

GRAVITY FROM SPACETIME THERMODYNAMICS

Goffredo Chirco



UNIVERSITEIT VAN AMSTERDAM

@ CENTRA - IST

Lisboa - June 25th



based on collaborations with: Liberati - SISSA, Eling, Sindoni, Oriti - AEI,
Vitagliano - IST/CENTRA

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OUTLINE

- ▶ **introduction and motivation**
 - ▶ BH thermodynamics => local spacetime thermodynamics
 - ▶ GR from thermodynamics of local horizons
 - ▶ non-equilibrium & dissipation
- ▶ **from GR to higher order gravity theories**
 - ▶ $f(R)$ and generalized Brans-Dicke gravity
 - ▶ Higher curvature and Entanglement Entropy and Noether charge
- ▶ **summary: thermodynamical equilibrium as a general principle of gravitational dynamics?**

A - the Einstein equation of state

► black hole thermodynamics

BH solutions can be described as dynamical systems in terms of a small number of parameters: M, J, Q_e (no hair theorem)

Israel 67, Christodoulou 71, Hawking 71, Bardeen, Carter 73

► mathematical analogy \to physical identity

from classical level

+

quantum level

0th κ constant along the horizon

1st $dM = \frac{\kappa}{8\pi G} dA + \Omega dJ + \Phi dQ$

2nd $dA \geq 0$

3rd $\kappa = 0$ unattainability

\Rightarrow

QM+SM

Bekenstein 73

$$S = \frac{A}{4l_p^2}$$

QFT

Hawking 75

$$T = \frac{\hbar\kappa}{2\pi} \Rightarrow S = \frac{A}{4G}$$

interplay



► how does GR know about T-k and S-A ?

Jacobson 95

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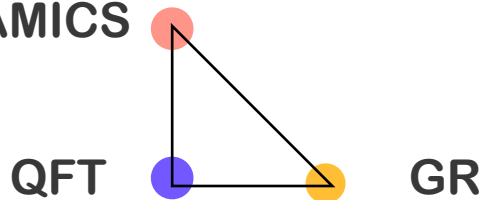
QFT

Hawking 75

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THERMODYNAMICS



semiclassical picture

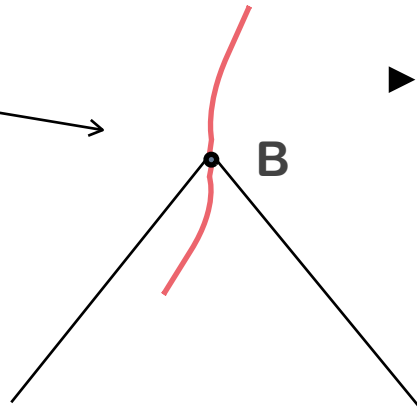
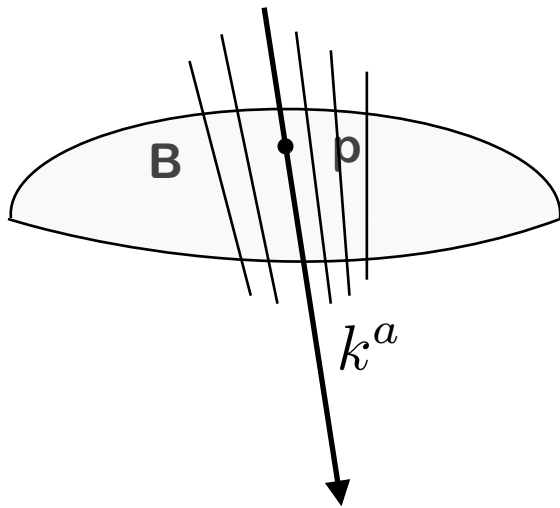
thermodynamics as a premise ?

from BH to local horizon thermodynamics

LOCAL CAUSAL HORIZON

- ▶ consider a local spacetime spacelike 2 surface patch B associated to an event p

Jacobson 95, Padmanabhan 02



- ▶ **CAUSAL HORIZON:**
boundary of the past of B = past directed null geodesic congruence normal to B

- ▶ at p the 2 surface can be characterized by the kinematical d.o.f. of the bundle of null geodesics: expansion, shear...

not enough...

- ▶ **QFT & THERMO** extended to local horizon physics

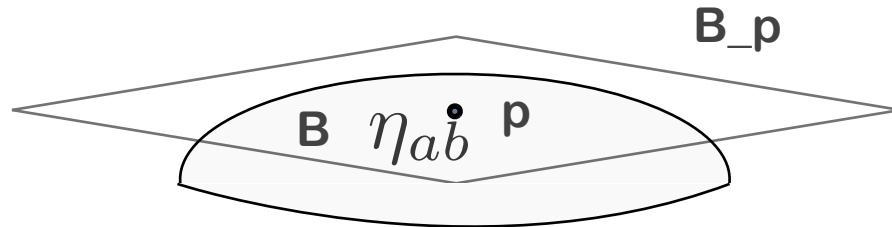
basic ingredients:

- stationarity : \exists time Killing vectors
- non inertial frames : Unruh effect
- horizon area-entropy relation

local horizon thermodynamics

STATIONARITY - LOCAL KILLING HORIZON

- ▶ introduce a **local inertial frame at p** (local Lorentz symmetry)



