Instituto Superior Tecnico
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Recent Developments in Eddington inspired Gravity

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CENTRA – Instituto Superior Técnico

http://blackholes.ist.utl.pt

P. Pani, V. Cardoso, T. Delsate Phys.Rev. Lett. 107 031101 (2011)

Collaborators

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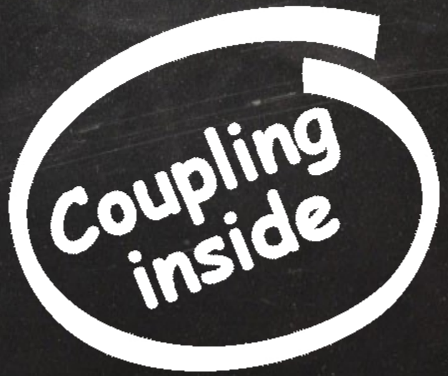
Outlook

- Motivation: exploring the gravity-matter coupling
- Eddington-inspired gravity
  - Action and equations
  - Linear structure of the theory
  - Dual interpretation
- The non relativistic approximation
  - Post-Newtonian parametrization
  - Modified Poisson law
  - Non relativistic collapse
  - Expectation for relativistic collapse
- Non relativistic stars
- Relativistic Stars

Motivations

- **Coupling to Matter rather unexplored**

- **Some singular problems in GR:**
  - **Big bang singularity**
  - **Naked singularity during dust collapse**
  - **...**



Eddington inspired gravity

$$S_{BI} = \frac{2}{\kappa} \int \left(\sqrt{|g_{ab} + \kappa R_{ab}|} - \lambda \sqrt{|g_{ab}|} \right) d^4x + S_M(g, \Psi_M),$$

- **Completely equivalent to GR in vacuum**
- **Minimal matter coupling to g**
- **Deviation only when coupled to matter :**
  - **Cosmology** [Banados, Ferreira PRL 105, 2010]
  - **Charged black hole** [Banados, Ferreira PRL 105, 2010]
  - **Neutron Stars and other compact objects** [Pani, Cardoso, Delsate, PRL 107, 2011]
  - **Sperically symmetric dust Collapse** [Pani, Cardoso, Delsate, in preparation]

Field Equations

$$q_{ab} = \kappa R_{ab}(q) + g_{ab},$$

$$\sqrt{-q}q^{ab} = \sqrt{-g}\left(\lambda g^{ab} - \kappa T^{ab}\right)$$

q is an auxiliary metric (~mathematical trick), Ricci tensor constructed with q.

q and g are the same in vacuum → Reduce to Einstein equations

coupled algebraic – dynamical system.

Stress tensor conservation with metric g :

$$\nabla_{(g)a}T^{ab} = 0$$

Small kappa limit

$$S_{BI} = \frac{2}{\kappa}\int \sqrt{|g_{ab}|}\,(\kappa R - (\lambda - 1))\, d^4x + S_M(g, \Psi_M) + \mathcal{O}(\kappa)^2,$$

→ **lambda : cosmological constant** $\Lambda = \frac{\lambda - 1}{2}$

- **Lambda = 1 : asymptotically flat space.**

$$R^{(q)}_{\mu\nu} \approx \Lambda g_{\mu\nu} + T_{\mu\nu} - \frac{1}{2}T g_{\mu\nu} + \kappa\left[S_{\mu\nu} - \frac{1}{4}S g_{\mu\nu}\right] + \ldots$$

Derivatives corrections

Quadratic in the matter fields

$$S_{\mu\nu} = T^{\alpha}_{\mu}T_{\alpha\nu} - \frac{1}{2}T T_{\mu\nu}$$

Structure of the equations

$$\delta q_{ab} = \delta g_{ab} + \kappa \delta R_{ab}$$

$$\delta g_{cd} = F(\delta q, \delta T)$$

- **linearly equivalent to GR, but different sources, easy to solve to linear order and general perturbation.**
- **Consequences :**
  - **Same PDE structure as GR for q.**
  - **2 ways of thinking the theory : for g or for q...**

Dual Theory

- **Assuming no matter, action is invariant under**

$$q_{ab} \leftrightarrow g_{ab}, \;\; \kappa \leftrightarrow -\kappa, \;\; \lambda \leftrightarrow 1/\lambda.$$

- **With matter, this translates to a highly non minimal coupling (g in terms of q):**

$$S_{dual} = \frac{2}{\kappa} \int \left(\sqrt{|q_{ab} + \kappa R_{ab}|} - \lambda \sqrt{|q_{ab}|} \right) d^4x$$
$$+ S_M(q_{ab} + \kappa R_{ab}, \Psi_M)$$

For example scalar field would have coupling of the form

$$R_{ab} \partial^a \phi \partial^b \phi$$

Leading to very complicated field equations.

Post Newtonian Expansion

Solve Einstein Equations for small velocities, small masses.

→ Book-keeping small parameters : ϵ

$$T^{ab} = (\epsilon^2 \rho + \epsilon^4 \rho \Pi + \epsilon^4 P) u^a u^b + \epsilon^4 P g^{ab}$$

$$u^a = u^0(-1, \epsilon v^i), \ i = 1, 2, 3$$

In standard GR, the PN expansion = solving order by order in ϵ

→ Express the metric in terms of potentials

$$g_{00} \approx -1 + 2\epsilon^2 U, \ g_{0i} \approx \epsilon^3 U_i, \ g_{i,j} \approx 1 + 2U\epsilon^2 \delta_{ij} + \epsilon^4 U_{ij}$$

U's: linear solutions to each order in ϵ **, with suitable gauge fixing; e.g.**

$$U(x) = \int \frac{\rho(x')}{|x - x'|} d^3x', \ U_i(x) = \int \frac{\rho(x') v_i(x')}{|x - x'|} d^3x', \ldots$$

Post Newtonian Expansion

Since we are (I am) a bit lazy, it would be sufficient to write down an Einstein-like equation and identify how the source term changes...

Method :

- Write down the 'PN' ansatz for both metric
- Solve for g in terms of q (*)
- Write down Ricci(q) = Source(T,g) → Potentials follow
- Read the potential for g using the dictionary (*)...

Results :

- q is the same than GR with effective pressure and internal energy :

$$P_{eff} = P + \frac{\kappa}{8}\rho^2, \; \Pi_{eff} = \Pi + \frac{5\kappa}{8}\rho.$$

- q and g differ only on compact support → Same PN expansion as GR !

