CENTRA Seminar

IST – Lisbon, 2 November 2012

Black-Hole Bombs and Photon-Mass Bounds

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http://blackholes.ist.utl.pt

PP, Cardoso, Gualitieri, Berti, Ishibashi

Phys.Rev.Lett. 109 (2012) 131102

Phys.Rev.D in press

Goal

Dynamics of light massive fields around spinning black holes

- **Dark matter candidates**
- **BHs as particle physics labs → simple objects, no coupling**
- **Open problems in BH perturbation theory**

BH superradiance

- Simple BH-matter interaction
- Kerr BH: wave amplified if

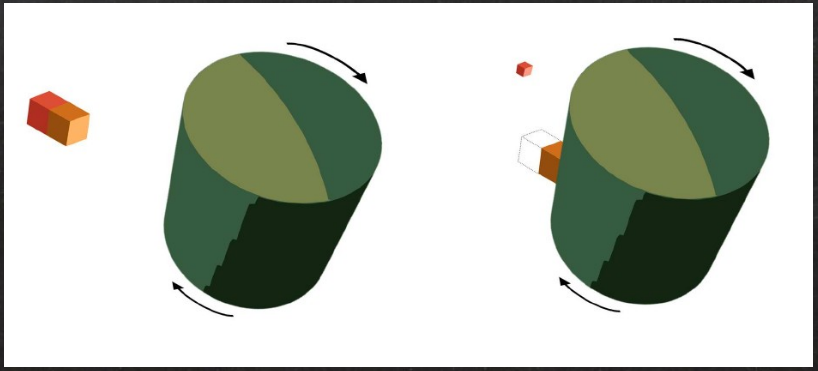
$$\omega < m\Omega_H$$

- Linear effect, but peek to backreaction
- Requires dissipation → needs an event horizon

[Thorne, Price, Macdonald's book]

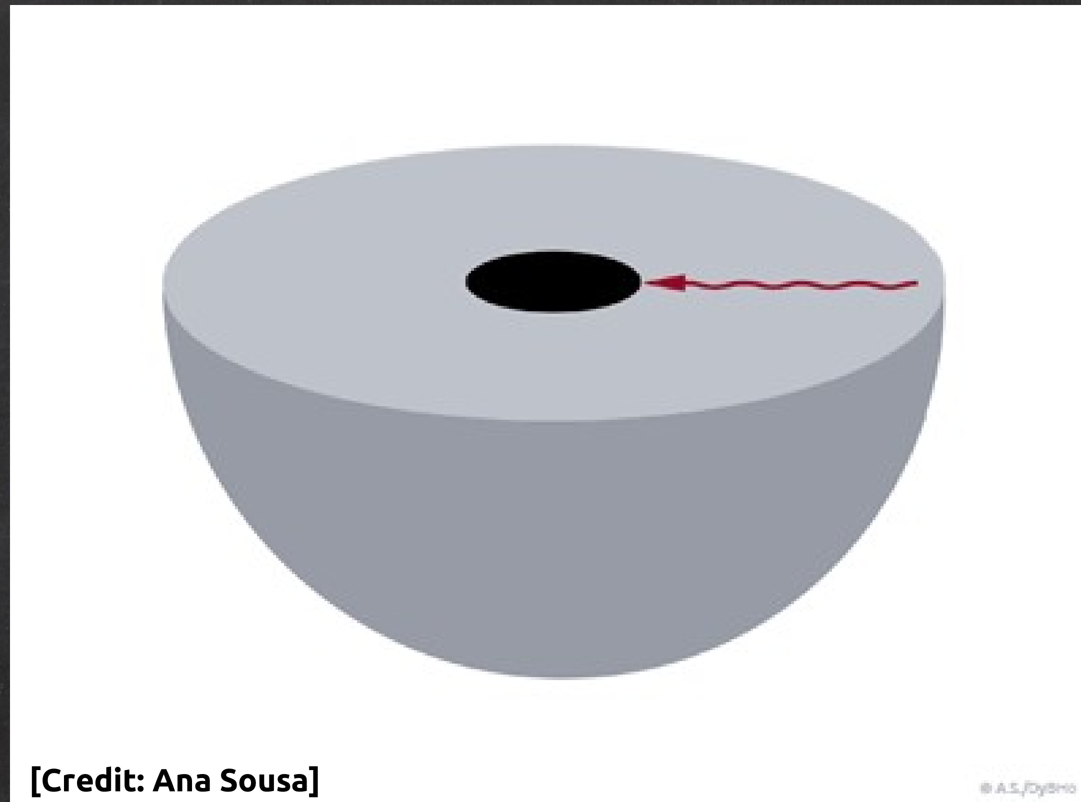
[Richartz et al. 2008]