- ▶ consider an approximate Killing field χ^a generating boosts orthogonal to B

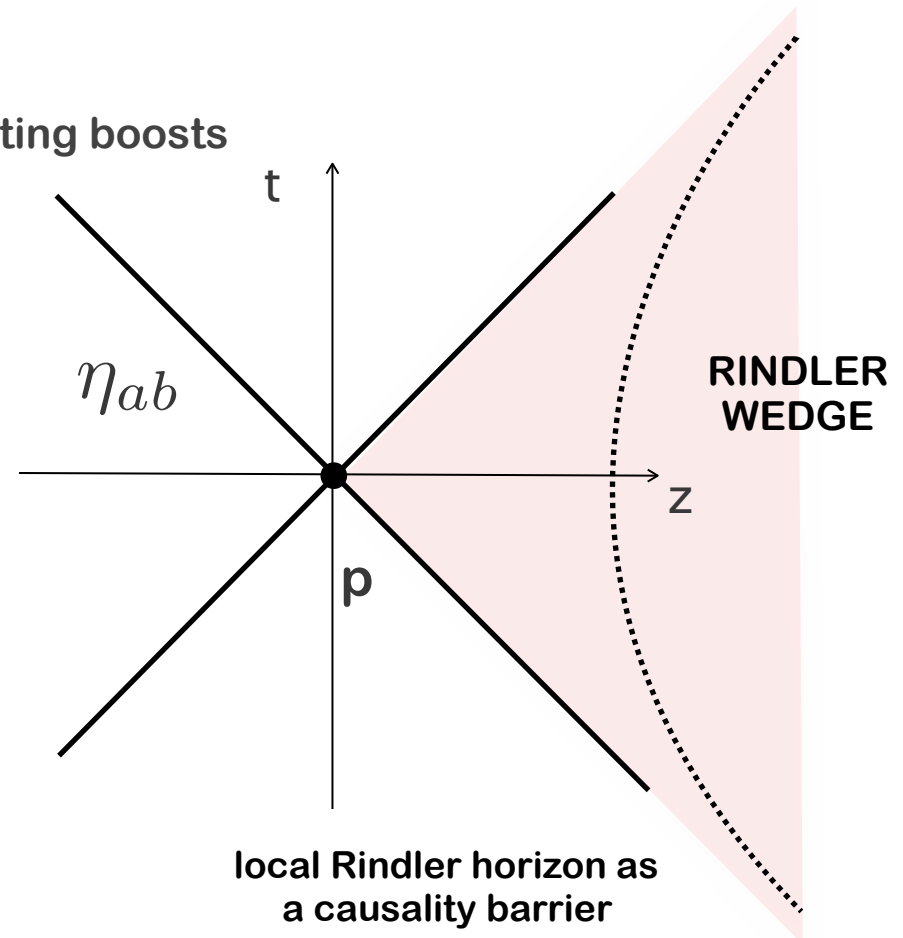
- ▶ Lorentz boost introduces a **local Rindler frame**

=> LOCAL RINDLER HORIZON

- ▶ arrange χ^a so that: $\chi^a = -\lambda k^a$

- + boost isometry => Killing horizon **stationarity**

uniformly accelerated frame....



local horizon thermodynamics

UNRUH EFFECT

- ▶ assume locally Minkowski vacuum
 - ▶ given local Lorentz symmetry + stability of the vacuum
- ⇒ when restricted to the Rindler wedge, the usual global Minkowski vacuum state $|0\rangle$ in quantum field theory turns out to be equivalent to a **Gibbs thermal state** with an Unruh-Tolman temperature

Unruh 76

- ▶ H operator generating Lorentz boost on the quantum fields noninertial frame translations in hyperbolic angle

$$\beta^{-1} \rightarrow T = \frac{1}{2\pi}$$

RINDLER WEDGE thermal system

$$\rho = \exp(-\beta H)/Z \quad \left| \begin{array}{l} \langle E \rangle = \text{Tr}(\rho H) \\ S = -\text{Tr}(\rho \ln \rho) \end{array} \right.$$

for $\delta\rho \ll \rho$

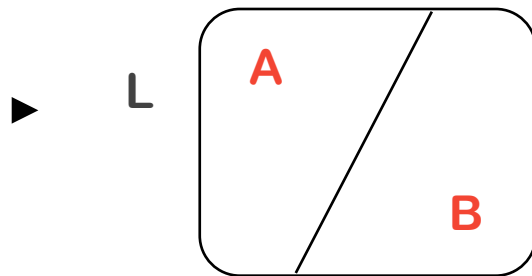
$$\Rightarrow \delta S = \beta \delta \langle E \rangle \quad \checkmark$$

EQUILIBRIUM

local horizon thermodynamics

HORIZON AREA - ENTROPY RELATION

some holographical assumption



$$\rho_L = |\Psi\rangle\langle\Psi| \quad \text{pure}$$

$$\rho_A = \text{tr}_B \rho \quad \text{mixed}$$

missing information - von Neumann entropy

entanglement entropy

$$S(\rho_A) = -\text{tr} \rho_A \ln \rho_A$$

+ Rindler horizon \Rightarrow entanglement entropy = thermal entropy

▶ the entanglement entropy scales with the area but is infinite. need UV regulation

$$S = \alpha A \quad \Rightarrow \quad S = f(\text{geometry})$$

we get horizon thermodynamics without any help from GR....