Modified Poisson equation

$$U_q \ : \ \Delta U_q = 4\pi G\rho \qquad U_g \ : \ \Delta\left(U_g - \frac{\kappa}{4}\rho\right) = 4\pi G\rho$$

Hydrostatic equilibrium modified too:

$$\frac{dP}{dr} + \frac{d}{dr}\frac{\kappa}{8}\rho^2 = -\frac{Gm(r)\rho}{r^2}$$

new pressure term

Usual attraction potential

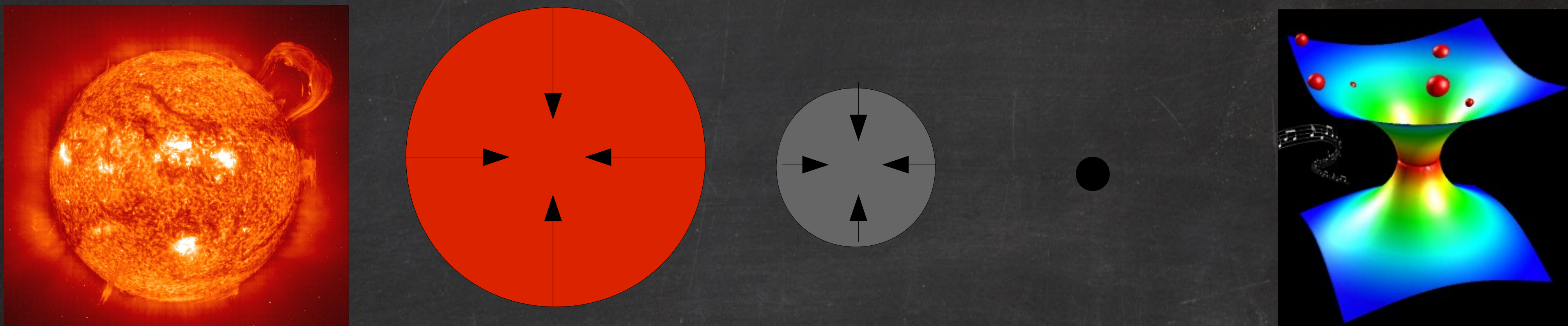
Consequence : Compact object with P = 0 !!!

$$\rho = \frac{\rho_c}{\omega r}\sin(\omega r), \ \omega = 4\sqrt{\pi/\kappa}.$$

→ Dust agglomerate ? Dark matter candidates?
→ Linearly stable !!!

Collapse in Eddington-inspired gravity

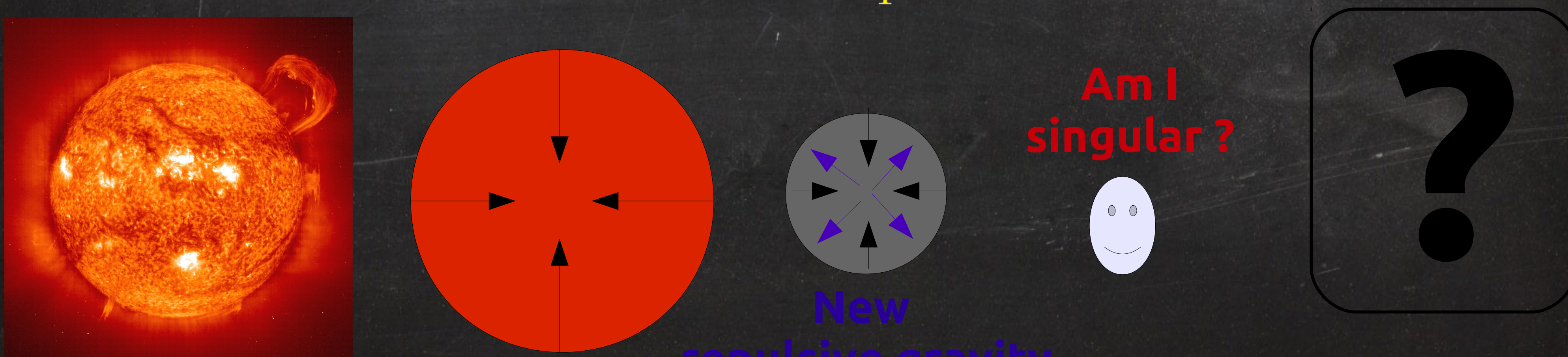
$$\nabla^2\Phi = 4\pi G\rho$$



Massive star — End of nuclear reactions — Collapse — Singularity — Black hole

time

$$\nabla^2\Phi = 4\pi G\rho + \frac{\kappa}{4}\nabla^2\rho$$



New
repulsive gravity

Collapse in Eddington-inspired gravity

$$\frac{\partial u(t,r)}{\partial t} + u(t,r)\frac{\partial u(t,r)}{\partial r} = -\frac{GM(t,r)}{r^2} - \frac{\kappa}{4}\frac{\partial \rho(t,r)}{\partial r}$$

Modified Euler equation

$$\frac{\partial \rho(t,r)}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho(t,r) r^2 u(t,r)\right]$$

$$\frac{\partial M(t,r)}{\partial r} = 4\pi r^2 \rho(t,r)$$

Continuity equation

- **Problems**
  - **Requires huge grid, CPU time**
  - **Appearance of shock waves → Code crashes**
- **Solutions**
  - **Lagrangian formulation of the equations (~ comoving coordinates)**
  - **Artifical viscosity in order to absorb locally the shock**

Lagrangian hydrodynamic

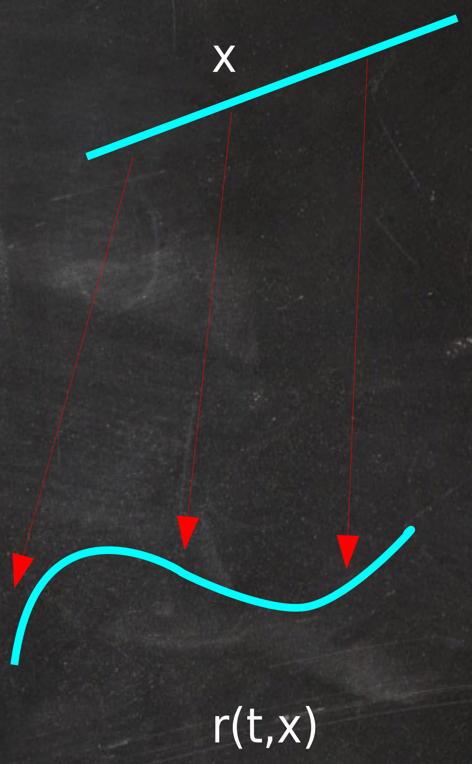
- **Frame attached to the fluid:**

$$u(t,x) = \frac{D}{Dt} r(t,x), \;\; \frac{D}{Dt} u(t,x) = a(t,x)$$

- **From Eulerian to Lagrangian:**

$$\frac{D}{Dt} \to \frac{d}{dt} + \vec{u}.\vec{\nabla}, \;\; r_{euler} \to r(t,x),$$

x : initial position of the fluid element.