[Cardoso & Pani, 2012]



Zel'dovich effect. [Credit: Ana Sousa]

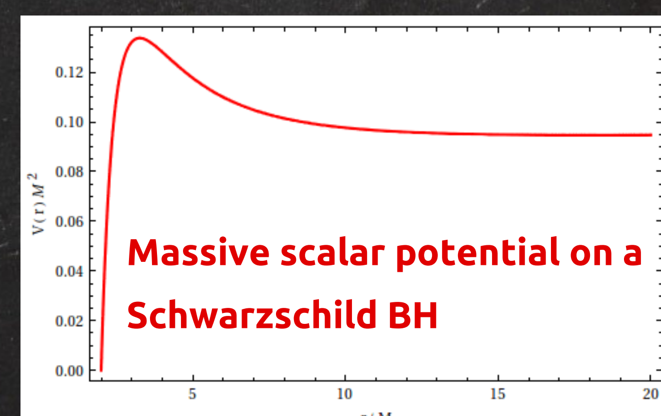
BH bomb [Press and Teukolsky '72]



- *"Nature may provide its own mirrors"*

  [Cardoso, Dias, Lemos, Yoshida, 2004]

  - AdS boundaries
  - Massive fields



Scalar fields & superradiance

$$\Box\phi - \mu^2\phi = 0$$

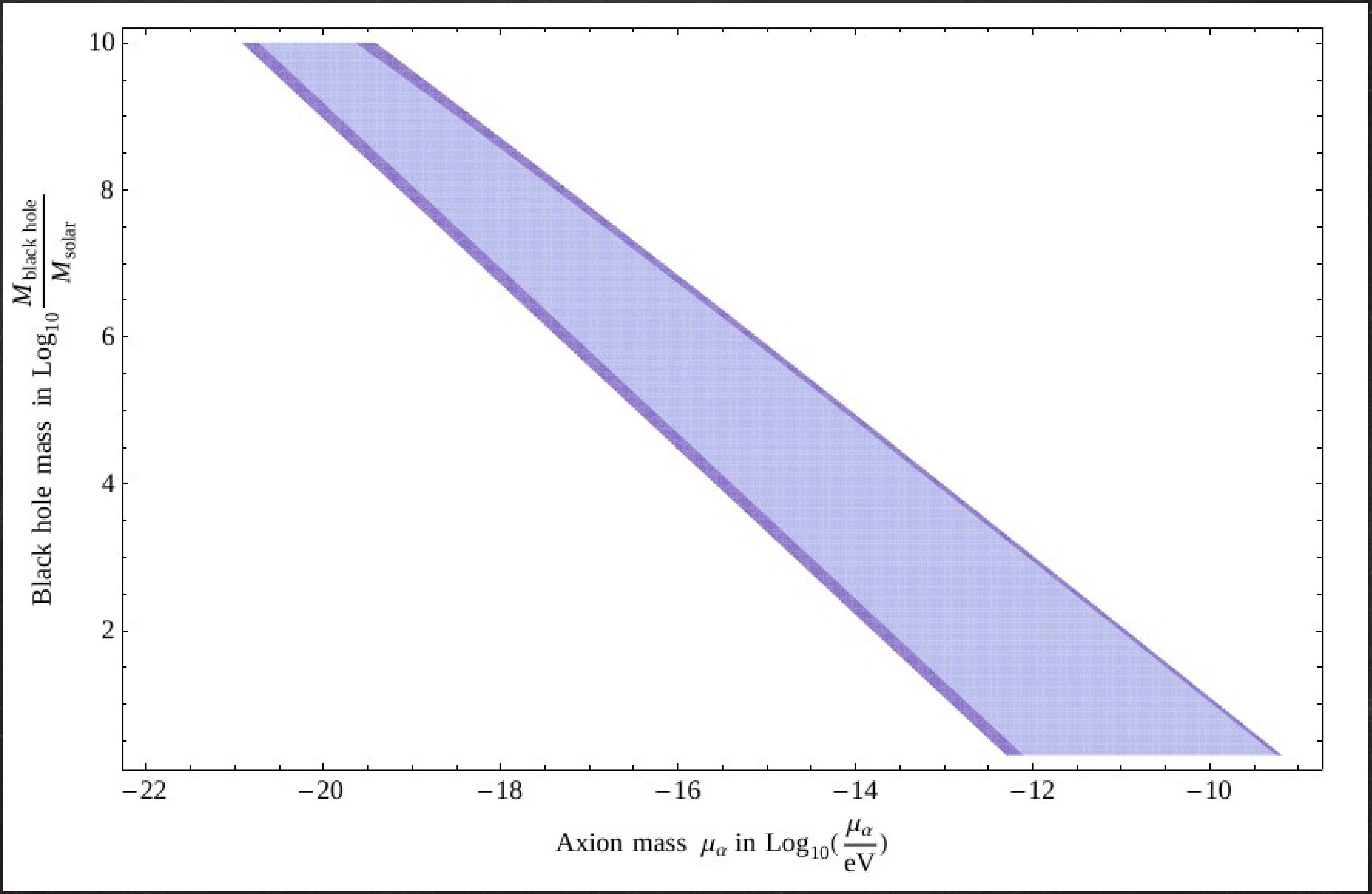
- Massive fields around spinning BHs are unstable
- Instability is well-studied in the scalar case
  - Strongest instability when μM ~1
    - Primordial BHs (10^{14} - 10^{23} kg) and SM particles
    - Ultra-light particles (m ~ 10^{-21} - 10^{-9} eV) and massive BHs
  - Axiverse scenario (QCD axions, Peccei-Quinn mechanism, etc...)
  - Bosenova (numerical simulations are challenging!)