MEM

the entropy density depends on the nature of the quantum fields and their interactions and can be some complicate function of the position in spacetime

GR from local horizon thermodynamics

Einstein equation of state: ANALOGY

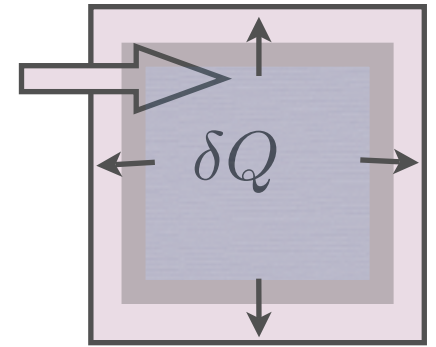
$$S(E, V) \Rightarrow dS = (\partial S / \partial E) dE + (\partial S / \partial V) dV$$
$$\delta Q = dE + p dV$$

equilibrium
entropy balance

$$\delta Q = T dS \Rightarrow$$

$$T^{-1} = (\partial S / \partial E)$$
$$p = T(\partial S / \partial V)$$

equation of state



IDEA

geometric entropy
functional

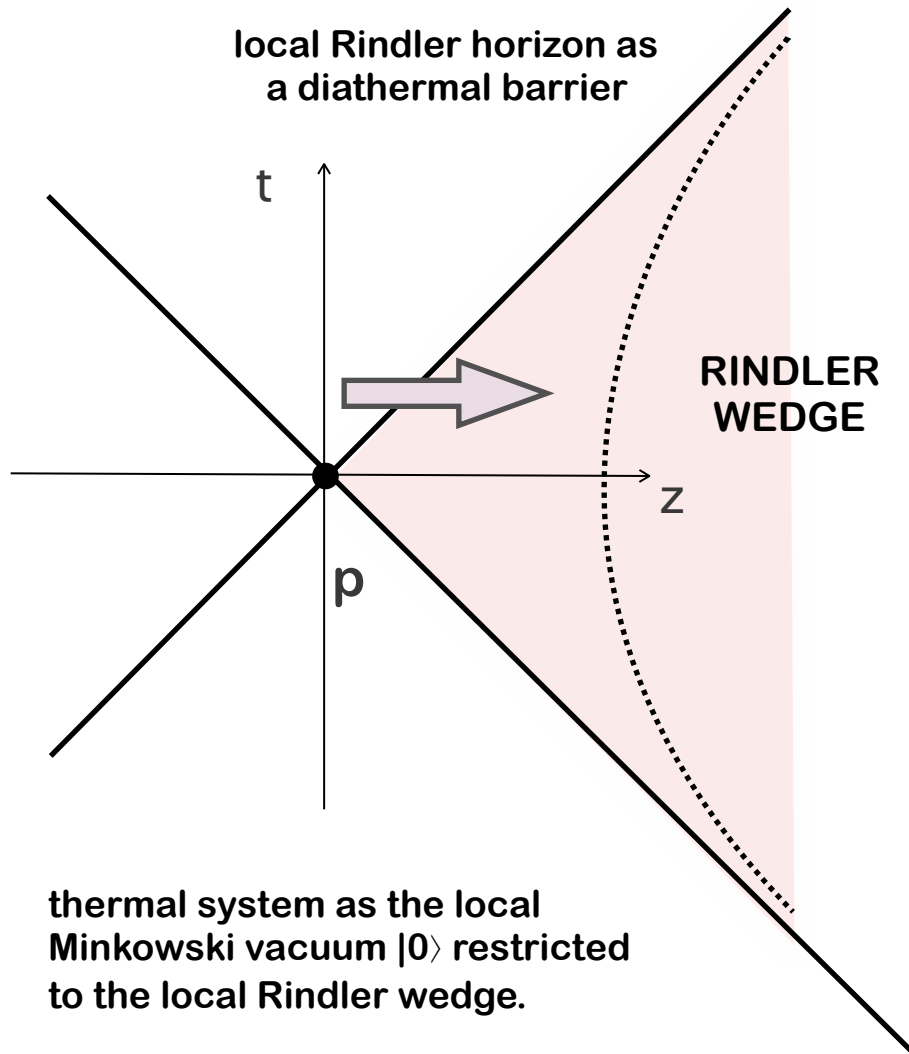
+

local matter-energy
perturbations

\Rightarrow

Einstein eq. as
equation of state

GR from local horizon thermodynamics



- ▶ for the horizon system the heat flow is the energy current of the matter as measured with respect to the boost Hamiltonian for which the state is thermal

$$\delta Q = \delta \langle E \rangle = \int T_{ab} \chi^a d\Sigma^b$$

energy crossing the horizon =
vacuum perturbation

for $\delta\rho \ll \rho \Rightarrow \delta S = \beta \delta \langle E \rangle \Rightarrow T \delta Q = \delta(\alpha A)$