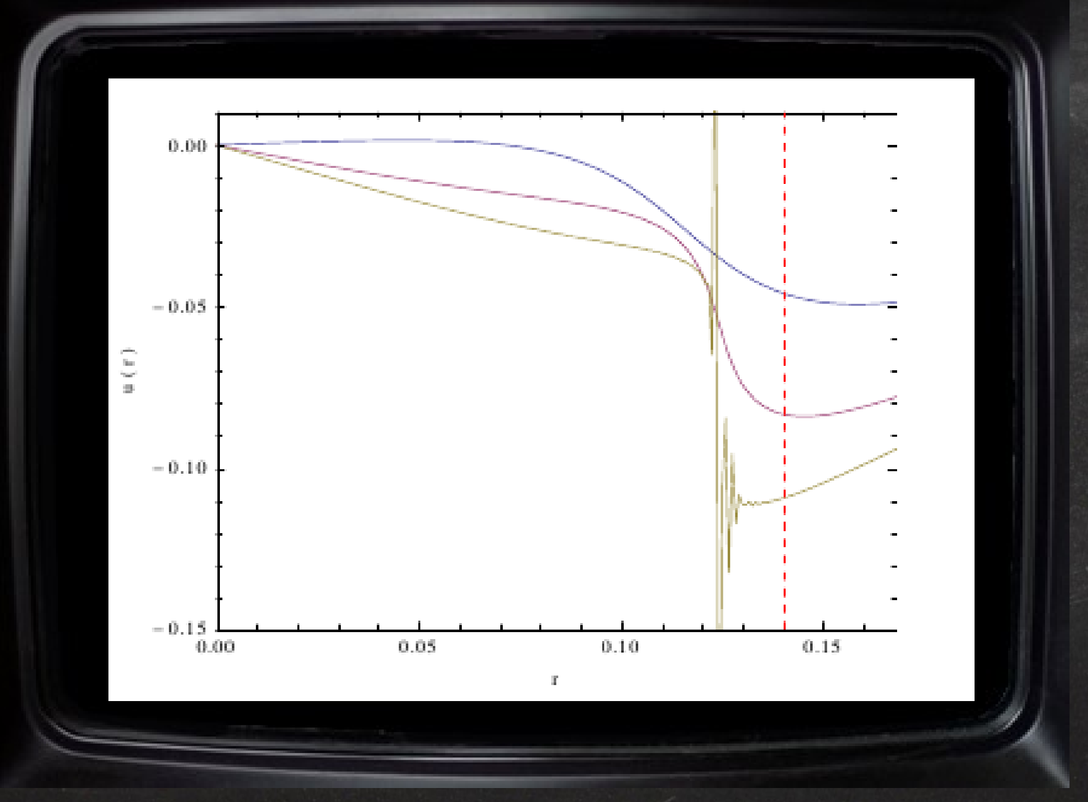
- **Price to pay: 1 extra equation.**

Shock waves for kappa != 0

Inner part wants to spread, outer part wants to collapse

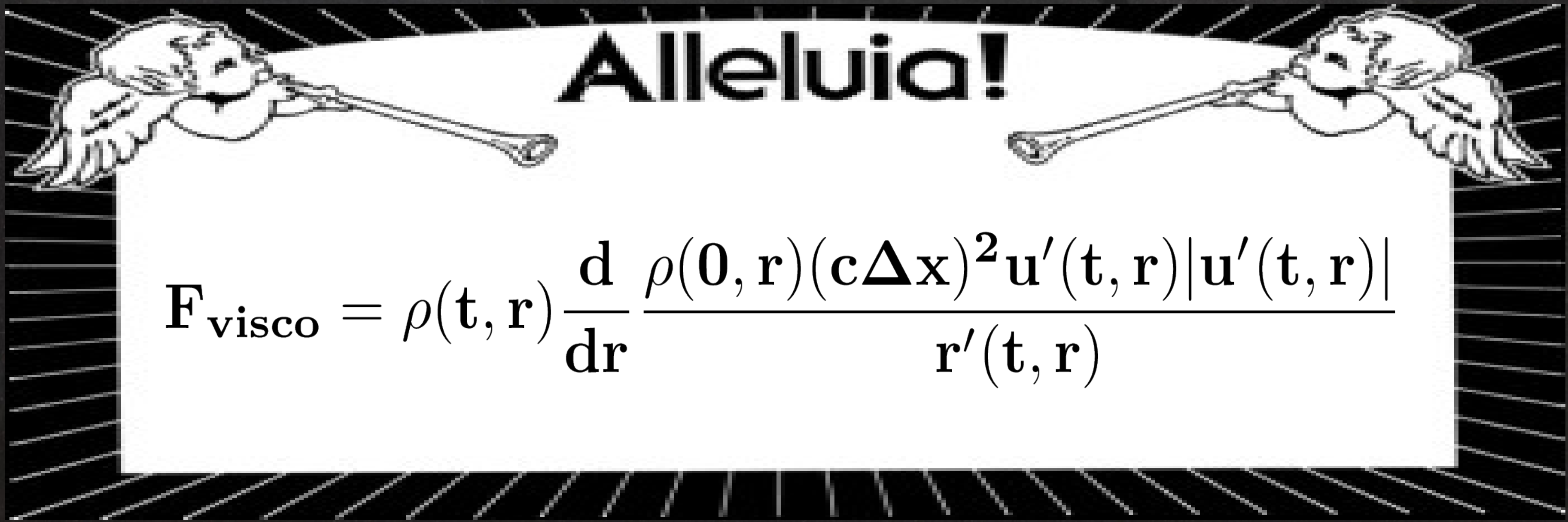
→ shockwave → infinite v' → spoils numerics

→ Code crashes before anything interesting happens.



Artificial viscosity

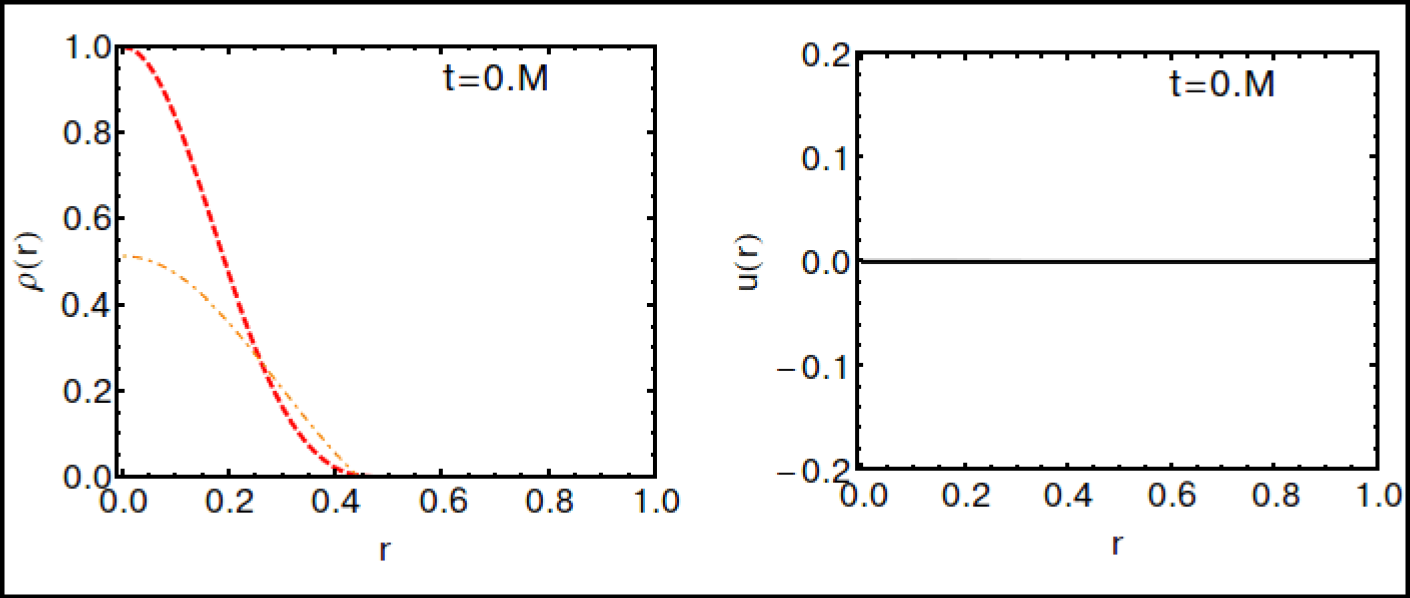
- **Solution: introduce a viscosity term**
- **Requirements:**
  - **Not affect the physics,**
  - **Regularize the discontinuity in u' (adapt to the mesh)**



$$\mathbf{F_{visco}} = \rho(\mathbf{t},\mathbf{r})\frac{\mathbf{d}}{\mathbf{dr}}\frac{\rho(\mathbf{0},\mathbf{r})(\mathbf{c}\Delta\mathbf{x})^{2}\mathbf{u}'(\mathbf{t},\mathbf{r})|\mathbf{u}'(\mathbf{t},\mathbf{r})|}{\mathbf{r}'(\mathbf{t},\mathbf{r})}$$

