Boson Absorption by Reissner-Nordström Black Holes

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Outline

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Introduction

- Scattering by black holes is a subject that has called much attention since long time ago, mainly in the 70's. In particular, the absorption cross section is directly related to their emission spectra.
- Today, it is believed that there is one black hole of million to billion solar masses on the center of each galaxy, further than millions stellar-mass black holes that populate them.
- Although charged black holes are not believed to be found in nature, many of their properties are very important theoretically. Many scientists are specifically interested in extreme Reissner-Nordström black holes.
- The first works based on the study of vector and tensor fields in Reissner-Nordström geometries analyzed the stability of charged black holes under these perturbations [Moncrief (1974)].

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Base Works

Here we show the computation of charged black holes absorption cross sections for massless particles of spins 0,1, and 2. We obtain results mainly for black holes with charges $|Q| = 0, 0.8M, M$. However, our computations could be generalized for any value of $0 \leq |Q| \leq M$. This presentation is based on the following works:

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Reissner Nordström Spacetimes

Charged Black Holes Configuration

The spacetime around static charged black holes is described by:

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where $f = 1 - 2M/r + Q^2/r^2$, with M being the black hole mass and Q the black hole charge.

There are basically three different possible configurations on this spacetime:

- $Q = 0 \implies$ Schwarzschild spacetime: uncharged black hole with event horizon radius $r_h = 2M$;
- $0 < |Q| < M \implies$ Typical Reissner-Nordström black hole: possess two horizons $r_\pm = M \pm \sqrt{M^2 - Q^2}$, being r_+ the external event horizon and r_- a Cauchy horizon;
- $|Q| = M \implies$ extremely charged black holes: both horizons coincide in $r_\pm = M$. This is the most interesting case and we show results mainly for it.

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High-Frequency Limit

Geodesics Approach

Null geodesics in Reissner-Nordström spacetimes behave according to:

$$\left(\frac{du}{d\phi}\right)^2 = \frac{1}{b^2} - u^2 + 2Mu^3 - Q^2u^4 \qquad (u = 1/r), \tag{1}$$

which integration yields

$$\Theta(b) = \frac{4}{\sqrt{Q^2(u_3 - u_1)(u_2 - u_0)}}\left[K(k) - F(z,k)\right] - \pi, \tag{2}$$

with $u_0 < 0$, $u_3 > u_2 \geq u_1 > 0$ being the roots of the RHS of Eq. (1). When $u_1 = u_2$, we have the critical orbit, from where it is possible to obtain the high-frequency absorption cross section:

$$\sigma_{\text{abs}}^{hf} = \pi b_c^2 = \pi \frac{\left(3M + \sqrt{9M^2 - 8Q^2}\right)^4}{8\left(3M^2 - 2Q^2 + M\sqrt{9M^2 - 8Q^2}\right)}. \tag{3}$$

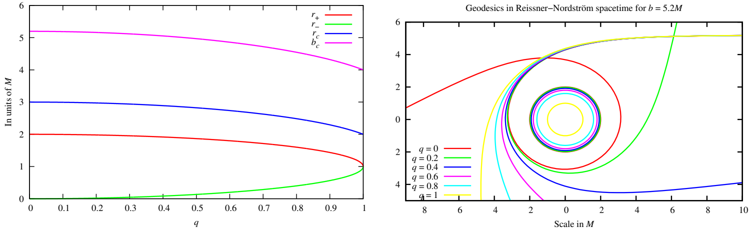
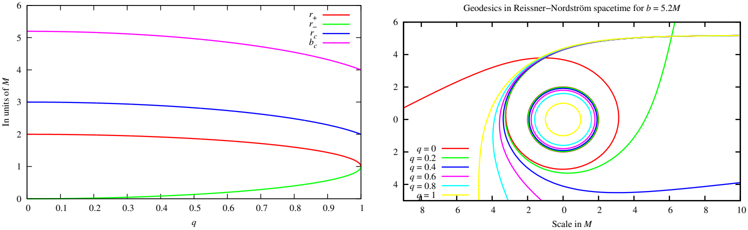


Figure: *Left*: some parameters of charged black holes; *Right*: Geodesic changing with the increasing of $q \equiv |Q|/M$.

