

Rastall's cosmology and the Λ CDM model

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General Relativity

The equations

- The Standard Cosmological Model is based on the General Relativity Theory:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu},$$

$$T^{\mu\nu}{}_{;\mu} = 0.$$

- The universe is isotropic and homogeneous at large scales. The geometry describing it is given by,

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right].$$

- $a(t)$ is the scale factor and k is the curvature of the spatial section.

General Relativity

The equations

- The equations of motion are:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_{i=0}^n \rho_i.$$

- Defining,

$$\Omega_i = \frac{8\pi G}{3H_0^2} \rho_i,$$

we have,

$$\frac{H^2}{H_0^2} = \sum_{i=0}^n \Omega_i.$$

The cosmic phases

Cosmology today

- The Standard Cosmological Model is called Λ CDM.
- This model considers four phases in the cosmic evolution:
 - A primordial phase, where inflation took place.
 - A phase dominated by radiation, called *radiative phase*.
 - A phase dominated by pressureless matter (including baryons).
 - An accelerated expansion phase dominated by dark energy.

Standard Cosmological Model

The different conditions

- Strictly speaking, the Λ CDM model concerns the two last phases.
- However, the two first phases are crucial to establish the initial conditions for Λ CDM.
- The two initial phases must give the following conditions:
 - The fractional quantities of baryon and radiation with respect to the critical density;
 - The abundance of light chemical elements;
 - The primordial spectrum of the perturbations;
 - The conditions for a homogeneous and isotropic universe (?).

The Λ CDM model

The content

- The Λ CDM model contains baryons, radiation, neutrino, dark matter and dark energy.
 - Dark matter is represented by a pressureless fluid: it is necessary in order to explain the formation and dynamics of local structures like galaxies and clusters of galaxies.
 - Dark energy is represented by the cosmological constant: simple and "natural".
- The Friedmann's equation:

$$H^2 = \Omega_{b0}(1+z)^3 + \Omega_{\gamma 0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\Lambda 0}.$$

Dark sector

The background tests

- The Λ CDM model agrees very well with the tests using the background equations, called *kinematical tests*:
 - 1 Supernova type Ia;
 - 2 Baryonique accoustic oscillations;
 - 3 The (differential) age of astronomical objects (galaxies);
 - 4 The position of the first accoustic pic in the CMB spectrum.

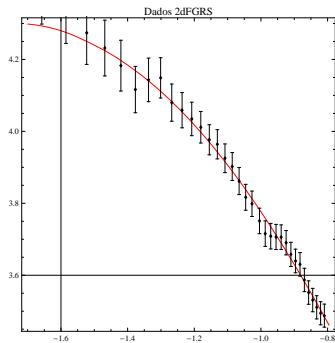
Dark sector

The perturbative tests

- The Λ CDM model agrees also very well with the tests using the perturbed equations, called *dynamical tests*:
 - 1 Matter power spectrum (agglomeration of matter in the universe);
 - 2 The full CMB spectrum.

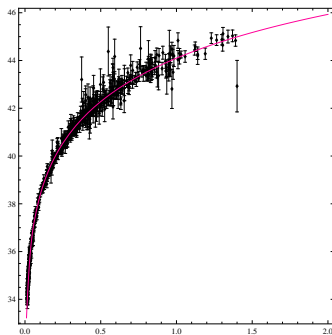
Observational test

Matter power spectrum



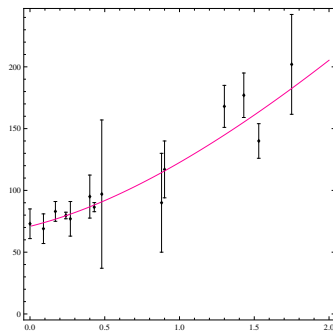
Observational test

Supernova type Ia



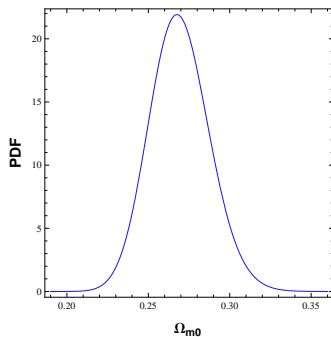
Observational test

$H(z)$



Observational test

Estimations for Ω_{m0}



The Λ CDM model

Composition

$$\Omega_{b0} \sim 0.04,$$

$$\Omega_{\gamma 0} \sim 5 \times 10^{-5},$$

$$\Omega_{m0} \sim 0.23,$$

$$\Omega_{\Lambda 0} \sim 0.73.$$

The Λ CDM model

Theoretical problems

- The candidates to dark matter (axions, neutralinos, etc.) remain hypothetical objects, without experimental support.
- Dark energy (the cosmological constant) can be interpreted as the vacuum energy. But, the predicted value (from quantum field theory) is tenths of order of magnitude larger than its observational value.
- Why the density of matter is comparable with the density of dark energy today, in spite of their different time behaviour?
Cosmic coincidence problem.