Artificial viscosity

Regularised shock waves forming for kappa != 0, then proagates:



Artificial viscosity

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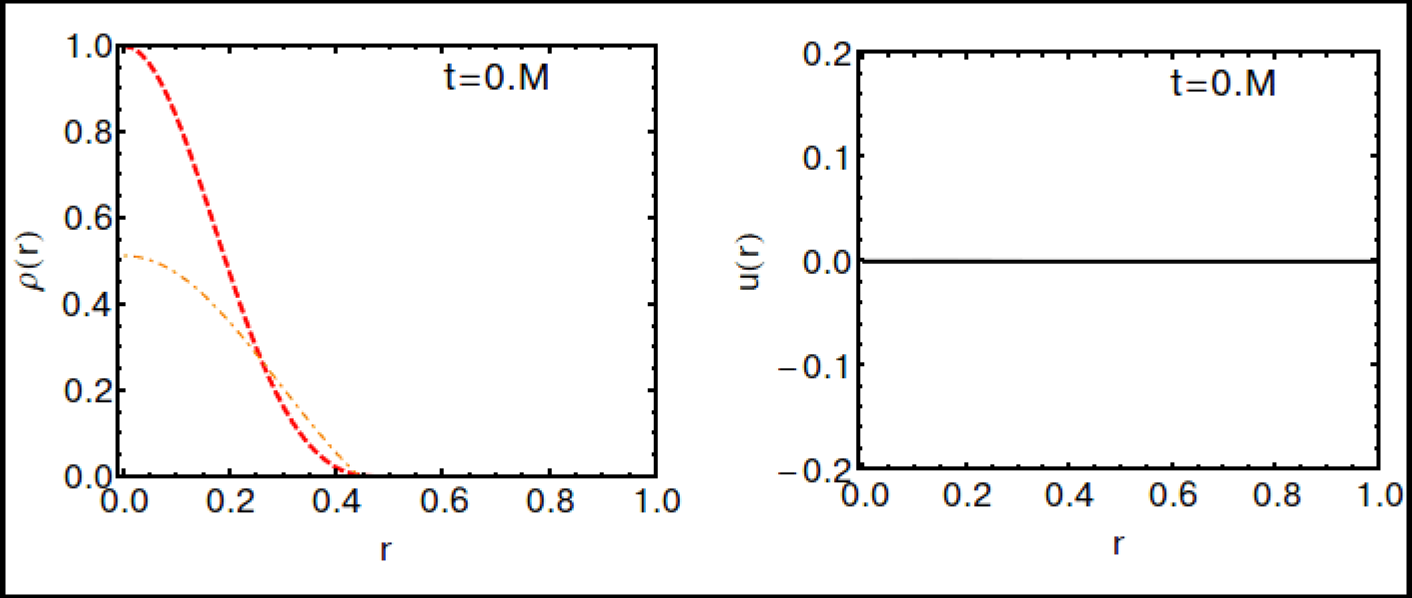


Results

- Pressureless initial configuration → no singularity for kappa>0
- Oscillates around P=0 static configuration, frequency predicted by linear analysis.

Results

- **Why is the axis changing ???**

Expectation for relativistic

- **Some regime, same results,**
- **What for realistic EOS ?**
- **Relativistic equivalent of P=0 → Maximal mass...**
  - **Small initial configuration mass → no collapse**
  - **Large initial configuration mass** ⇒

Singularity ?

Evolving solution ?

Matter emission ?

???

→ Pulsation or stability analysis [in progress]

→ *Compact stars in EI gravity*

- **Non-relativistic theory**
  - Pressureless stars, stability analysis, modified Chandrasekhar model
- **Relativistic theory**
  - Realistic Neutron Stars, slowly rotating models

Have we tested Newtonian gravity enough?

- **Parametrized Post-Poissonian approach:**

Most general Poisson's eq. which is covariant, perturbative, 2nd derivatives and reduces to Laplace eq. in vacuum:

$$\underbrace{\nabla^2\Phi = 4\pi G\rho}_{\text{standard}} + \underbrace{\frac{\kappa}{4}\nabla^2\rho}_{\text{Linear corrections}} + \alpha_g \epsilon^{ij}\nabla_i\Phi\nabla_j\rho +$$

$$\underbrace{+\eta\rho^2 + \gamma\nabla\rho\cdot\nabla\rho + \epsilon_1\nabla\Phi\cdot\nabla\rho + \epsilon_2\Phi\nabla^2\rho + \epsilon_3\rho\nabla^2\Phi + \ldots}_{\text{Quadratic corrections}}$$

- **Tests of the equivalence principle constrain many terms**
- **Can we constrain these corrections using current observations?**
- **Precise measurements of solar neutrinos and helioseismology**

[Casanellas, Pani, Lopes, Cardoso, ApJ (in press) astro-ph.SR/1109.0249]

Stars in modified Newtonian gravity

$$\nabla^2\Phi = 4\pi G\rho + \frac{\kappa}{4}\nabla^2\rho$$

- **Newtonian stars**

Modified hydrostatic equilibrium:

$$\frac{dP}{dr} = -\frac{Gm(r)\rho}{r^2} - \frac{\kappa}{4}\rho\rho' \quad = -G_{\rm eff}(r)\frac{m(r)\rho}{r^2}$$

$$G_{\rm eff}(r) = G + \frac{\kappa}{4}\frac{\rho'(r)r^2}{m(r)}$$

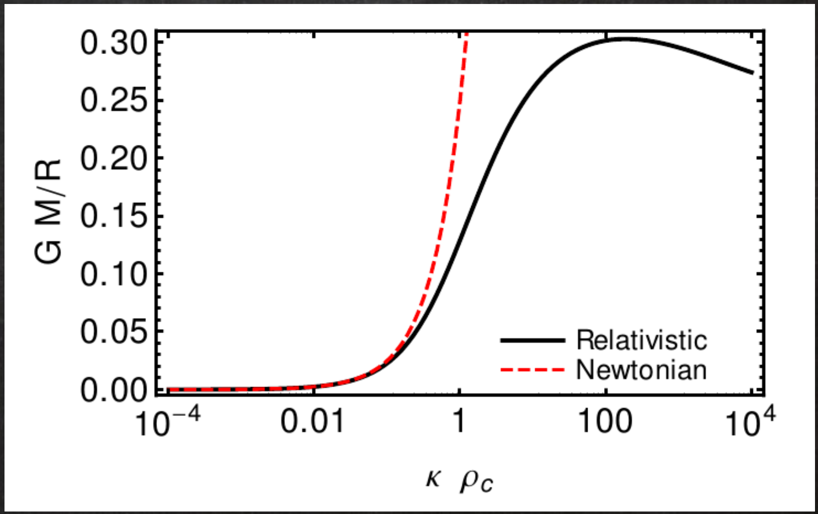
Equivalent to standard gravity with a polytropic EOS:

$$P(\rho) = \frac{\kappa}{8}\rho^2$$

- **The theory admits "dark matter stars"**
  - **P=0 and κ=0**

$$\rho(r) = \rho_c\frac{\sin\varpi r}{\varpi r} \qquad \varpi = 4\sqrt{\frac{\pi G}{\kappa}}$$