As we increase the black hole charge intensity, it seems that the attraction of neutral particles decreases. Therefore, the absorption cross section in the high-frequency limit decreases as we increase the black hole charge:

- $q = 0 \Longrightarrow \sigma^{hf}_{\text{abs}} = 27\pi M^2$;
- $q = 1 \Longrightarrow \sigma^{hf}_{\text{abs}} = 16\pi M^2$.



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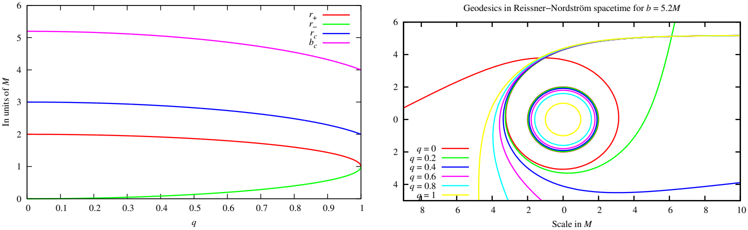

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Wave Equations

The results valid for arbitrary values of frequency are obtained through the partial wave method.

The equations we need are

Klein-Gordon

$$\Box\Phi = \nabla_\mu \nabla^\mu \Phi = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi\right) = 0. \qquad (4)$$

Einstein-Maxwell

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \qquad (5)$$

- Analytical solutions $\Longrightarrow$ low-frequency limit absorption[1];
- Numeric computations $\Longrightarrow$ absorption valid for arbitrary values of frequency.

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Massless Scalar Waves

We can expand the the massless scalar field in terms of the spherical harmonics. In this case we get a radial equation similar to the Schröndinger one:

$$\frac{d^2}{dr_*^2}\psi_{\omega l}(r_*) + \left(\omega^2 - V_{\rm RN}\right)\psi_{\omega l}(r_*) = 0, \qquad (6)$$

where r_* is the tortoise coordinate and the effective potential is

$$V_{\rm RN}(r) = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\left[\frac{2M}{r^3} - \frac{2Q^2}{r^4} + \frac{l(l+1)}{r^2}\right]. \qquad (7)$$

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Asymptotic Solution

Scalar Case

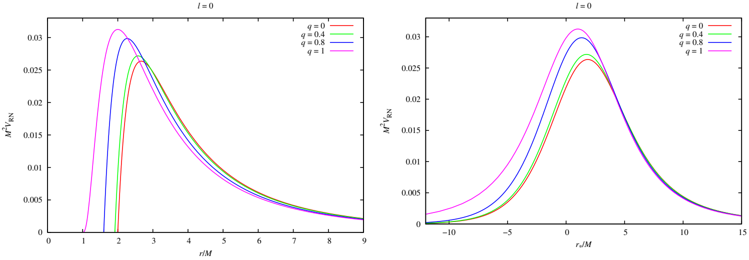

Figure: Effective potential behavior. *Right*: in terms of r; *Left*: in terms of r_*.

With the potential behavior, it is straight to determine asymptotic solutions to the radial function:

$$\psi_{\omega l}(r_*) \sim \begin{cases} A^{\text{tr}}_{\omega l} e^{-i\omega r_*} & (r_* \to -\infty); \\ A^{\text{in}}_{\omega l} e^{-i\omega r_*} + A^{\text{ref}}_{\omega l} e^{i\omega r_*} & (r_* \to \infty). \end{cases} \quad (8)$$

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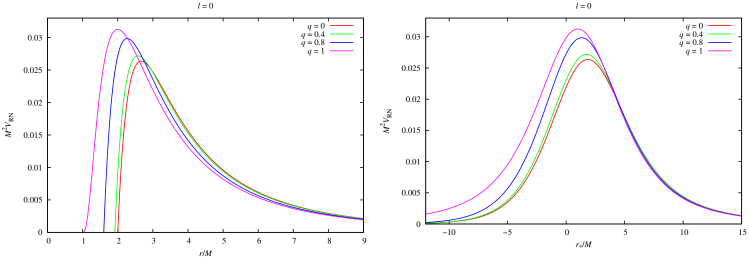

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Absorption Cross Section

Partial Wave Method

The absorption cross section is defined as:

$$\sigma_{\text{abs}} \equiv -\frac{\text{wave flux at infinity}}{\text{incident wave current}}. \tag{9}$$

This leads to:

$$\sigma_{\text{abs}} = \sum_{l=s}^{\infty} \sigma_{\text{abs}}^{(l)} = \sum_{l=s}^{\infty} \frac{\pi}{\omega^2}(2l+1)|T_{\omega l}|^2, \tag{10}$$

where s is the particle spin and $|T_{\omega l}|^2 = |A_{\omega l}^{\text{tr}}|^2/|A_{\omega l}^{\text{in}}|^2$ is the rate of absorbed energy.