The Λ CDM model

Observational problem

- The Λ CDM model predicts a excess of power at galactic scales, not confirmed by observations.
- It can not explain the Tully-Fisher law.
- The supernova can be considered as a solid test? Many calibration tensions.

The Λ CDM model

Alternatives

- The difficulties with the Λ CDM model gave birth to many other alternatives:
 - Quintessence - Dark energy is represented by a self-interacting scalar field.
 - *K*-essence - The kinetic term for the scalar field is generalized in a non-canonical way.
 - Interaction models for the dark sector - there is a decaying of dark energy into dark matter or vice-versa.

The Λ CDM model

Difficulties with the alternatives

- Problems with the alternatives
 - Quintessence: the mass of the scalar particle is of the order of $m_\phi = 10^{-33} \text{ eV}$.
 - K -essence: stability is not always assured.
 - Interactions model: thermodynamic constraints are not always obeyed.

Unifying dark matter and dark energy

The Chaplygin gas

- There is an interesting possible unification of dark matter and dark energy into a single fluid: the Chaplygin gas.
- Equation of state:

$$p = -\frac{A}{\rho}.$$

- Negative pressure: it can accelerate the universe.
- Positive (squared) sound velocity: it can have a well-defined perturbative behaviour.
- A.Y. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. **B511**, 265(2001).

- Conservation equation:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0.$$

- Solution for the Chaplygin gas:

$$\rho(a) = \sqrt{A + \frac{B}{a^6}}.$$

- Asymptotic behaviour:

$$a \rightarrow 0 \Rightarrow \rho \propto a^{-3} \quad \text{Pressureless matter.}$$

$$a \rightarrow \infty \Rightarrow \rho \propto \text{cte} \quad \text{Cosmological constant.}$$

- The Chaplygin gas equation of state can be obtained from the Nambu-Goto action:

$$S_c = \int d^2x \sqrt{-G} G^{ab} \eta_{\mu\nu} X_{;a}^{\mu} X_{;b}^{\nu}.$$

- Under some conditions, this action can be rewritten as the *DBI* action:

$$S_{DBI} = \int d^4x \sqrt{-g} V(T) \sqrt{1 - T_{;\rho} T^{;\rho}}.$$

- This action can lead to the Chaplygin gas equation of state.

- The confrontation with the observations indicates that the Chaplygin gas is not competitive with the Λ CDM model: χ^2 is much higher.
- New proposal: the generalized Chaplygin gas:

$$p = -\frac{A}{\rho^\alpha}.$$

- Price to pay: there is not anymore a connection with string theory, even if a generalized *DBI* action may be written.

- There is one more parameter - it is natural that good results, concerning the confrontation with observation, can be obtained.
- On the other hand, there is a tension between the background and perturbative observational tests due to a problem with sound speed.

Observational tension

The sound speed

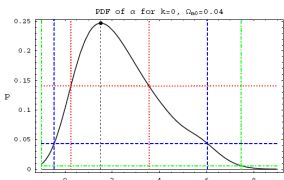
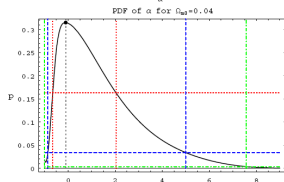
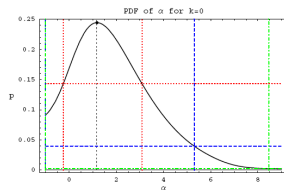
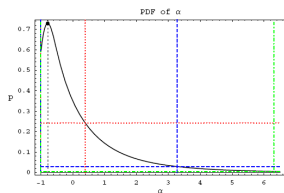
- The background tests ask for a negative α .
- But, if α is negative, the sound speed is imaginary:

$$v_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{\alpha A}{\rho^{\alpha+1}}.$$

- $\alpha < 0$ implies $v_s^2 < 0$.
- The model becomes unstable.