[Damour et al. 1976]
[Detweiler, 1980]
[Earley & Zouros 1979]
[Cardoso & Yoshida 2005]
[Dolan 2007]
[Rosa 2010, 2012]

[Arvanitaki et al. 2010-2011]

[Kodama & Yoshino 2011-2012]
[Witek et al, in preparation]

Scalar fields & superradiance



[Arvanitaki et al. 2010-2011]

Vector fields & superradiance

$$\nabla_\sigma F^{\sigma\nu} - \mu^2 A^\nu = 0$$

- The massive spin-1 around Kerr BHs still uncharted territory
  - Massive hidden U(1) vector fields are generic features of extensions of SM [Goodsel et al. 2009]
  - Non-rotating case: conjecture of a stronger instability? [Rosa & Dolan 2011]
  - Perturbation equations do not separate (?) → set of PDEs [Konoplya 2006] [Herdeiro, Sampaio, Wang 2012]
- Approaches
  - Full nonlinear evolution
  - Linear time evolution on a fixed background
  - Frequency domain on a fixed background

BH perturbations. Spherical symmetry

[Kokkotas & Schmidt 1998]
[Berti et al. 2009]
[Konoplya & Zhidenko 2011]

$$ds^2 = \underbrace{-f(r)dt^2 + h(r)^{-1}dr^2 + r^2 d\Omega_2}_{\text{background}} + \underbrace{(\delta_{\rm RW} g_{\mu\nu})e^{-i\omega t}dx^\mu dx^\nu}_{\text{perturbations}}$$

- Regge-Wheeler formalism:

Polar — Axial

$$\|\delta_{\rm RW} g_{\mu\nu}\| = \begin{bmatrix} f(r)H_0(r)Y_{lm} & H_1(r)Y_{lm} & -h_0(r)\frac{1}{\sin\theta}\frac{\partial Y_{lm}}{\partial\varphi} & h_0(r)\sin\theta\frac{\partial Y_{lm}}{\partial\theta} \\ * & \frac{H_2(r)Y_{lm}}{h(r)} & -h_1(r)\frac{1}{\sin\theta}\frac{\partial Y_{lm}}{\partial\varphi} & h_1(r)\sin\theta\frac{\partial Y_{lm}}{\partial\theta} \\ * & * & r^2K(r)Y_{lm} & 0 \\ * & * & * & r^2\sin^2\theta K(r)Y_{lm} \end{bmatrix}$$

- The axial and polar sectors decouple:

$$\mathcal{A}_\ell = 0 \qquad \mathcal{P}_\ell = 0$$

Linear equations involving axial *or* polar perturbations only

- Solved with suitable boundary conditions → eigenvalue problem

- Any spherically symmetric background, any theory, any field

Non-separable (?) problems

- Separability in Kerr is almost a miracle!
- Four dimensions
  - Massive vector (Proca) fields on a Kerr background
  - Gravito-EM perturbations of Kerr-Newman BHs
  - Rotating objects in alternative theories
- Higher dimensions
  - Myers-Perry BHs
  - Other rotating solutions
- Stability, greybody factors, quasinormal modes?