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Absorption Cross Section

Low-frequency Regime

In order to obtain the low-frequency absorption cross section, we have to solve the radial equation for $\omega = 0$ and then match the solution with the asymptotic forms in the regime $\omega r \ll 1$.

We also have to to:

- choose previously the black hole charge (here we show the development just for the extreme Reissner-Nordström case);
- find a better approximation for $r \gg r_+$. This also helps to obtain better numerical results.

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Low-Frequency Solutions

The general solution for $r \gg r_+$ is

$$\psi_{\omega l}(r_*) \approx \omega r_* \left[(-i)^{l+1} A^{\text{in}}_{\omega l} h_l^{(1)*}(\omega r_*) + i^{l+1} A^{\text{ref}}_{\omega l} h_l^{(1)}(\omega r_*)\right], \qquad (11)$$

that in the regime $\omega r_* \ll 1$ reduces to:

$$\psi_{\omega l} \approx (-i)^{l+1} A^{\text{in}}_{\omega l} \frac{2^{l+1} l!}{(2l+1)!} (\omega r)^{l+1}. \qquad (12)$$

For extreme Reissner-Nordström black holes, the effective potential falls slower to zero. Then, near the event horizon, it is possible to obtain:

$$\psi_{\omega l}(r_*) \approx i^{l+1} A^{\text{tr}}_{\omega l} \sqrt{-\frac{\pi \omega r_*}{2}} H^{(1)}_{l+1/2}(-\omega r_*), \qquad (13)$$

which, in the low-frequency limit becomes:

$$\psi_{\omega l} \approx i^l A^{\text{tr}}_{\omega l} \frac{\Gamma(l+1/2)}{\sqrt{\pi}} \left(\frac{2}{M^2 \omega}\right)^l (r-M)^l. \qquad (14)$$

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By the other side, the solution of the radial equation when $\omega = 0$ is

$$\psi_{0l} = b_1 r(r-M)^l + b_2 r(r-M)^{-l-1}. \tag{15}$$

- Taking the behavior of Eq. (15) for $r \gg r_+$, and comparing with the asymptotic solution at the low-frequency limit, we conclude that:

$$\psi_{\omega l} \approx (-i)^{l+1} A^{\text{in}}_{\omega l} \frac{2^{l+1} l!}{(2l+1)!} \omega^{l+1}\, r(r-M)^l. \tag{16}$$

- Comparing this result with the approximation we obtained near the event horizon for low frequencies, we get:

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The absorption cross section at low frequencies can now be directly obtained:

$$\sigma_{abs}^{ext} \approx \frac{\pi}{\omega^2}\sum_{l=0}^{\infty}(2l+1)|T_{\omega l}|^2$$

$$\sigma_{abs}^{ext} \approx \pi M^2 \sum_{l=0}^{\infty}(2l+1)\left[\frac{2^{2l+1}l!\,\sqrt{\pi}}{(2l+1)!\Gamma(l+1/2)}\right]^2 (M\omega)^{4l}. \quad (18)$$

The non-negligible contribution to the total absorption cross section comes from the $l = 0$ mode. In this case we obtain:

$$\sigma_{abs}^{ext} \approx \sigma_{abs}^{ext(0)} \approx 4\pi M^2 = 4\pi r_+^2 = A_{ext}. \quad (19)$$

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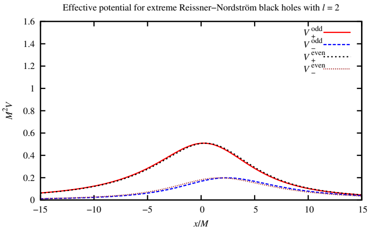
Non-Zero Spin Cases

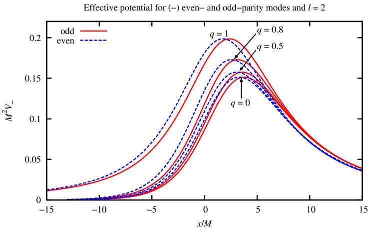
Decoupled Equations

It is not possible to obtain separated equations for electromagnetic and gravitational waves in the Reissner-Nordström spacetime. However, it is possible to obtain decoupled equations if we define two new functions. The equations are

$$\frac{d^2}{dr_*^2}\varphi_\pm^\lambda + \left(\omega^2 - V_\pm^\lambda\right)\varphi_\pm^\lambda = 0, \tag{20}$$

where λ stands for the polarization (axial, polar).