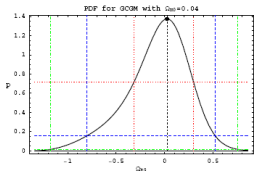
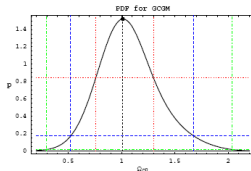
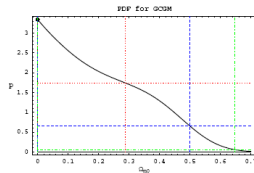
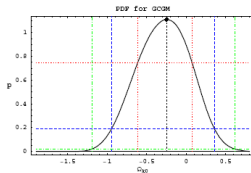
Supernova

Estimations for α - R. Colistete and J.C. Fabris, Class.Quant.Grav. **22**,2813(2005)



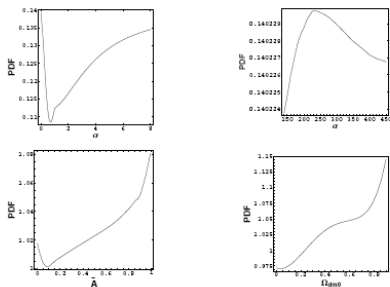
Observational tension

Estimations for Ω_{m0}



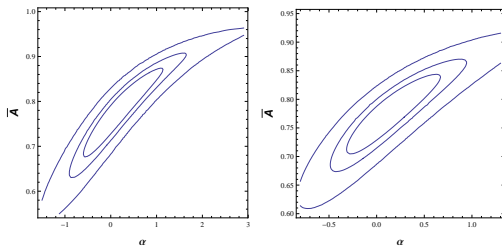
Observational tension

Matter power spectrum - J.C. Fabris, H.E.S. Velten and W. Zimdahl, Phys.Rev. D81,087303(2010)



Observational tension

$H(z)$, J.C. Fabris, P.L.C. de Oliveira and H.E.S. Velten, European Physical Journal C **71**, 1773(2011).



Scalar models for the unification program

Canonical scalar field

- Let us consider the energy-momentum tensor of a self-interacting scalar field:

$$T_{\mu\nu} = \frac{1}{2}\phi_{;\mu}\phi_{;\nu} - \frac{1}{2}g_{\mu\nu}\phi_{;\rho}\phi^{;\rho} + g_{\mu\nu}V(\phi).$$

- The energy and the pressure associated to this field are,

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi),$$

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi).$$

Scalar models for the unification program

The sound speed

- The sound speed can be obtained as:

$$v_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{\partial_\chi p}{\partial_\chi \rho} = 1 \quad , \quad \chi = \frac{\phi^2}{2}.$$

- Hence, no instability problem.
- But, it is impossible to form the structures.
- The structure formation process asks for a zero sound speed.

The Rastall's theory

The conservation law

- The relativistic gravitation theories are based on the null divergence of the energy-momentum tensor:

$$T^{\mu\nu}{}_{;\mu} = 0.$$

- This means conservation of matter?
- In the cosmological framework,

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0.$$

- There is an exchange of energy between matter and gravitational field.

The Rastall's theory

The conservation law - P. Rastall, PRD **6**, 3357(1972).

- The Rastall's theory is based on the idea that the conservation laws have been verified only in the Minkowski space-time.
- The conservation law may change in curved space-time. For example,

$$T^{\mu\nu}{}_{;\mu} = \kappa R^{i\nu}.$$

- This remember strongly the changing in the classical conservation law due to quantum effects.
- In some cases, considering a quantum scalar field, the conservation law reads,

$$T^{\mu\nu}{}_{;\mu} = \frac{1}{48\pi} R^{i\nu}.$$

The Rastall's theory

The field equations

- The field equations can be written as,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G \left\{ T_{\mu\nu} - \frac{\gamma - 1}{2}g_{\mu\nu}T \right\},$$

$$T^{\mu\nu}{}_{;\mu} = \frac{\gamma - 1}{2}T^{;\nu}.$$

- There is one free parameter: γ .
- When $\gamma = 1$, General Relativity is recovered.

The Rastall's theory

Coupled matter

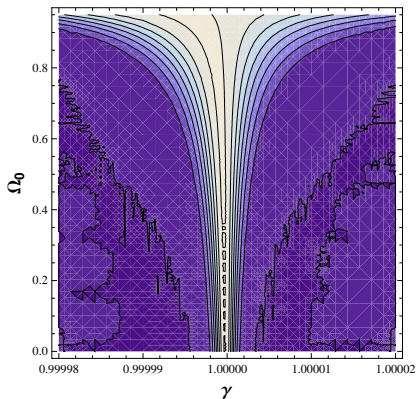
- In the universe there are many matter component.
- In order to have structure formation, at least one component must have effective zero pressure.
- Let us consider two fluids such that,