- ▶ then the assumption of entropy balance (Clausius law) provides a **local matter/geometry constitutive relation**

GR from local horizon thermodynamics

► assume α constant $\Rightarrow \delta S = \alpha \delta A \quad \delta A = \int_H \tilde{\epsilon} \theta d\lambda$

► **equilibrium** = Rindler horizon bifurcation surface

$$\delta S = \alpha \delta A = 0 \quad \Rightarrow \quad \theta_p = 0$$

► **perturbation** via heat flux

$$\theta \approx \theta_p + \lambda \left. \frac{d\theta}{d\lambda} \right|_p + \mathcal{O}(\lambda^2) \quad \text{by Raychaudhuri}$$

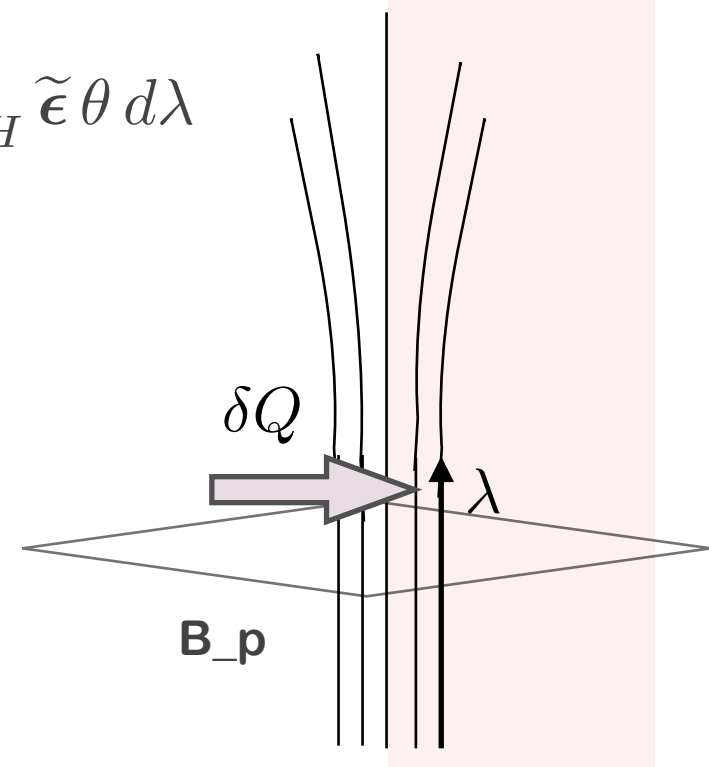
$$\Rightarrow dS = \alpha \int_H \tilde{\epsilon} d\lambda [\theta - \lambda(1/2 \theta^2 + \|\sigma\|^2 + R_{ab} l^a l^b)]_p$$

► **equilibrium recovery** via entropy balance law $\delta Q = T dS$

$$T dS = \alpha \frac{\kappa \hbar}{2\pi} \int_H \tilde{\epsilon} d\lambda \left[\underbrace{\theta - \lambda(1/2 \theta^2 + \|\sigma\|^2)}_{=0} + \underbrace{R_{ab} l^a l^b} \right]_p =$$

$$= \int_H \tilde{\epsilon} d\lambda (-\lambda \kappa) \underbrace{T_{ab} l^a l^b} = \delta Q$$

δA has local and non-local contributions



GR from local horizon thermodynamics

LOCAL LEVEL:

for all null k^a $\frac{2\pi}{\hbar\alpha} T_{ab} = R_{ab} + \Phi g_{ab}$ local constitutive relation

to define $\Phi = -\frac{1}{2}R - \Lambda$ • local energy conservation $\nabla^b T_{ab} = 0$

• Bianchi identity $\nabla^b R_{ab} = \frac{1}{2}\nabla_a R$

$$\Rightarrow 8\pi G T_{ab} = R_{ab} - \frac{1}{2}R g_{ab} - \Lambda g_{ab}$$

Jacobson 95

$$\text{if only } \alpha = \frac{1}{4G}$$

UV cutoff \Leftrightarrow G
INDUCED GRAVITY

the equation is then extended to the whole spacetime via EP

NON-LOCAL
LEVEL:

$$dS_i = -\alpha \int_H \tilde{\epsilon} d\lambda \lambda \|\sigma^2\|$$

How to get rid of the
non local terms ?

B - the non-equilibrium regime

non-equilibrium horizon thermodynamics

- ▶ **impossible!** it is associated to an arbitrary kinematical d.o.f. of the null congruence, associated to the arbitrary choice of B
- ▶ the non-local entropy term can be written as an **internal entropy production** term

=>

$$dS = \underline{dS_{eq}} + \underline{dS_i}$$

LOCAL = EQUILIBRIUM

NON-LOCAL = NON-EQUILIBRIUM

GENERALIZED CLAUSIUS EQUATION

Eling 2006, Chirco 2009

- ▶ to recover the eom for GR one needs a generalization of the entropy balance relation where the non-local entropy term is interpreted as **unmatched heat**

$$dS = \frac{\delta Q}{T} + \delta N$$

irreversible processes

=> microscopic level ?

non-equilibrium horizon thermodynamics

- ▶ the unmatched heat for GR coincides with the expression for the tidal heating dissipation in BH (Hawking-Hartle)