[Teukolsky ~ 1973]
[Teukolsky and Press]
[Chandra's book]

Part I

Perturbations of slowly-rotating BHs:

General framework

Method. *Perturbations of slowly rotating spacetimes*

- **Slowly-rotating background metric:**

$$ds_0^2 = -F(r)dt^2 + B(r)^{-1}dr^2 + r^2 d^2\Omega - 2\varpi(r)\sin^2\theta d\varphi dt$$

- **Expand any equation (scalar, vector, tensor...) in spherical harmonics**

[Kojima 1992, 1993, 1997]

$$\delta X_{\mu_1\ldots}(t,r,\vartheta,\varphi) = \delta X_{\ell m}^{(i)}(r)\mathcal{Y}_{\mu_1\ldots}^{\ell m\,(i)} e^{-i\omega t}$$

- **For any metric, any theory and any perturbations: system of radial ODEs:**

$$\mathcal{A}_{\ell m} + \tilde{a}m\bar{\mathcal{A}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{P}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{P}}_{\ell+1m}) = 0$$

$$\mathcal{P}_{\ell m} + \tilde{a}m\bar{\mathcal{P}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{A}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{A}}_{\ell+1m}) = 0$$

- **Zeeman splitting**
- **Laporte-like selection rule**
- **Propensity rule**

$$\mathcal{Q}_{\ell m} = \sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}}$$

$\mathcal{A}, \mathcal{P} \rightarrow$ **Linear combinations of axial and polar perturbations**

Perturbative scheme

$0 = \mathcal{A}_\ell$

Zeroth order: decoupled

$0 = \mathcal{P}_\ell$

$\mathcal{P}_{L+3}$

$\mathcal{A}_{L+2}$

$\mathcal{P}_{L+1}$

$\mathcal{A}_L$

$\mathcal{P}_{L-1}$

$\mathcal{A}_{L-2}$

$\mathcal{P}_{L-3}$

Perturbative scheme

$$0 = \mathcal{A}_\ell$$

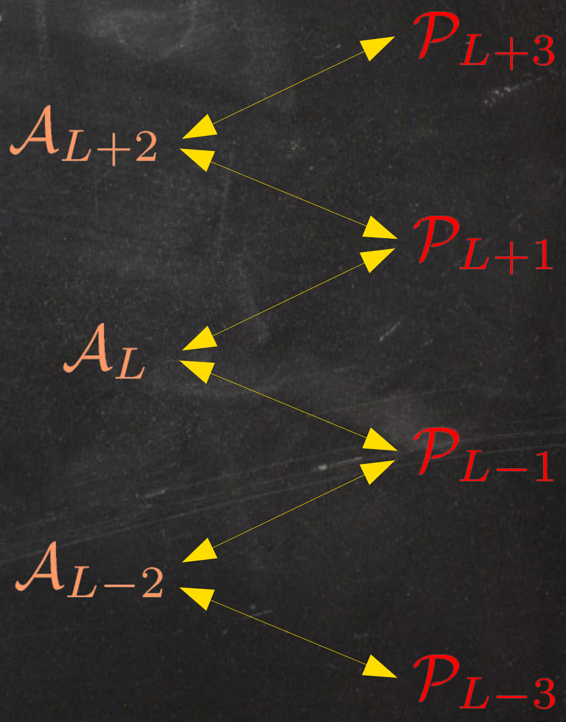
$$+\tilde{a} m \bar{\mathcal{A}}_\ell + \tilde{a}(\mathcal{Q}_\ell \tilde{\mathcal{P}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{P}}_{\ell+1})$$

$$0 = \mathcal{P}_\ell$$

$$+\tilde{a} m \bar{\mathcal{P}}_\ell + \tilde{a}(\mathcal{Q}_\ell \tilde{\mathcal{A}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{A}}_{\ell+1})$$