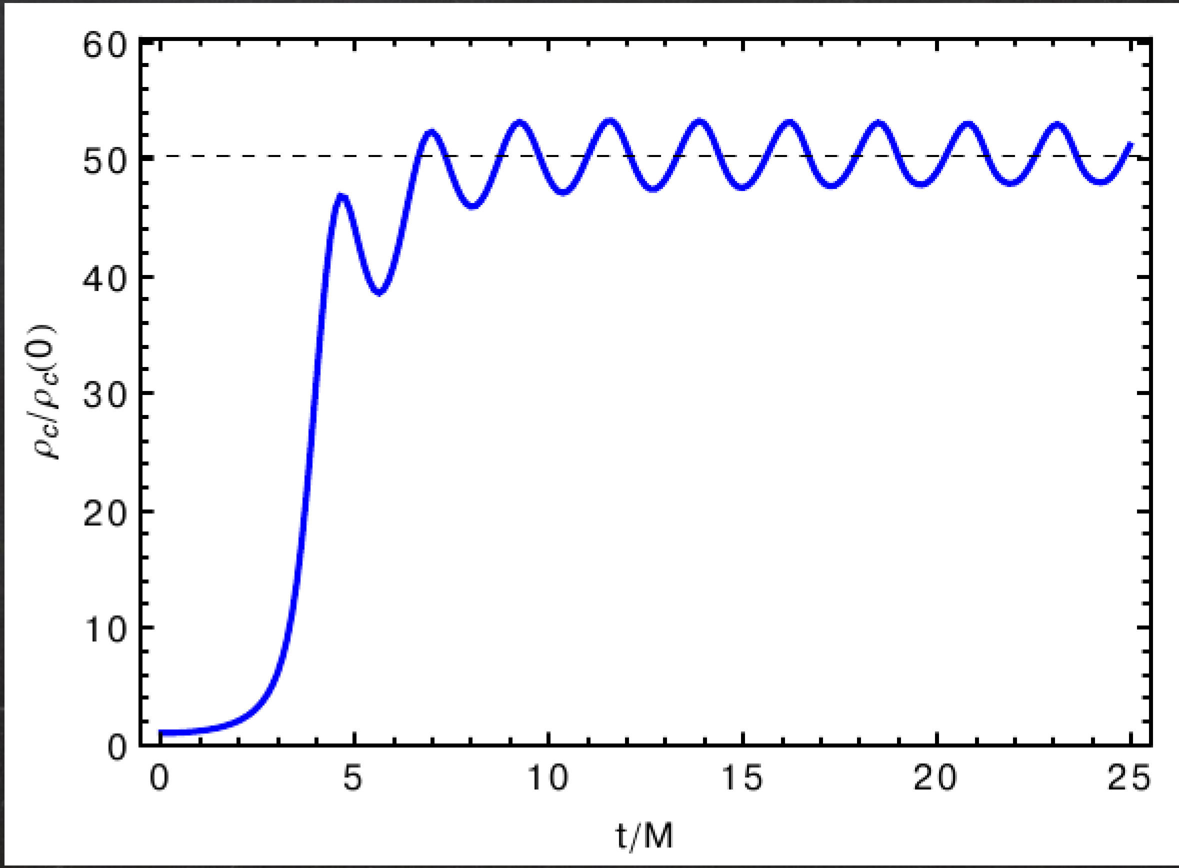
Pressureless stars



Stars in modified Newtonian gravity

$$\nabla^2\Phi = 4\pi G\rho + \frac{\kappa}{4}\nabla^2\rho$$

- Pressureless stars are the end-point of non-relativistic collapse



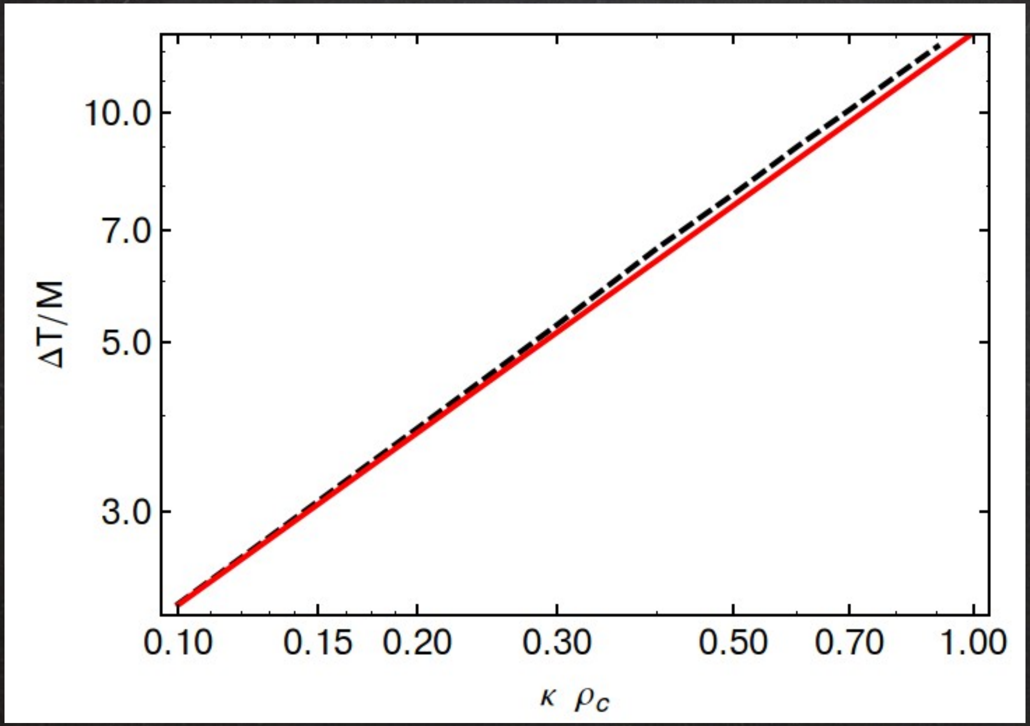
- Dissipation would lead the system to a stationary configuration