Chirco 2009

internal entropy production term must be associated to some horizon viscosity

membrane paradigm : horizon congruence dynamics is well described by 2+1 viscous fluid equations

Damour 79, Thorne 86

$$dS_i = \frac{2\eta}{T} \hat{\sigma}_{\mu\nu} \hat{\sigma}^{\mu\nu} + \frac{\xi_B}{T} \hat{\theta}^2$$

$$T dS_i = 2\eta \int_H \tilde{\epsilon} dv \|\sigma^2\| = \frac{1}{8\pi G} \int_H \tilde{\epsilon} dv \|\sigma^2\|$$

- ▶ spacetime viscosity will be related to the UV cut-off scale of the theory, through the entropy density

=> introduce a horizon viscosity coefficient

$$\eta = \frac{T\alpha}{2} = \frac{\hbar\alpha}{4\pi} = \frac{1}{16\pi G}$$

- > as for BH, the shear viscosity to entropy density ratio for the Rindler horizon system saturate the **Kotvun Son Starinets bound**

$$\eta/\alpha = \frac{1}{4\pi}$$

focussing on the irreversible sector

- ▶ in AdS/CFT the KSS bound is interpreted as a universal lower bound for all strongly coupled field theories with gravity dual

Policastro 2001, Starinets, Kotvun 2003

INTERESTING...

KSS ratio seem to be rooted in gravitational physics, but the Rindler wedge is a subregion of Minkowski space time: **NO GRAVITY AT ALL**

- ▶ interpretation of dissipative effects as a consequence of an underlying fluctuating behaviour of spacetime at the UV cut-off scale (Candelas-Sciama and AdS/CFT)

> **ENTANGLEMENT VISCOSITY**

Chirco 2010

if one could calculate η directly from the fluctuations of the matter fields in the thermal vacuum, that would **characterize the KSS bound as a fundamental property of quantum entanglement and its associated holography**

entanglement viscosity

PROBLEM how to relate a phenomenological transport coefficient η from a **fluid-wise** description (membrane) of the **horizon** to the quantum **vacuum state on the bulk ??**

MAIN IDEA
(AdS/CFT)

- ▶ on large spatial and time scale the thermal vacuum can be **effectively** described by **hydrodynamics**
- ▶ calculate the hydrodynamics transport coefficient from microscopic theory using KUBO FORMULA **involving the Green's function of the energy momentum tensor for the matter fields in the wedge**

Kotvun 03, Son 07, Starinets 09

▶ no holographic duality like AdS/CFT in Rindler wedge ... **but PRE-HOLOGRAPHY**

area scaling behaviour of entanglement entropy : quantum degrees of freedom of the wedge seem to be packed on the stretched horizon surface

>

TRY a lower dimensional description of the vacuum fields associated to the horizon

lower dimensional description of bulk fields

WHAT WE WANT:

dual lower dimensional description of the vacuum state in terms of a strongly coupled thermal CFT effectively living on a (D-1) Minkowski (horizon membrane)

RECIPT

- ▶ start with the canonical energy momentum tensor for the Rindler wedge

$$T_{(R)\nu}^{\mu} = \frac{\partial L_R}{\partial(\partial_{\mu})\psi} \partial_{\nu}\psi - \delta_{\nu}^{\mu} L_R \quad \text{where } L_R = \sqrt{-g} L_M$$

ANSATZ: on large scales, the holographic vacuum state is described by a conserved lower dimensional SET

$$\langle \hat{T}_{\mu\nu}^{(D-1)} \rangle = Z^{-1} \text{Tr}(\rho \hat{T}_{\mu\nu}^{(D-1)}) = \langle 0 | \hat{T}_{\mu\nu}^{(D-1)} | 0 \rangle$$

Minkowski vacuum expectation value

thermal average at Tolman-Unruh temperature $T_{(R)\nu}^{\mu} = \kappa \xi T_{(M)\nu}^{\mu}$

- ▶ **DIMENSIONAL REDUCTION**

$$\langle \hat{T}_{\mu\nu}^{(D-1)} \rangle = \int_{l_c}^{\infty} d\xi \langle \hat{T}_{\mu\nu}^{(R)} \rangle = \int_{l_c}^{\infty} d\xi \kappa \xi \langle \hat{T}_{\mu\nu}^{(M)} \rangle$$

Kubo like formula for the horizon viscosity

II

apply the formalism of viscous hydrodynamics and calculate the shear viscosity through a **Green-Kubo approach** in terms of the effective lower dimensional SET

- ▶ consider a metric perturbation $h_{\mu\nu}$ associated to the bulk vacuum perturbation $\delta\langle E\rangle$ as source for the (D-1) field theory operator $\hat{T}_{\mu\nu}^{D-1}$
- ▶ assuming the perturbation is small, from **linear response theory**, one can calculate the change of the expectation value of $\hat{T}_{\mu\nu}^{D-1}$

$$\langle \delta\hat{T}_{\mu\nu}^{D-1}(k^0, \vec{k}) \rangle = G_R(k^0, \vec{k}) h_{\mu\nu}(k^0, \vec{k})$$

where G_R is the retarded 2-point thermal Green's function of $\hat{T}_{\mu\nu}^{D-1}$

$$G_R(k^0, \vec{k}) = \int d\tau d^{D-2}x e^{ik^0\tau} e^{-i\vec{k}\vec{x}} \langle [\hat{T}_{\mu\nu}^{D-1}(\tau, \vec{x}) \hat{T}_{\mu\nu}^{D-1}(0, \vec{0})] \rangle$$