Zeroth order: decoupled

First order: polar-axial l±1



Perturbative scheme

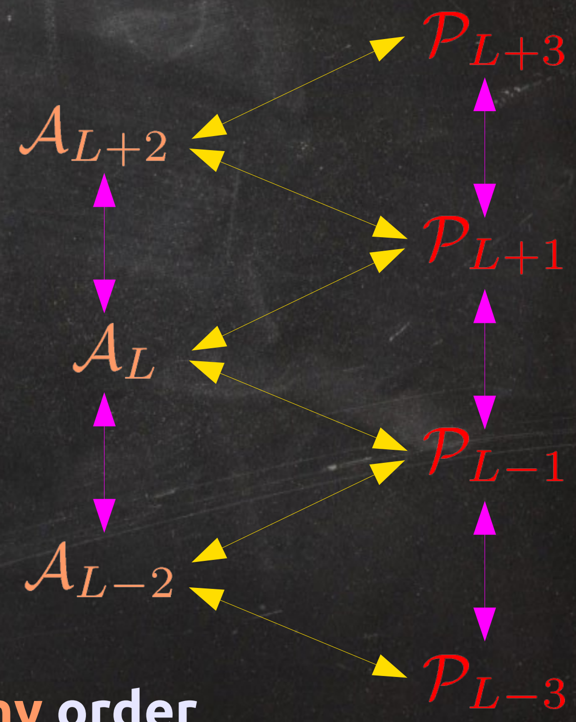
$$0 = \mathcal{A}_\ell$$
$$+\tilde{a}m\bar{\mathcal{A}}_\ell + \tilde{a}(\mathcal{Q}_\ell\tilde{\mathcal{P}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{P}}_{\ell+1})$$
$$+\tilde{a}^2\left[\hat{\mathcal{A}}_\ell + \mathcal{Q}_{\ell-1}\mathcal{Q}_\ell\breve{\mathcal{A}}_{\ell-2} + \mathcal{Q}_{\ell+2}\mathcal{Q}_{\ell+1}\breve{\mathcal{A}}_{\ell+2}\right]$$

Zeroth order: decoupled

First order: polar-axial l±1

Second order: l±2

$$0 = \mathcal{P}_\ell$$
$$+\tilde{a}m\bar{\mathcal{P}}_\ell + \tilde{a}(\mathcal{Q}_\ell\tilde{\mathcal{A}}_{\ell-1} + \mathcal{Q}_{\ell+1}\tilde{\mathcal{A}}_{\ell+1})$$
$$+\tilde{a}^2\left[\hat{\mathcal{P}}_\ell + \mathcal{Q}_{\ell-1}\mathcal{Q}_\ell\breve{\mathcal{P}}_{\ell-2} + \mathcal{Q}_{\ell+2}\mathcal{Q}_{\ell+1}\breve{\mathcal{P}}_{\ell+2}\right]$$