Stars in modified Newtonian gravity

- **Oscillation modes and linear stability analysis**

$$\ddot{\xi} - \frac{1}{\rho}\left[\frac{\gamma P}{r^2}(r^2\xi)'\right]' + \frac{4}{\rho r}\xi P' = -\frac{\kappa}{4}\left[\frac{2}{r}\xi\rho' - \xi'\rho' - \left[\frac{\rho}{r^2}(r^2\xi)'\right]'\right]$$

$\xi \sim e^{i\omega t} \rightarrow$ **Lagrangian displacement** $\gamma \rightarrow$ **Adiabatic index of perturbations**



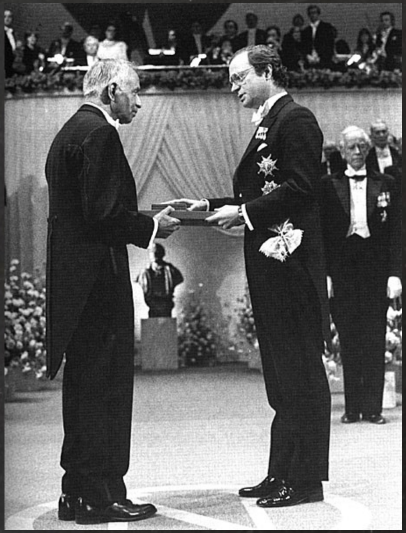
Pressureless stars are stable!

Oscillation period:

$$\frac{\Delta T}{M} = \frac{\pi^{5/4}}{2\alpha}\left(\frac{\kappa}{M^2}\right)^{3/4}$$

perfectly agrees with the oscillations of our simulations

Modified Chandrasekhar model



- Ultra-relativistic matter $P=K\rho^{4/3}$

- Energy:

$$\nabla^2\Phi = 4\pi G\rho + \frac{\kappa}{4}\nabla^2\rho$$

$$E = E_F + E_G$$

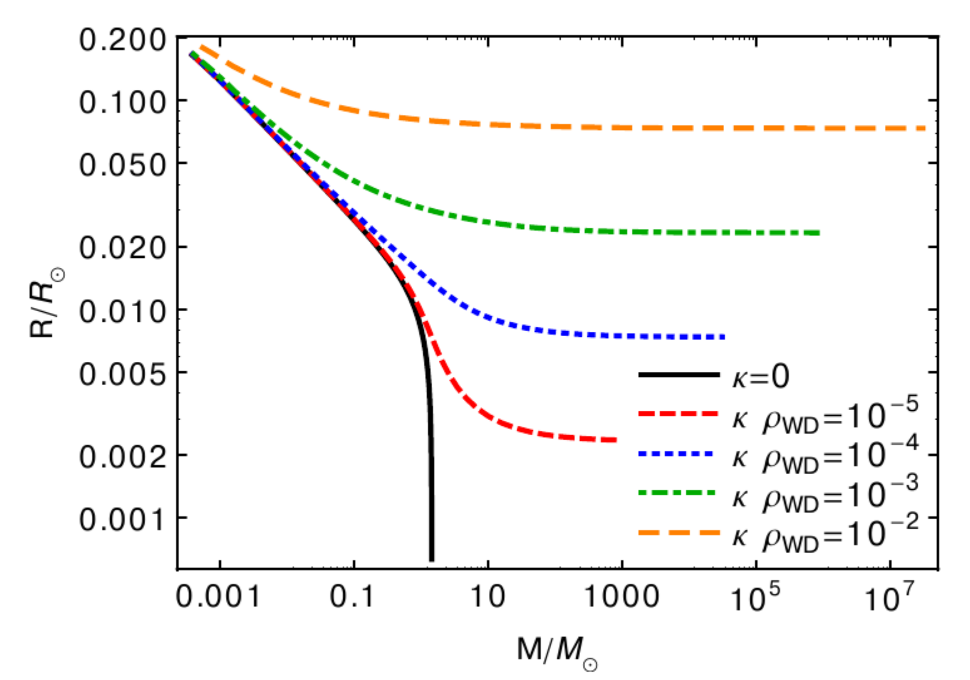
$$\approx \underbrace{\frac{\hbar c N^{1/3}}{R}}_{\text{Fermi energy}} \underbrace{- \frac{GNm_b^2}{R} + \frac{3\kappa N m_b^2}{16\pi R^3}}_{\text{Gravitational energy per fermion}}$$

- If $\kappa=0$ (Chandra's result)

$$N_{\max} \approx \left(\frac{c\hbar}{Gm_b^2}\right)^{3/2} \Rightarrow M_{\max} \approx 1.4 M_\odot$$

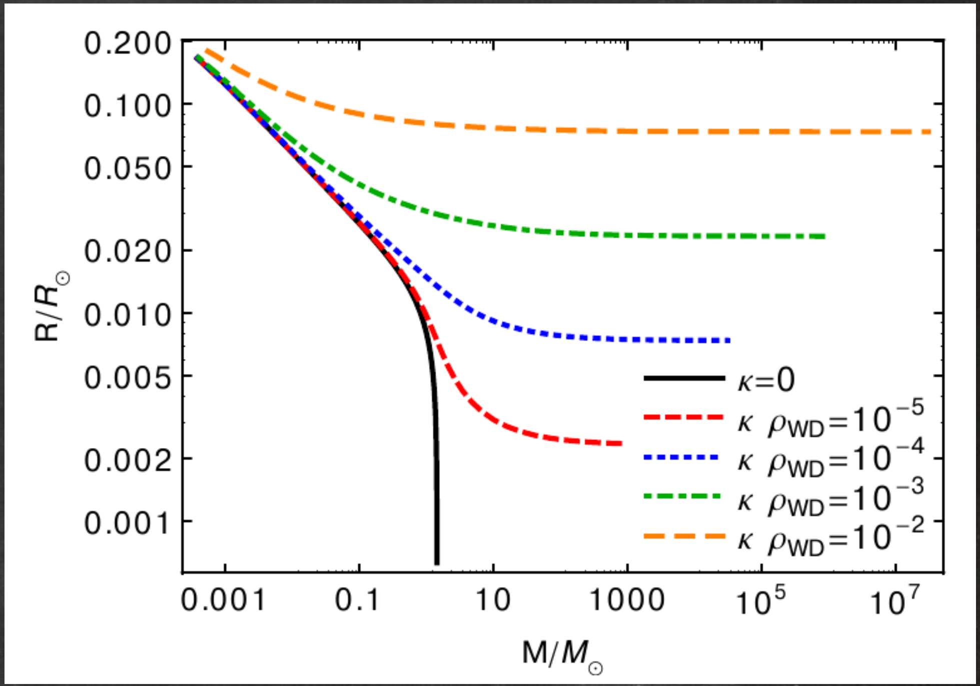
- If $\kappa>0$ (Eddington-inspired gravity)

$$R_{\min} \approx \frac{3}{4}\sqrt{\frac{\kappa}{\pi G}}\sqrt{1 - \frac{N_{\max}^{2/3}}{N^{2/3}}}$$



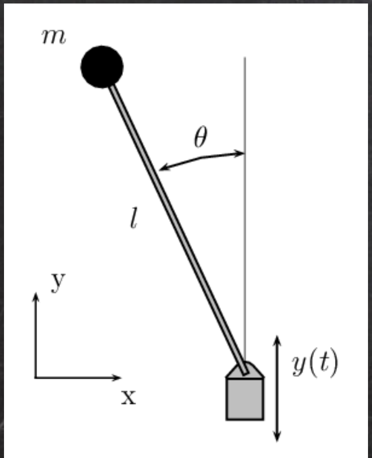
Open issues

- Collapse when M>1.4 Msun?