- ▶ in the limit $(k^0, \vec{k}) \rightarrow 0$ $\langle \hat{T}_{xy}^{D-1}, \hat{T}_{xy}^{D-1} \rangle (k^0, \vec{k} \rightarrow 0) = i\eta k^0 - P + \mathcal{O}(\omega^2)$

from which one gets the **quantitative** expression for the shear viscosity

$$\eta = \lim_{k^0 \rightarrow 0} \frac{1}{k^0} G_R^{xy,xy}(k^0, 0)$$

KSS bound for the Rindler horizon

- ▶ in our particular case we have

$$\eta = \lim_{k^0 \rightarrow 0} \frac{1}{k^0} \int_{l_c}^{\infty} \xi' \int_{l_c}^{\infty} \xi \int d\tau d^{D-2}x e^{ik^0\tau} \theta(\tau) \kappa^2 \xi \xi' < [T_{xy}^D(\tau, x, y, \xi), T_{xy}^D(0, \xi')] >$$

where $< [T_{xy}^D(\tau, x, y, \xi), T_{xy}^D(0, \xi')] >$

is the **Minkowski 2-point correlator of the bulk field theory**

Chirco 2010

THEN

for a free minimally coupled scalar field in 4D Rindler spacetime is

$$\eta = \frac{1}{1440\pi^2 l_c^2}$$

**AREA SCALING
ENTANGLEMENT
VISCOSITY**

- ▶ from the thermal description of the dimensionally reduced vacuum fields

$$\epsilon = \frac{\pi^2 T^4}{30} = \frac{1}{480\pi^2 \xi^4} \quad \text{--->} \quad \epsilon_r = \frac{\kappa}{960\pi^2 l_c^2}$$

$$s = \frac{2\pi^2}{45} T^3 = \frac{1}{180\pi \xi^3} \quad \text{--->} \quad s = \frac{1}{360\pi l_c^2}$$

all area scaling quantities

4D - Planckian form :
massless free scalar field = $\epsilon = 3P$
ultrarelativistic boson gas

$$> \quad \eta/s = \frac{1}{4\pi}$$

KSS bound satisfied by just entanglement quantities !!!

KSS bound for the Rindler horizon

NON equilibrium thermodynamical description:

- ▶ propagation of purely gravitational dof associate with macro dissipative effects
- ▶ a microscopic description for the macro shear viscosity in terms of the fluctuations of the Rindler wedge thermal state in a finite temperature QFT

KSS ratio from entanglement:

local Rindler horizon system

NO GRAVITY

NO HOLOGRAPHIC DUALITY LIKE ADS/CFT

GOAL

KSS ratio may be a fundamental holographic property of spacetime and quantum entanglement

support for the hypothesis that semi-classical gravity on macroscopic scales is **induced as an effective theory** of some lower dimensional strongly coupled quantum system with a large number of degrees of freedom

Does it work for generalized gravity theories ?

interestingly successful enough to try to go from GR to higher order gravity theories...

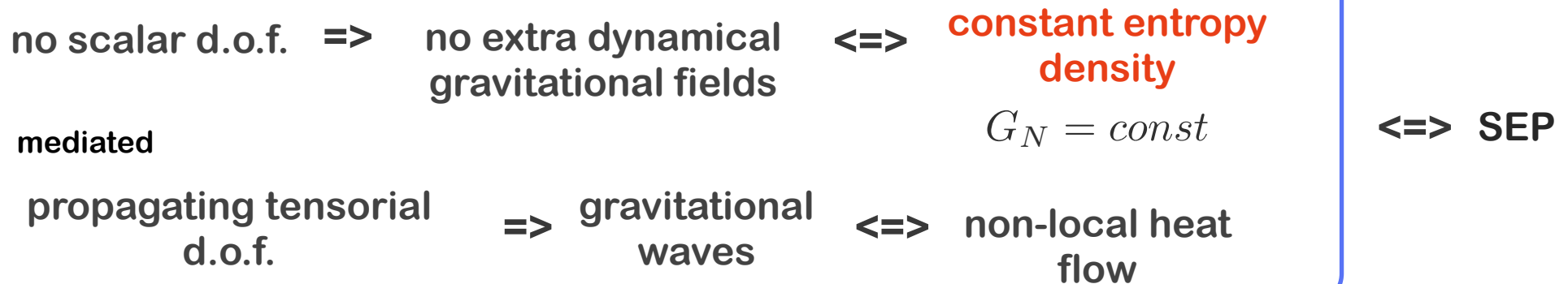
Is there a general principle for gravitational dynamics ?

Discriminant factor for gravity theory: a metatheory of gravity ?

extension to generalized gravity theories

GR is effectively purely geometrical. very special.