Decoupling Function

The functions $\varphi^{\lambda}_{\pm}$ are related to the electromagnetic and gravitational perturbations respectively as:

$$F^{\lambda} = \varphi^{\lambda}_{+} \cos\psi - \varphi^{\lambda}_{-} \sin\psi, \quad (21)$$

$$G^{\lambda} = \varphi^{\lambda}_{+} \sin\psi + \varphi^{\lambda}_{-} \cos\psi, \quad (22)$$

where

$$\sin(2\psi) = -2PQ\frac{[(l-1)(l+2)]^{1/2}}{\Omega}, \quad (23)$$

with

$$\Omega = \left[9M^2 + 4Q^2(l-1)(l+2)\right]^{1/2}. \quad (24)$$

$P = +1$ (-1) for polar (axial) perturbations.

Asymptotic Conditions

The potential behaviors tell us that:

$$\varphi_{\pm}^{\lambda} \propto \begin{cases} e^{-i\omega r_*} + A_{\pm}^{\lambda} e^{i\omega r_*}, & (r_* \to +\infty); \\ B_{\pm}^{\lambda} e^{-i\omega r_*}, & (r_* \to -\infty). \end{cases} \tag{25}$$

We have two different problems to deal with:

- The first is when the incident wave is purely electromagnetic. In this case, we have at infinity:

$$F^{\lambda} \approx F_{\text{in}}^{\lambda} e^{-i\omega r_*} + F_{\text{out}}^{\lambda} e^{i\omega r_*}; \tag{26}$$

$$G^{\lambda} \approx G_{\text{out}} e^{i\omega r_*}. \tag{27}$$

- The second case is for a incident wave that is purely gravitational:

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In both cases, part of the incident wave will be converted into the other radiation type. Therefore, we can define the conversion coefficient:

$$C^{\lambda}_{\omega l} = G^{\lambda}_{\text{out}}/F^{\lambda}_{\text{in}} = F^{\lambda}_{\text{out}}/G^{\lambda}_{\text{in}} = [\sin(2\psi)/2](A^{\lambda}_{+} - A^{\lambda}_{-}). \qquad (30)$$

The amount of converted energy will then be $|C^{\lambda}_{\omega l}|^2$.
The reflection and transmission coefficients are

- For pure electromagnetic incoming wave:

$$R^{e,\lambda}_{\omega l} \equiv F^{\lambda}_{\text{out}}/F^{\lambda}_{\text{in}} = A^{\lambda}_{+}\cos^2\psi + A^{\lambda}_{-}\sin^2\psi, \qquad (31)$$

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Absorption Cross Section

The absorption cross section will have the following forms:

- Electromagnetic

$$\sigma^{e}_{\text{abs}} = \sum_{l=1}^{\infty} \sigma^{e(l)}_{\text{abs}} = \frac{\pi}{2\omega^2} \sum_{l=1}^{\infty} (2l+1) \sum_{\lambda} |T^{e,\lambda}_{\omega l}|^2 \quad (35)$$

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The procedure to find the absorption cross section here is very similar to the one for the scalar case. We consider:

- extreme Reissner-Nordström black holes. (Results related to Schwarzschild black holes can be found in [Page (1976)].);
- only axial modes, since the absorption must no depend on the wave polarization.

In this cases, we have:

$$B_{+,\omega l} = \frac{2^{2l+1}(l+2)(l!)^2}{l\,[(2l+1)!]^2}(M\omega)^{2l+2}, \qquad (l \geq 1) \qquad (37)$$

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Electromagnetic Case

Using the previous coefficients, it is possible to obtain that:

$$|T^{e,\lambda}_{\omega,l=1}|^2 \approx \frac{4}{9}(\omega M)^8, \tag{39}$$

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We call attention to the fact that the $l = 1$ and $l = 2$ modes give contribution of the same order to the low-frequency electromagnetic absorption cross section (no other case known!). Considering these two main contributions, the absorption cross section reads:

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We could directly compute the low-frequency gravitational absorption cross section from the results we have for $B_{\pm}^{\lambda}$. But, there is a way to relate electromagnetic and gravitational absorption cross sections for the extreme Reissner-Nordström case. In [Onozawa et al. (1996)] it was shown that

$$V_{+,l}(r_*) = V_{-,l+1}(-r_*). \tag{42}$$

This results to:

$$|B_{+,l}|^2 = |B_{-,l+1}|^2. \tag{43}$$

With this relation, it can be shown that electromagnetic and gravitational absorption cross sections for extremely charged black holes are equal (not just in the low-frequency limit)![2] They read:

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that should be compared with

$$\sigma^{e}_{\text{abs}} \approx \frac{64}{3}\pi M^2 (M\omega)^2, \qquad (Q = 0) \tag{46}$$

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$$\sigma^{g}_{\text{abs}} \approx \frac{256}{45}\pi M^2 (M\omega)^4, \qquad (Q = 0). \tag{47}$$

We should emphasize that this equality will be valid for arbitrary values of the wave frequency. However, it only holds for the total absorption cross section. The results for the partial absorption cross sections are different. This is easy to note if we remember that the $l = 1$ mode exists in the electromagnetic case, but is absent in the gravitational case.

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Outline

Numerical Computations

Analytical solutions to the main equations can only be obtained in the low-frequency regime. Therefore, to obtain results for

- low frequencies, we use the partial wave method together with the radial equation solution for $\omega = 0$;
- high frequencies, we use geodesics analysis;
- arbitrary frequencies, we have to develop the main equations numerically. Our method bases on developing the radial equations solutions from $r = (1 + \varepsilon)r_+$, being ε a very small number, to $r \gg r_+$. The result is then matched with the asymptotic solutions in order to obtain the appropriate coefficients.

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Absorption Cross Section

Scalar Case

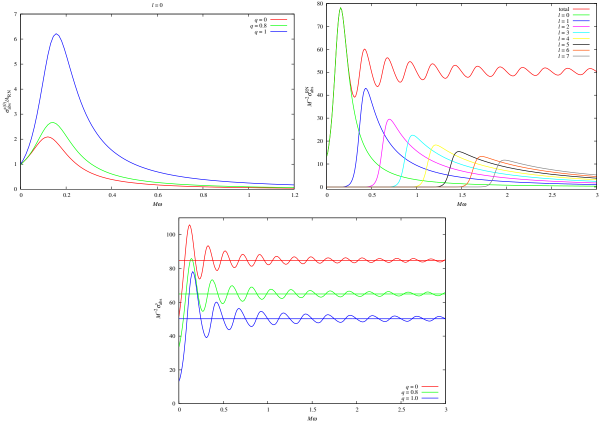

Figure: Scalar absorption cross section for Reissner-Nordström black holes. *Top-left*: partial absorption for the $l = 0$ mode in terms of the black hole area; *Top-right*: total and partial absorption in the extreme case; *Bottom*: total absorption for $q = 0, 0.8, 1$.

Coefficients

Conversion

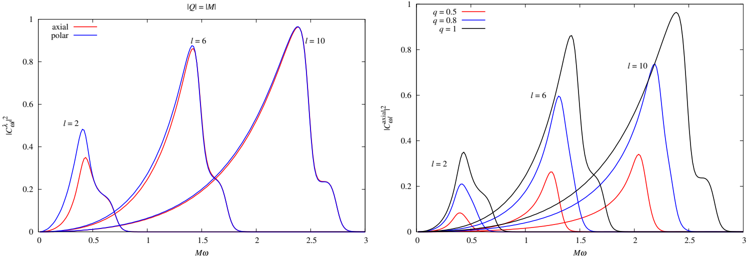

Figure: Coefficients for the conversion of waves in Reissner-Nordström spacetimes [Chandrasekhar (1983)]. *Left*: comparison for the two kinds of polarizations around extremely charged black holes. *Right*: axial modes for $q = 0.5, 0.8, 1$ [Olson & Unruh (1974)].

Of course, there is no conversion around Schwarzschild black holes, neither for the $l = 1$ mode.