$$T_x^{\mu\nu}{}_{;\mu} = \frac{\gamma - 1}{2} T_x^{;\nu},$$

$$T_m^{\mu\nu}{}_{;\mu} = 0.$$

The Rastall's theory

Power spectrum estimations for γ



The Rastall's theory

Scalar field

- One possibility is to describe the *Rastall's component* by a self-interacting scalar field.
- This scalar field must obey a modified Klein-Gordon equation:

$$\begin{aligned}
 R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= \phi_{;\mu}\phi_{;\nu} - \frac{2-\gamma}{2}g_{\mu\nu}\phi_{;\rho}\phi^{;\rho} \\
 &+ g_{\mu\nu}(3-2\gamma)V(\phi), \\
 \square\phi + (3-2\gamma)V_{\phi} &= (1-\gamma)\frac{\phi^{;\rho}\phi^{;\sigma}\phi_{;\rho;\sigma}}{\phi_{;\alpha}\phi^{;\alpha}}.
 \end{aligned}$$

The Rastall's theory

The scalar field - sound speed

- Now, the expression for the sound speed is,

$$v_s^2 = \frac{2 - \gamma}{\gamma}.$$

- For $\gamma = 2$, the sound speed is zero.
- A good model for the unification program of the dark sector?
- C. Gao, M. Kunz, A.R. Liddle and D. Parkinson, PRD **81**, 043520(2010).

The Rastall's theory

Scalar field

- For $\gamma = 2$, we have,

$$T_{\mu\nu}^{\phi} = \phi_{;\mu}\phi_{;\nu} + g_{\mu\nu}V(\phi).$$

- The energy and the pressure associated with this non-canonical scalar field are,

$$\rho_{\phi} = \dot{\phi}^2 + V(\phi),$$

$$p_{\phi} = -V(\phi).$$

The Rastall's theory

Scalar field

- If we want that this scalar field represents the generalized Chaplygin gas model, then,

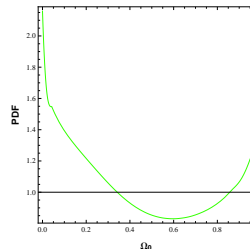
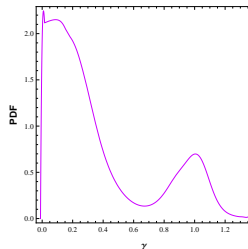
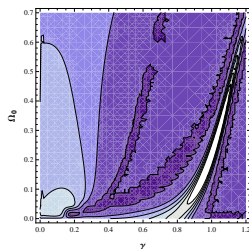
$$\dot{\phi}(a) = \sqrt{3\Omega_{c0}} \sqrt{g(a)^{1/(1+\alpha)} - \bar{A}g(a)^{-\alpha/(1+\alpha)}},$$

$$V(a) = 3\Omega_{c0}\bar{A}g(a)^{-\alpha/(1+\alpha)},$$

$$g(a) = \bar{A} + \frac{(1 - \bar{A})}{a^{3(1+\alpha)}}.$$

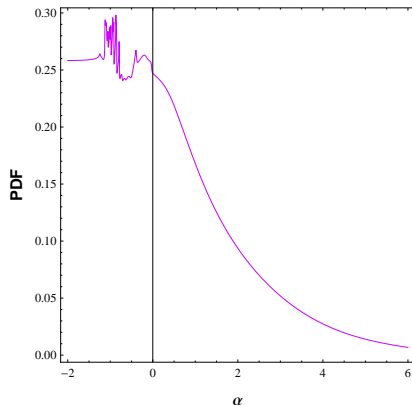
Rastall's scalar formulation for the Chaplygin gas

Results - J.C. Fabris, T.C.C. Guio, M. Hamani Daouda and O.F. Piattella,
Gravitation&Cosmology, **17**, 259(2011)



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Problems with the scalar formulaton

Perturbed equations in the Newtonian gauge

- Let us consider perturbations in the Newtonian gauge

$$ds^2 = a^2(1 - 2\Phi)d\eta^2 - a^2(1 + 2\Phi)\gamma_{ij}dx^i dx^j,$$

- The perturbed equations read:

$$\begin{aligned} \nabla^2\Phi - 3\mathcal{H}(\mathcal{H}\Phi + \Phi') + \gamma(\mathcal{H}^2 - \mathcal{H}')\Phi &= \\ 4\pi G[\gamma\phi'_0\delta\phi' + (3 - 2\gamma)a^2V_{,\phi}\delta\phi], & \\ \mathcal{H}\Phi_{,i} + \Phi'_{,i} &= 4\pi G\phi'_0\delta\phi_{,i}, \\ \Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2)\Phi + (2 - \gamma)(\mathcal{H}^2 - \mathcal{H}')\Phi &= \\ 4\pi G[(2 - \gamma)\phi'_0\delta\phi' - (3 - 2\gamma)a^2V_{,\phi}\delta\phi]. & \end{aligned}$$