BHs are vacuum solutions, but can be formed in dynamical scenarios?

~ inverted pendolum:



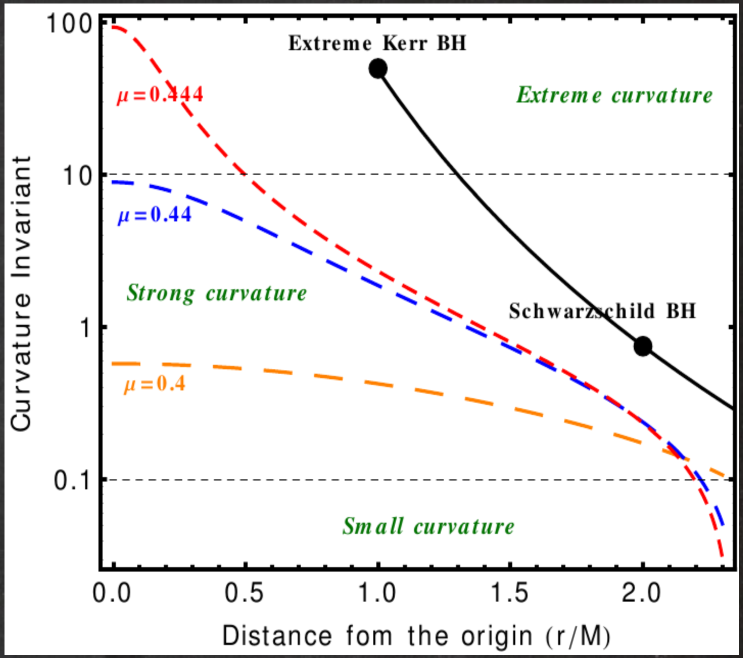
- Are BHs stable?

To answer these questions, we need a fully relativistic theory

Eddington-inspired gravity

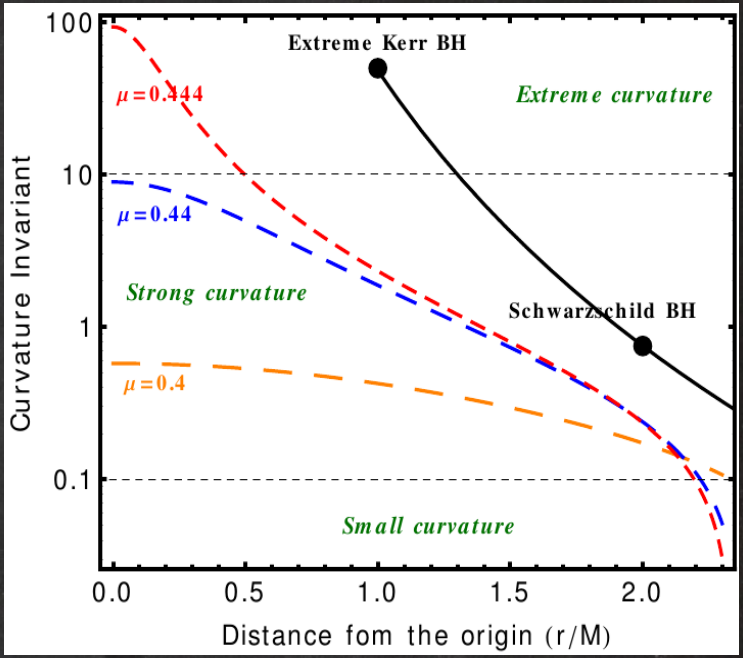
Compact stars VS black holes

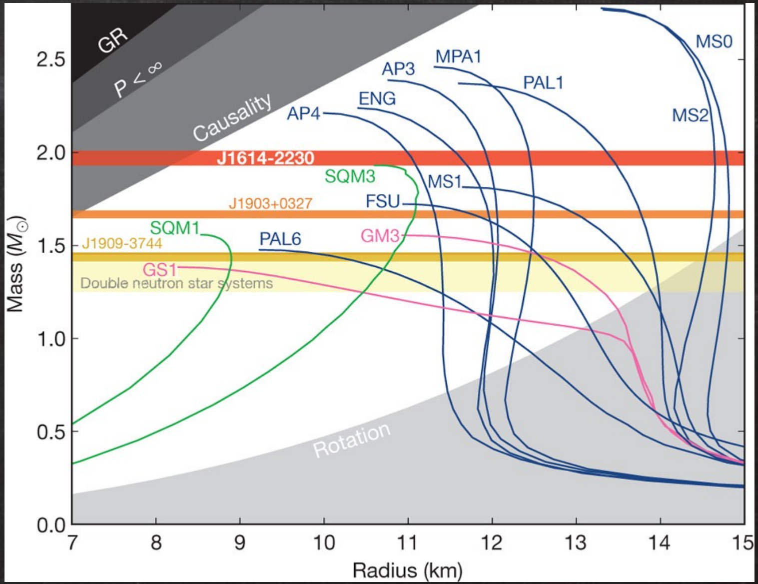
- Intimately related: collapse, Chandra, etc..
- Even stronger curvatures
- New physics even at non-relativistic level
- Neutron stars (NSs) are common objects
- More accessible than black holes (BHs)



Compact stars VS black holes

- **Intimately related:** collapse, Chandra, etc..
- Even **stronger curvatures**
- New physics even at **non-relativistic level**
- Neutron stars (NSs) are **common objects**
- **More accessible** than black holes (BHs)
- **However:**
  - BHs are simple objects, NSs are not!
  - Equation of state of a NS?
- Future experiments **(NICER)**
- **Theoretical insights may be EOS independent**





Demorest et al. Nature 2010

Relativistic stellar models

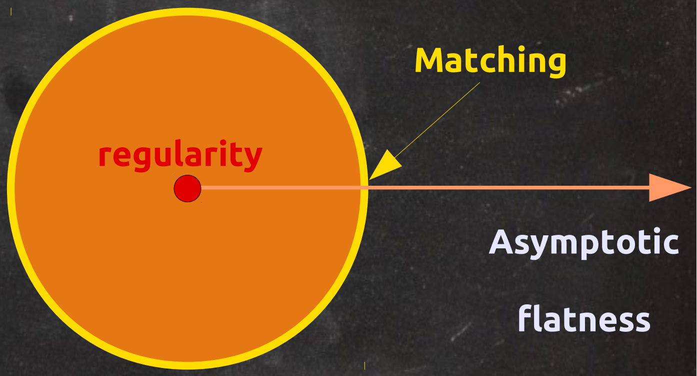
- **It's not "just" modified gravity → many subtleties**
  - **Well-posedness** of the field equations (cf. Palatini f(R) theories)
  - **Matching conditions** at the stellar surface
- **Relativistic stellar collapse?**