Coefficients

Reflection & Transmission

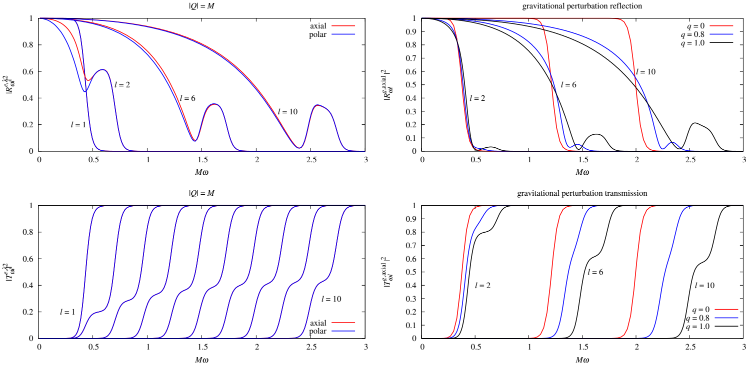

Figure: Reflection and transmission coefficients for (*left*) electromagnetic radiation for the two different polarizations and (*right*) gravitational axial perturbations for $q = 0, 0.8, 1$.

Main Features

Critical Frequency

The interesting features (concavity changes) present in the coefficients are related to the critical impact parameter. In the semiclassical description of scattering, it is possible to write [Ford & Wheeler (1959)]:

$$b = (l + 1/2)/\omega. \tag{48}$$

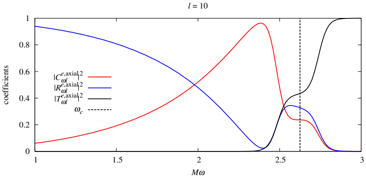

Figure: We can define the critical frequency as $\omega_c = (l + 1/2)/b_c$ (for extreme Reissner-Nordström, $b_c = 4M$). Therefore, the main features in the coefficients happen around ω_c.

Absorption Cross Section

Extreme Case

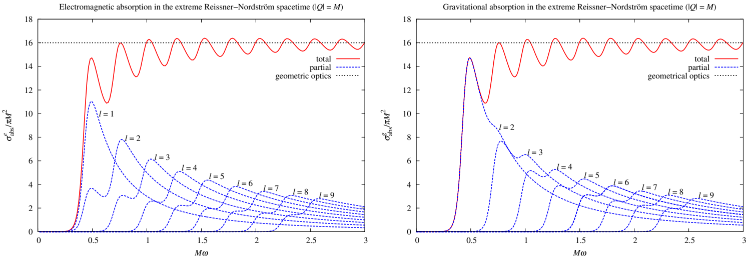

Figure: Total and partial absorption cross section comparison of extremely charged black holes for (*left*) the electromagnetic case and (*right*) the gravitational case.

Although the partial absorption cross sections have different values, their sum give the same result.

Absorption Cross Section

Different Charges

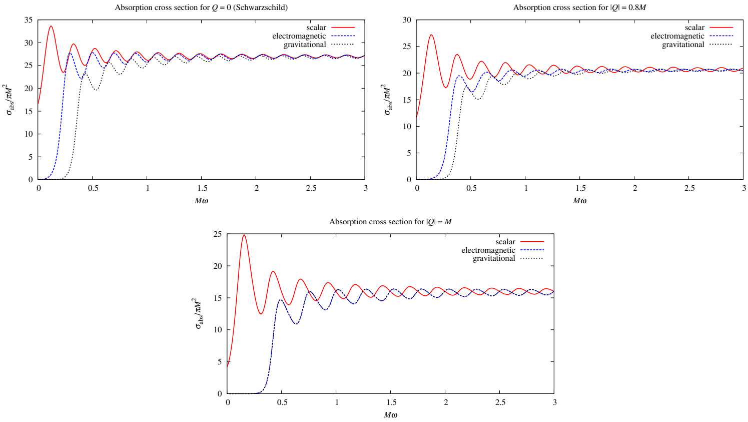

Figure: Bosonic absorption cross section of charged black holes for (*top-left*) $Q = 0$, (*top-right*) $q = 0.8$, and (*bottom*) $q = 1$.

Outline

Summary

- Changing the black hole charge means changing the main parameters of the spactime, as the radius of the critical orbit for null geodesics. These parameters changes have strong effects on the absorption process.
- The conversion of waves is a very interesting subject. It also plays very important roles on the absorption process. For extremely charged black holes, it leads to a very interesting result, that is the equality between electromagnetic and gravitational absorption cross sections.
- Our numeric results are in excellent agreement with the analytical ones.
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  - Electromagnetic and gravitational scattering process (in progress);
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