RATIO the entropy functional may be constrained by the different formulations of the **equivalence principle**



=>

changing the assumed entropy functional would change the associated gravitational field equation

How does the thermodynamical derivation work in the case ?

extension to generalized Brans-Dicke gravity

ENTROPY

- ▶ promote the entropy density to be an independent field

$$\alpha \rightarrow \phi(x) \quad \Rightarrow \quad S = \int d^4x \sqrt{h} \phi(x)$$

- ▶ again, consider the variation of S along the horizon geodesic bundle

$$\delta S = \int \sqrt{h} \left(\phi \theta + \frac{d\phi}{d\lambda} \right) d\lambda d^2x$$

- ▶ Taylor expand around p ($\lambda = 0$)

$$\delta S = \int \sqrt{h} \left[\left(\phi \theta + \frac{d\phi}{d\lambda} \right) + \lambda \left(\theta \frac{d\phi}{d\lambda} + \phi \frac{d\theta}{d\lambda} + \frac{d^2\phi}{d\lambda^2} + \phi \theta^2 + \theta \frac{d\phi}{d\lambda} \right) \right]_p$$

Raychaudhuri

- ▶ **equilibrium** $\mathcal{O}(\lambda^0)$ $\Rightarrow \theta_p = -\phi^{-1} \frac{d\phi}{d\lambda}$

the kinematical d.o.f. θ_p is linked to the derivative of the spacetime scalar field

- ▶ at $\mathcal{O}(\lambda)$

$$\delta S = \int \sqrt{h} d\lambda d^2x \lambda \left(-\phi R_{\mu\nu} k^\mu k^\nu + k^\mu k^\nu \nabla_\mu \nabla_\nu \phi - \frac{3}{2} \phi \theta^2 - \phi \sigma_{\mu\nu} \sigma^{\mu\nu} \right)$$

extension to generalized Brans-Dicke gravity

EXTRA ENTROPY

- ▶ **new non-equilibrium contributions?**

$$\delta S = \int \sqrt{h} d\lambda d^2 x \lambda (-\phi R_{\mu\nu} k^\mu k^\nu + k^\mu k^\nu \nabla_\mu \nabla_\nu \phi - \frac{3}{2} \phi \theta^2 - \phi \sigma_{\mu\nu} \sigma^{\mu\nu})$$

$$dS_i = \frac{2\eta}{T} \hat{\sigma}_{\mu\nu} \hat{\sigma}^{\mu\nu} + \frac{\xi_B}{T} \hat{\theta}^2$$

the additional scalar d.o.f. may appear as a new gravitational channel for dissipating energy

- ▶ **not correct:** given the equilibrium condition

$$\frac{3}{2} \phi \theta^2 = \frac{3}{2\phi} k^\mu k^\nu \nabla_\mu \phi \nabla_\nu \phi \quad \Rightarrow$$

unlike the shear term, the bulk term is LOCAL

- ▶ after the k^μ are peeled of, local terms at p are **frame independent**

PUZZLE !!

Jacobson, Padmanabhan

these terms exist for any observer in the local spacetime patch

\Rightarrow

they will end up describing the dynamics of the global spacetime

extension to generalized Brans-Dicke gravity

HEAT

- ▶ principle of **background independence** $\Rightarrow \phi(x)$ must be varied like other fields
- ▶ must contribute to the total Lagrangian

$$L_{matt}(g_{\mu\nu}, \psi) + L_{scalar}(g_{\mu\nu}, \phi) \Rightarrow \delta Q \sim k^\mu k^\nu (T_{\mu\nu}^M + T_{\mu\nu}^\phi)$$

most general contribution to the heat flux:

assume the action constructed out of first derivatives of the scalar field

$k^\mu k^\nu$ projection \Rightarrow no relevant contribution from interaction terms

$$\Rightarrow \delta Q_{scalar} \sim \frac{\Omega(\phi)}{\phi} k^\mu k^\nu \nabla_\mu \phi \nabla_\nu \phi$$

- ▶ restricting to $\Omega(\phi) \equiv \Omega$ constant, for simplicity:

$$\Rightarrow \frac{\delta Q}{T} = - \int d^4x \sqrt{h} \lambda \left(2\pi T_{\mu\nu}^M k^\mu k^\nu + \left(\frac{\Omega}{\phi} \right) k^\mu k^\nu \nabla_\mu \phi \nabla_\nu \phi \right)$$

extension to generalized Brans-Dicke gravity

ENTROPY BALANCE

- ▶ at local level:

$$\phi R_{\mu\nu} - \nabla_{\mu}\nabla_{\nu}\phi + \left(\frac{3/2 - \Omega}{\phi}\right) \nabla_{\mu}\phi\nabla_{\nu}\phi + \Phi g_{\mu\nu} = 2\pi T^M{}_{\mu\nu}$$

\Rightarrow by defining the Dicke constant $\omega = \Omega - 3/2$ one obtains the constitutive relations capturing any Brans-Dicke theory

- ▶ Φ come from local matter-energy conservation + Bianchi identity +

$$\begin{aligned} \nabla_{\nu}\Phi &= \nabla_{\nu} \left(\square\phi - \frac{1}{2}\phi R + \frac{\omega}{2\phi} \nabla_{\mu}\phi\nabla^{\mu}\phi \right) + \\ &+ \left(\frac{1}{2}R + \frac{\omega}{2\phi^2} \nabla_{\mu}\phi\nabla^{\mu}\phi + \frac{\omega}{\phi} \square\phi \right) \nabla_{\nu}\phi \end{aligned}$$

