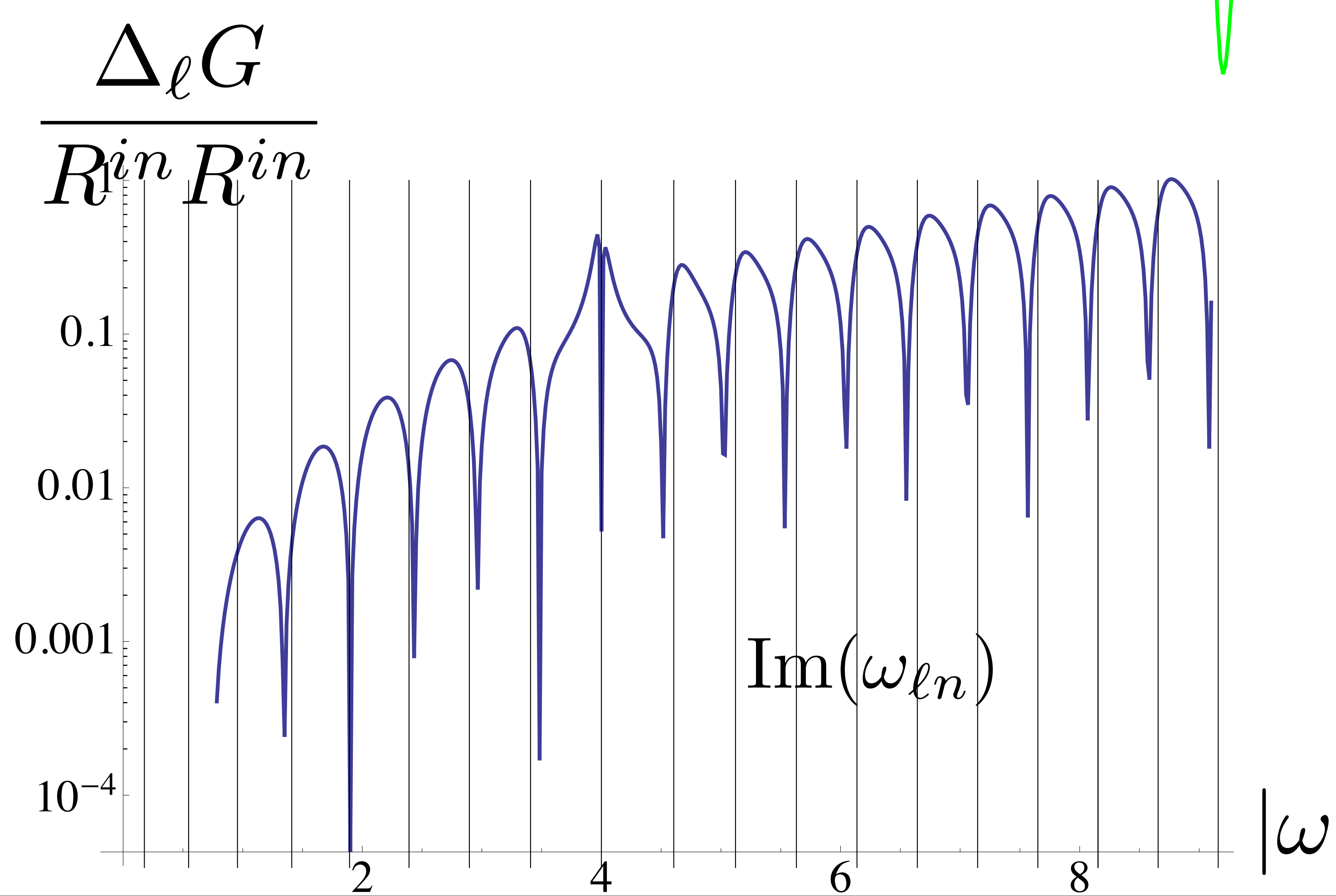
$s = 0,\ \ell = 1$

$s = 1,\ \ell = 1$



- **Connexion between QNMs and BC**