Relativistic stellar models

- It's not "just" modified gravity → many subtleties
  - Well-posedness of the field equations (cf. Palatini f(R) theories)
  - Matching conditions at the stellar surface
- Relativistic stellar collapse?
- Let us start with static configurations:

$$ds_q^2 = q_{ab}dx^a dx^b = -p(r)dt^2 + h(r)dr^2 + r^2 d\Omega^2$$

$$ds_g^2 = g_{ab}dx^a dx^b = -F(r)dt^2 + B(r)dr^2 + A(r)r^2 d\Omega^2$$



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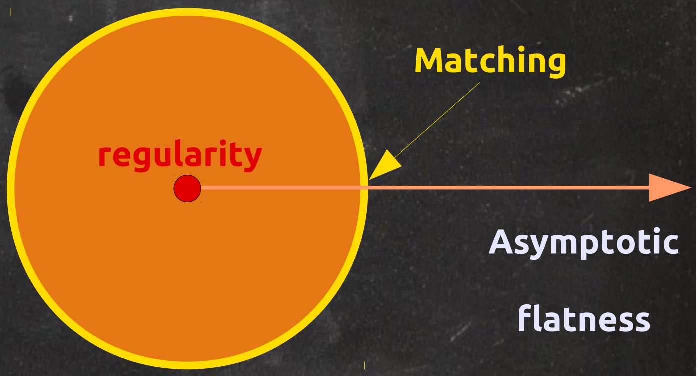


- Slowly-rotating models

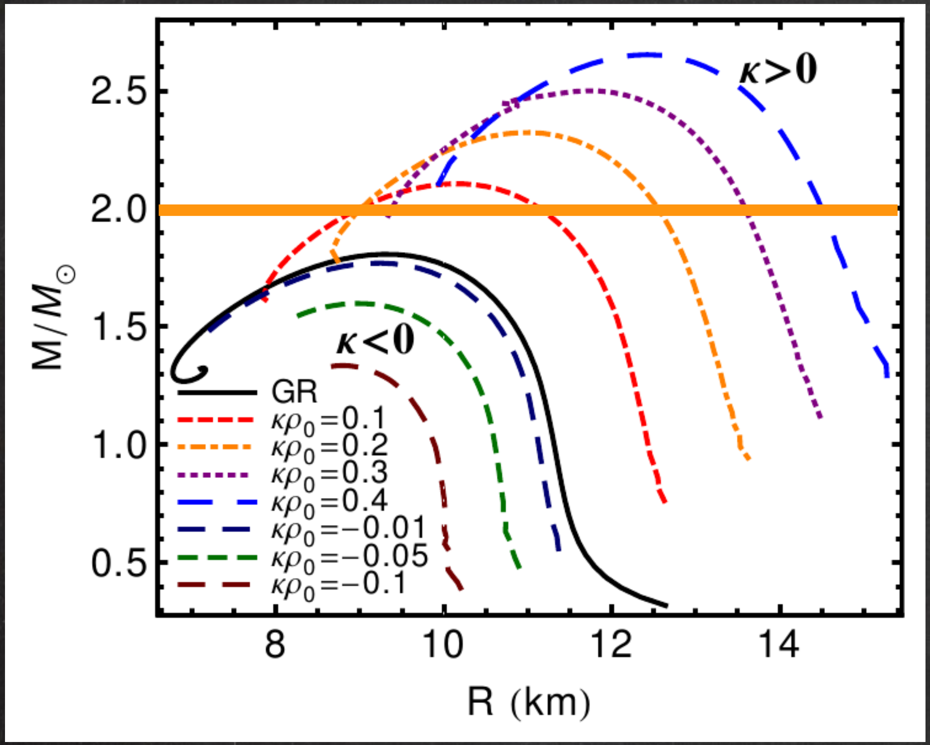
$$q_{t\varphi} = -\eta(r)r^2 \sin^2\theta$$

$$g_{t\varphi} = -\zeta(r)r^2 \sin^2\theta$$

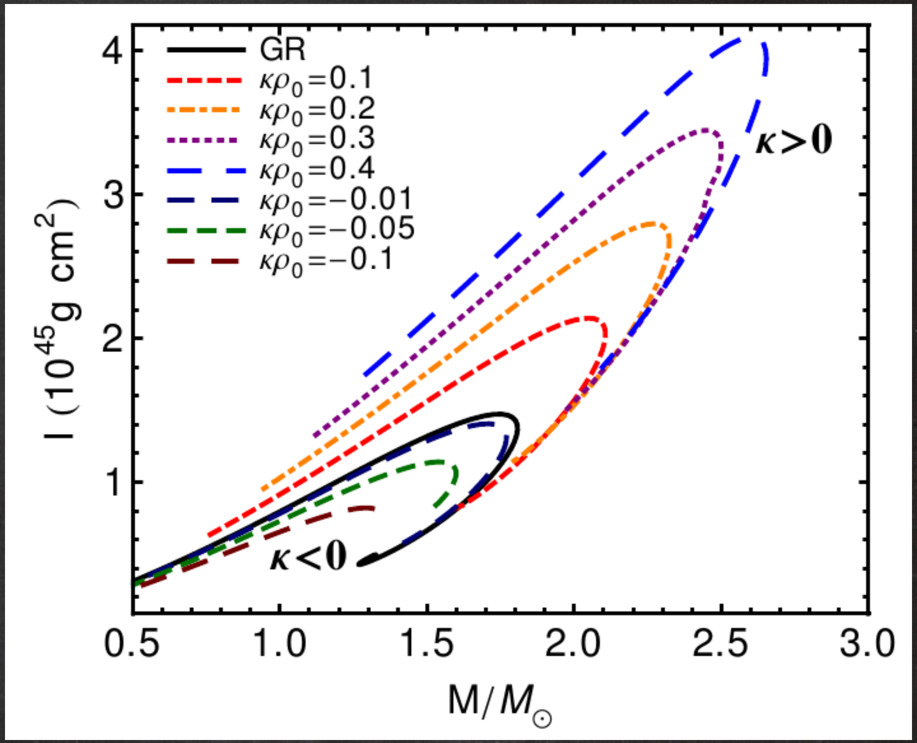
→ Field eqs can be solved perturbatively [Hartle '67]

Standard neutron stars in EI gravity:

Mass-radius relation



Moment of inertia



Positive κ contribute to enhance the relativistic effects

- **Degeneracy** between different equations of state!
- Can explain recent observations **without assuming exotic EOS**
- **No compact objects** when:

$$\begin{cases} P_c\kappa < 1 & \kappa > 0 \\ \rho_c|\kappa| < 1 & \kappa < 0 \end{cases}$$

Conclusion

- Did we test the matter-gravity sector of GR enough??
- Singularities in GR can be avoided modifying the coupling to matter
- Rich and viable phenomenology even in the non-relativistic limit
- Eddington inspired gravity has a very appealing features

  - Non-singular cosmology
  - Stable dark matter stars
  - Modified non-relativistic limit
  - Non-singular Newtonian collapse
  - Higher maximum mass in neutron stars
  - Constraints from solar physics

- Important to understand the relativistic collapse
- Currently hidden sectors of GR will be tested in the near-future
- Non-linear, strong-field effects are “smoking guns” for next experiments

Boas ferias!