... plus an **integrability condition** on the last term $(\dots)\nabla_{\nu}\phi = \nabla_{\nu}V(\phi)$

then
$$\frac{dV}{d\phi} = R + \frac{\omega}{\phi^2} \nabla_{\mu}\phi\nabla^{\mu}\phi + 2\frac{\omega}{\phi} \square\phi$$

$$\Rightarrow \Phi = \square\phi - \frac{1}{2}\phi R + \frac{\omega}{2\phi} \nabla_{\mu}\phi\nabla^{\mu}\phi + \frac{1}{2}V(\phi)$$

extension to generalized Brans-Dicke gravity

... therefore: from $S = \int d^4x \sqrt{h} \phi(x)$

$$\Rightarrow I_{gen} = \frac{1}{4\pi} \int \sqrt{-g} d^4x \left[(\phi R - \frac{\omega}{\phi} \nabla_\mu \phi \nabla^\mu \phi + V(\phi)) + L_{matt} \right]$$

- ▶ the entropy functional holds for the general Brans-Dicke theory
- ▶ **IMPORTANT:** the information about the dynamics of the scalar d.o.f. is encoded in the **integrability condition** together with the trace of the metric field equation

$\omega = 0 \Rightarrow$ metric $F(R)$

$$3\Box\phi + 2V(\phi) - \phi \frac{dV}{d\phi} = (2\pi) T^M{}_\mu{}^\mu \Rightarrow \text{propagating scalar sourced by matter}$$

$\omega = -3/2 \Rightarrow$ Palatini $F(R)$

$$2V(\phi) - \phi \frac{dV}{d\phi} = (2\pi) T^M{}_\mu{}^\mu \Rightarrow \text{scalar and matter in algebraic relation: non propagating scalar}$$

summary

- ▶ the thermodynamical derivation works for the simplest cases of scalar tensor higher order gravity, naturally providing the dynamics for the extra dynamical scalar d.o.f.
- ▶ **but... $f(R)$ is** still special since they are trivially related to GR coupled to a scalar field by a field dependent conformal rescaling of the metric: no new lesson

MAIN POINT

can we capture higher curvature corrections to GR with the local thermodynamic reasoning ?

- NO** no clear thermodynamical interpretation for higher curvature corrections to entanglement and high: thermal derivation from entanglement is limited to theories where **$S \sim A$**

Fursaev, Solodukhin

- YES** **Noether charge entropy** functional naturally brings the extra information about curvature corrections (e.g. Lovelock gravity): it works!

Parikh 98, Padmanabhan, Jacobson 2011

summary

For stationary horizons, Wald entropy for $L = R + aR^2 + \dots$ is

$$S_{bh} = \frac{A}{4\hbar G_N} + \text{curvature terms} = \frac{2\pi}{\hbar} \int_{\Sigma} Q^{ab}[\chi] N_{ab} dA$$

$$Q^{ab}[\chi] = W^{abc} \chi_c + X^{abcd} \nabla_c \chi_a$$

Noether potential for the horizon generating Killing flow

For $L = L[g_{ab}, R_{abcd}]$, can choose $X^{abcd} = \frac{\partial L}{\partial R_{abcd}}$

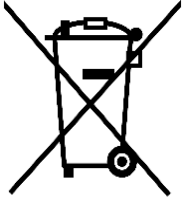
but... and $W^{abc} = 2\nabla_d X^{abcd}$

Cardoso 99

- ▶ not complete thermodynamical interpretation : problems with 2nd Law
- ▶ the causal structure of higher curvature theories is generally not the metric light cone
- ▶ Noether charge is associated to a gravitational Lagrangian: **Entropy loses its statistical interpretation**

it seems that the **local** thermodynamical derivation can only capture the leading order area term in the entropy....

thermodynamical equilibrium as a general principle ?



in fact the local character makes the derivation very general: can we consider the entropy balance equation as a general principle for deriving the gravitational dynamics ??

further investigations :

- ▶ confirm the validity of the approach at the GR level in a **different formalism** \Rightarrow Plebanski formulation of GR in terms of self dual forms
 - > **BF theory with constraints**
 - ▶ extend the approach with more classical geometric variables: **new degrees of freedom** \Rightarrow
 - metric affine theories
 - non zero torsion: Einstein-Cartan
 - > **Poincare' gauge gravity**
- \Rightarrow **characterize possible continuous limits in an emergent geometry perspective**

thermodynamical equilibrium as a general principle ?

interesting relations and hints :

- ▶ **fluid/gravity duality in flat spacetime**

Bredberg 2011, Compere 2011, Chirco 2011

- ▶ **holographic entanglement entropy and spacetime reconstruction**

Raamsdonk 2012, Takayanagi 2012

⇒ **provide a dynamical principle for the geometry
'emerging' from some holographic quantum theory**

intrinsic local character:

**just a failure or telling something deep on the
holographic principle ...?**

obrigado !