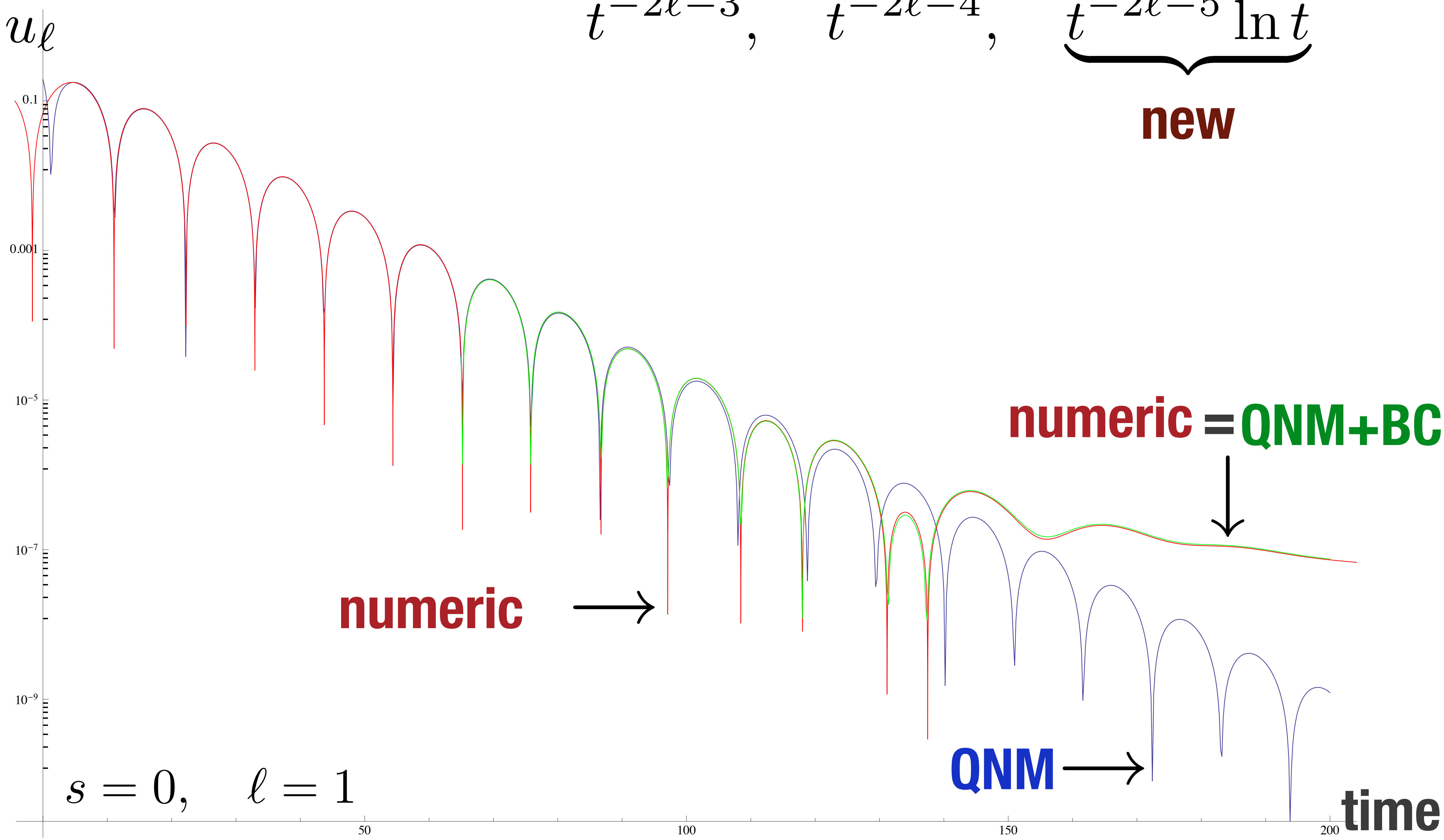
$s = \ell = 2$



Results: BC

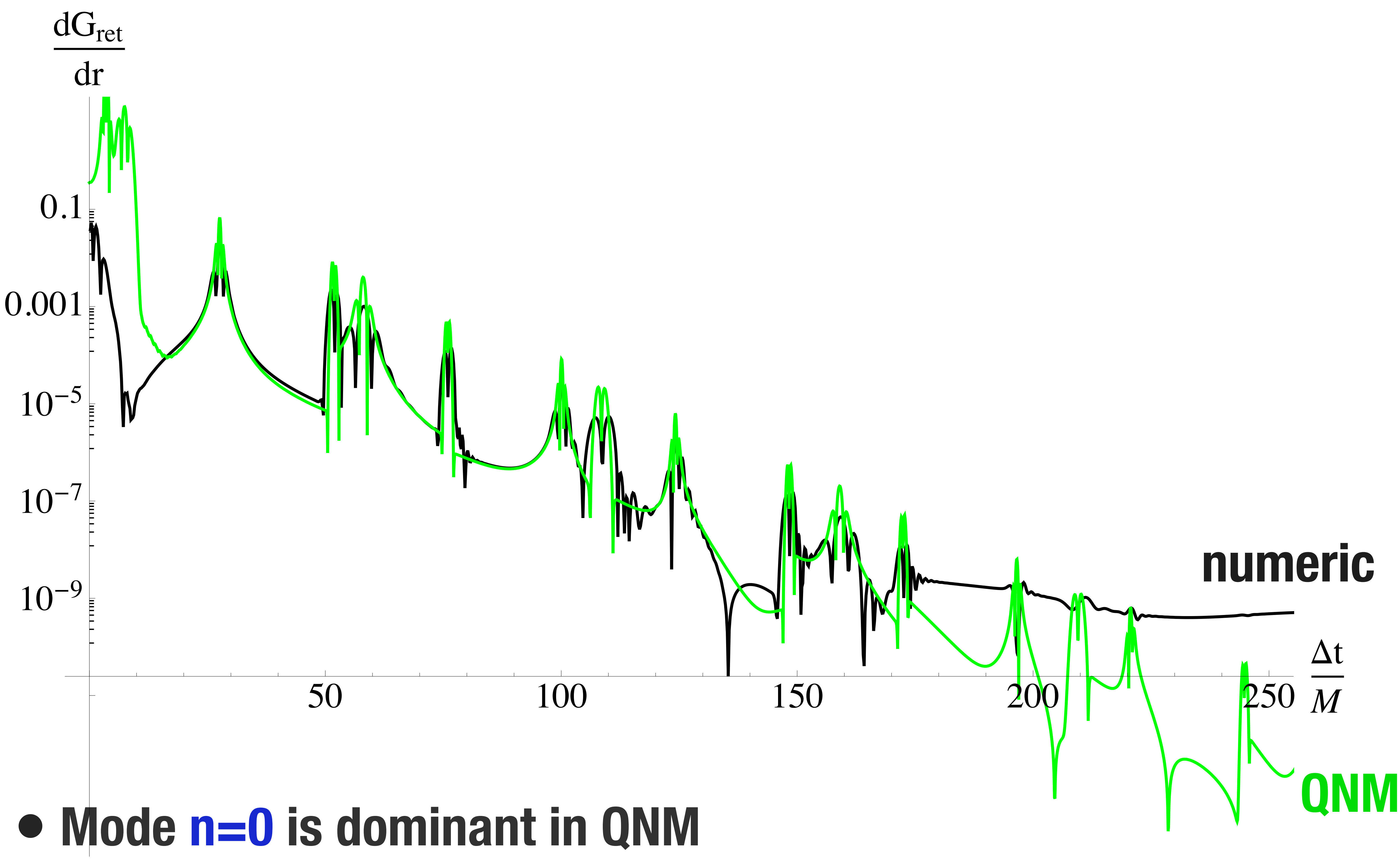
- **BC gives late-time decay of initial data:**

$$t^{-2\ell-3}, \quad t^{-2\ell-4}, \quad \underbrace{t^{-2\ell-5}\ln t}_{\text{new}}$$