Problems with the scalar formulation

Perturbed equations in the newtonian gauge

- Two different combinations of the perturbed equations lead to two different single equation for the potential Φ :

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2) \Phi = \frac{2 - \gamma}{\gamma} [-k^2\Phi - 3\mathcal{H}(\mathcal{H}\Phi + \Phi')] - \frac{2V_{,\phi}a^2}{\gamma\phi'_0} (3 - 2\gamma) (\mathcal{H}\Phi + \Phi') ,$$

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2) \Phi = -k^2\Phi - 3\mathcal{H}\frac{2 - \gamma}{\gamma} (\mathcal{H}\Phi + \Phi') - \frac{2V_{,\phi}a^2}{\gamma\phi'_0} (3 - 2\gamma) (\mathcal{H}\Phi + \Phi') .$$

- These equations are identical only if $\gamma = 1$, that is, the General Relativity limit.

Problems with the scalar formulation

Perturbed equations in the newtonian gauge

- Possible solution: to add matter. In this case, all equations are compatibles for all values of γ .
- J.C. Fabris, M. Hamani Daouda and O.F. Piattella, Physics Letters B, to appear.

The Λ CDM model revisited

Another coupling for the fluid components

- Let us again consider again a two-fluid model, but with a different coupling:

$$T_x^{\mu\nu}{}_{;\mu} = \frac{\gamma - 1}{2} T^{;\nu},$$

$$T_m^{\mu\nu}{}_{;\mu} = 0.$$

- T is the total trace for the two components.

The Λ CDM model revisited

Another coupling for the fluid components

- Choose $p_m = 0$ and $p_x = -\rho_x$.
- In a *FLRW* background, the equations of motion are,

$$H^2 = \frac{8\pi G}{3} \left\{ (3 - 2\gamma)\rho_x + \frac{-\gamma + 3}{2}\rho_m \right\},$$

$$\dot{\rho}_m + 3H\rho_m = 0,$$

$$(3 - 2\gamma)\dot{\rho}_x = \frac{\gamma - 1}{2}\dot{\rho}_m.$$

- For the densities, we have:

$$\rho_m = \frac{\rho_{m0}}{a^3},$$

$$\rho_x = \frac{\rho_{x0}}{3 - 2\gamma} + \frac{\gamma - 1}{2(3 - 2\gamma)}\rho_m.$$

The Λ CDM model revisited

Another coupling for the matter components

- The final equations are:

$$H^2 = \frac{8\pi G}{3}(\rho_{x0} + \rho_m),$$
$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = 8\pi G\rho_{x0}.$$

- They are the same that those for the Λ CDM model.
- But we have in addition,

$$\rho_x = \frac{\rho_{x0}}{3 - 2\gamma} + \frac{\gamma - 1}{2(3 - 2\gamma)}\rho_m.$$

Le modèle Λ CDM retrouvé

Un autre couplage pour la matière

- At perturbative level (synchronous gauge), we find:

$$\ddot{\delta}_m + 2\frac{\dot{a}}{a}\delta_m - 4\pi G\rho_m\delta_m = 0.$$

- Again, the same equation as in the Λ CDM model.
- But, now,

$$\delta\rho_x = \frac{\gamma - 1}{2(3 - 2\gamma)}\delta\rho_m.$$

- Dark energy now agglomerates.

The Λ CDM model revisited

The spherical collapse

- Using the spherical collapse model we have:

$$\left(\frac{\dot{a}}{a_i}\right)^2 = H_i^2 \left(\Omega_p(t_i) \frac{a_i}{a} + 1 - \Omega_p(t_i) \right),$$

$$\Omega_p = 1 + \frac{2(3 - 2\gamma)}{5 - 3\gamma} \delta_m + \frac{\gamma - 1}{5 - 3\gamma} \delta_x.$$

- At non-linear level, the equivalence with the Λ CDM model must be broken.

Conclusions

- For the moment, the Λ CDM model is the best one: it reproduces the observations with few parameters.
- But, it must face difficulties, theoretical and observational, mainly at non-linear level.
- The unified models for the dark sector are very interesting, but they must face many serious drawbacks.
- These difficulties may be surmounted partially if the conservation laws are modified, as proposed by the Rastall's theory.
- The Rastall's theory may reproduce the succes of the Λ CDM model, giving at same time new features at non-linear level.
- Rastall's theory is a serious alternative? To be verified deeply.