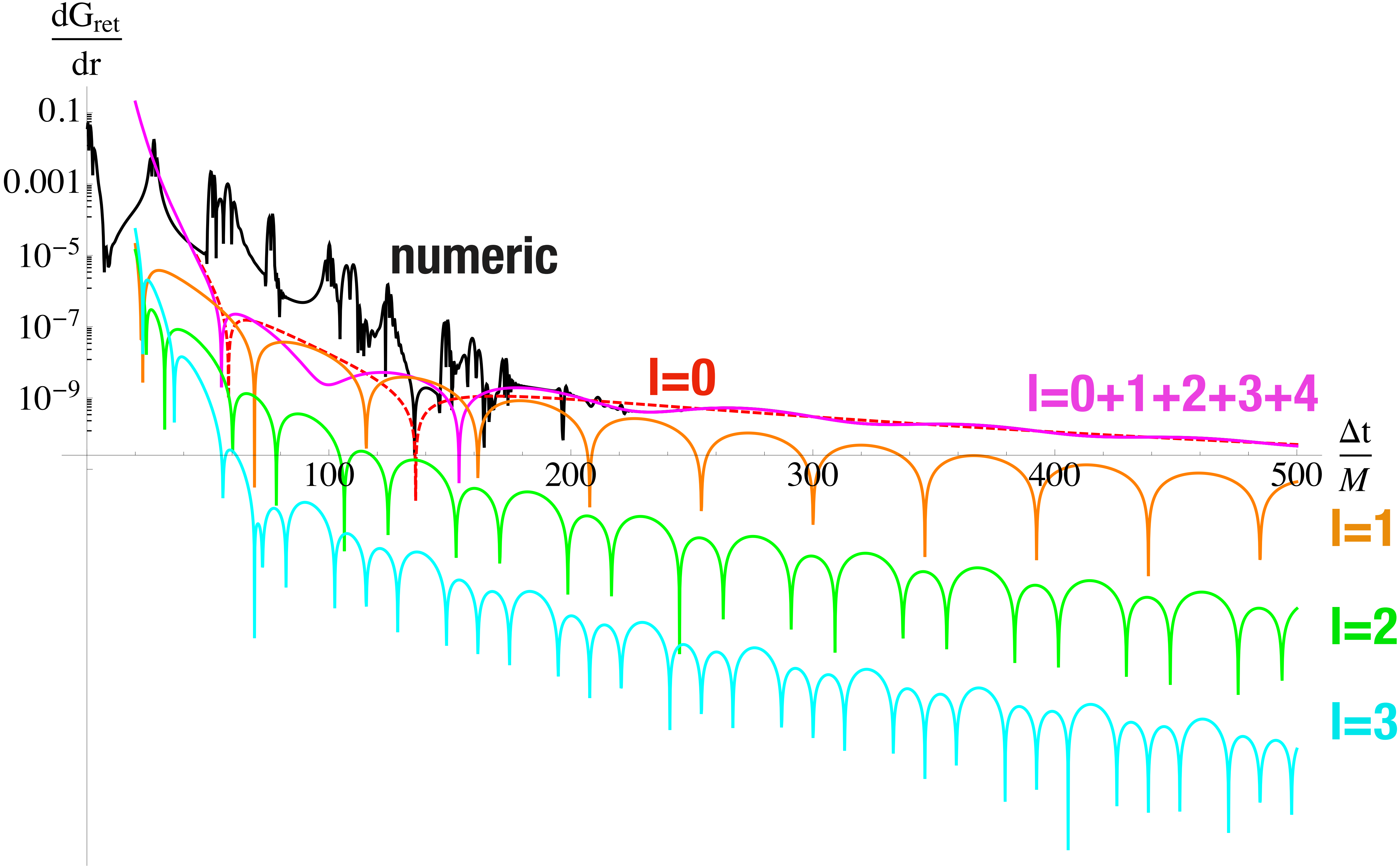
Results: Green Function, QNMs

- **Scalar** charge on **circular** geodesic at r=6M



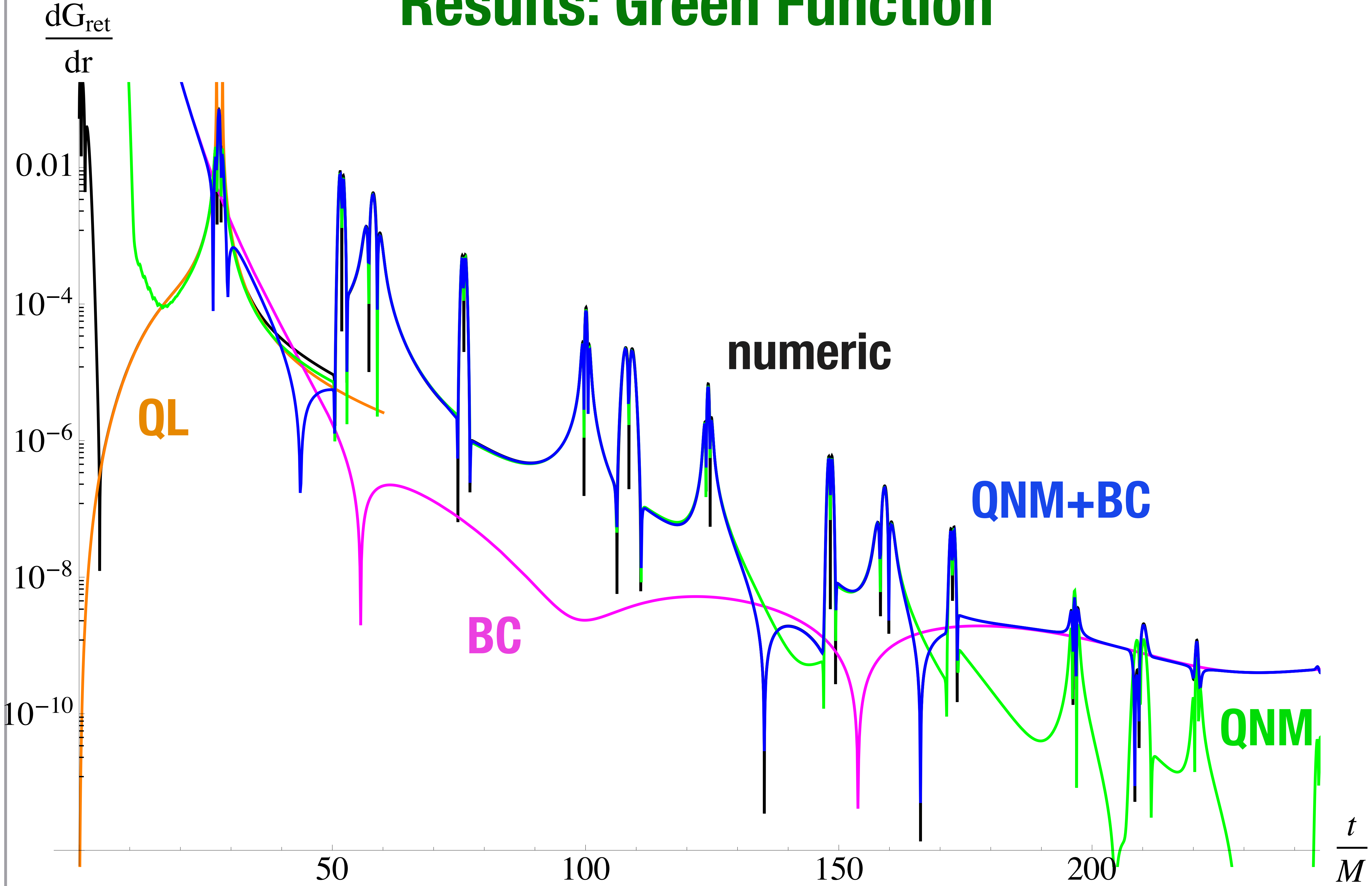
- Mode **n=0** is dominant in QNM

Results: Green Function, BC



- Mode l=0 is dominant in BC

Results: Green Function



- **Numeric = QL + QNM + BC**

Results: Self-Force

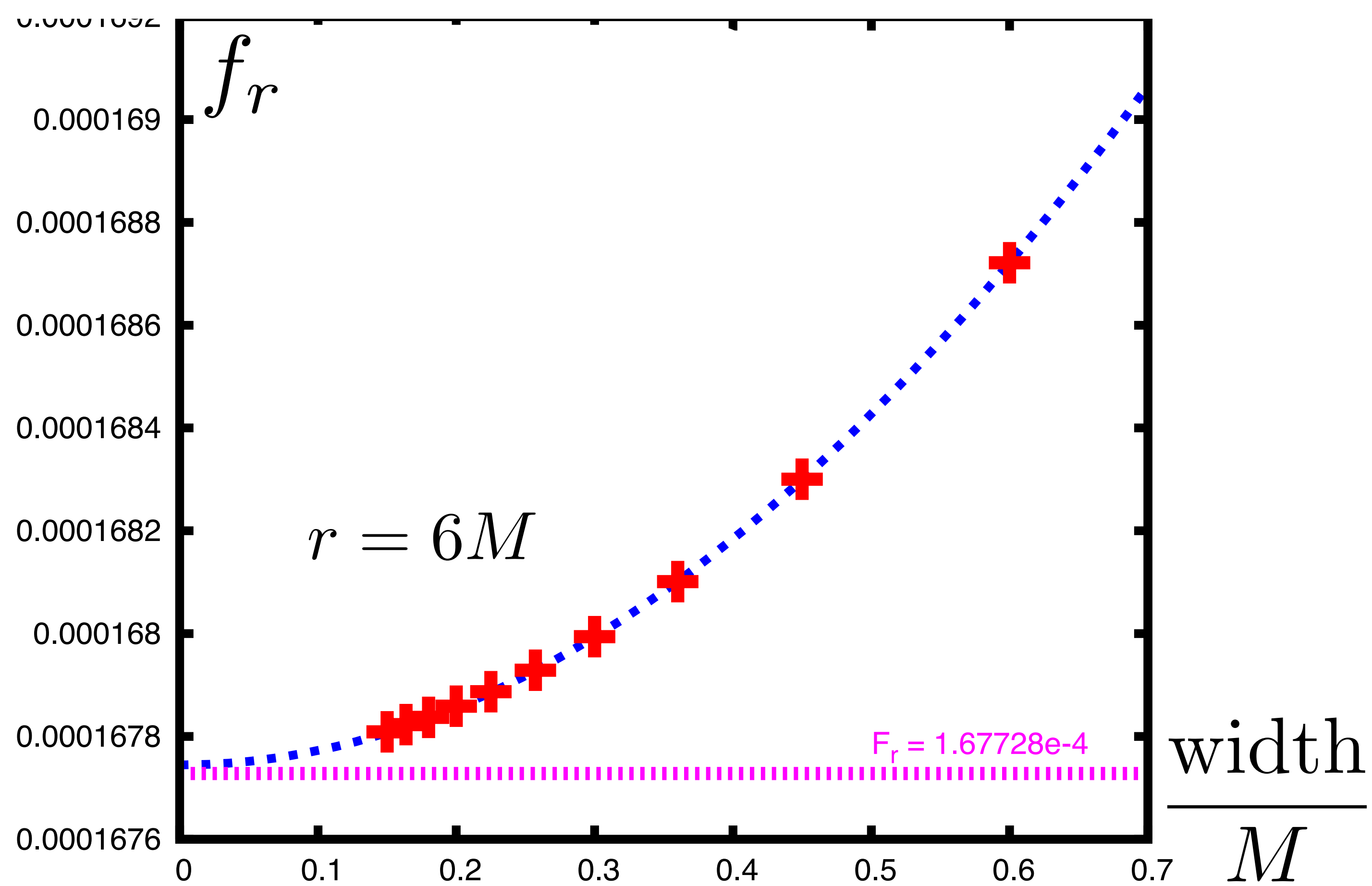
- **Radial S-F** for scalar charge $(q=1)$ on circular geodesic

$$f_r = \int_{-\infty}^{\tau^-} \partial_r G_{ret}(z(\tau), z(\tau'))d\tau'$$

- **Exact** value (method of mode-sum regularization): 0.000167728

- **Numerical** method (peaked Gaussian) + QL:.

  Rel.err. $\approx 0.01\%$



- Method of **matched expansions** (QL+QNM+BC): Rel.err. $\approx 0.001\%$

Summary

- **Spectroscopy:**
  - **QNMs**: four-fold **singularity structure** of Green function
  - First method for the **BC**: new logarithmic **tail**
- New method for calculation of S-F using **peaked Gaussian**: quick and accurate
- Method of **matched expansions**: trivial regularization, physical insight and accurate S-F results. Only requires n=0 for QNM and l=0 for BC

Next

Next

- Gravitational case

Next

- **Gravitational** case
- **Kerr**

Next

- Gravitational case
- Kerr

- S-F stops formation of naked singularities....

Next

- Gravitational case
- Kerr
- S-F stops formation of naked singularities....

  so, can it stop the end of the world on 21/12/2012 (Mayan arxiv)?

Tortuguero monument, Mexico (7th century AD)