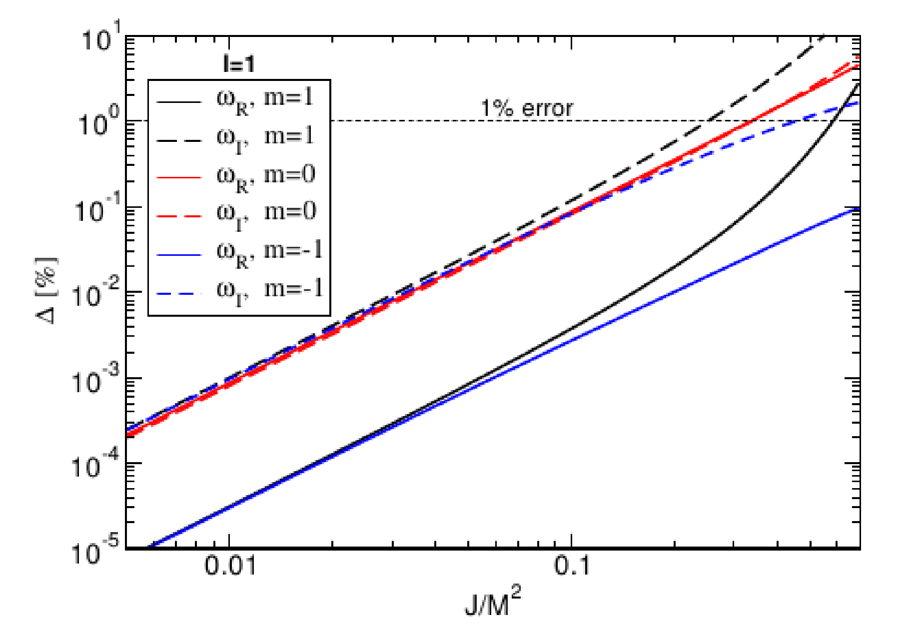


- **Generic: any metric, any perturbation, any theory, any order**

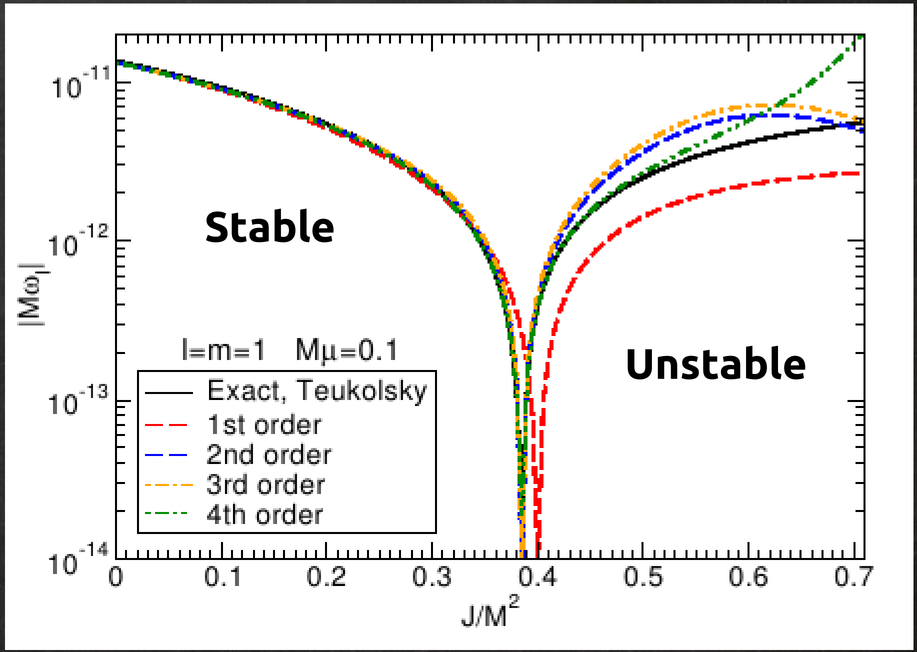
Slow-rotation method. *tests*

Numerics in the slow-rotation scheme are "easy" to perform

- direct integration (bound states)
- continued fractions (QNMs, bound states)
- Breit-Wigner method (QNMs, bound states)
- WKB (?)



EM (massless) QNMs of a Kerr BH



Massive scalar modes of a Kerr BH

- Good results even for moderately large spin

Part II

Proca perturbations of a Kerr BH

Proca equation


Alexandru Proca

$$\nabla_\sigma F^{\sigma\nu} - \mu^2 A^\nu = 0 \qquad m = \hbar\mu/c$$

Mass

$$\Longrightarrow \quad \nabla_\sigma A^\sigma = 0\,, \qquad \Box A^\nu - \mu^2 A^\nu = 0$$

- (apparently) nonseparable in a Kerr background
- Note that EM (massless) perturbations in Kerr-(A)dS are separable!

$$\nabla_\sigma F^{\sigma\nu} = 0 \quad \Longrightarrow \quad \Box A^\nu - \nabla^\nu(\nabla_\sigma A^\sigma) + \Lambda A^\nu = 0$$

- However → role of the gauge freedom → massless fields propage 2 DOF
- Proca eq. implies Lorenz condition → no more freedom → 3 DOF
- Equivalent to gravitational theories with higher-curvature terms:

[Buchdahl '70]

[Vitagliano, Sotiriou, Liberati 2010]

$$\mathcal{L} = \sqrt{-g}\left(R + \alpha R_{[ab]} R^{[ab]}\right)$$

Proca in slowly-rotating Kerr

- The Proca problem becomes tractable in the slow-rotation approximation
- Let us decompose the Proca field in vector spherical harmonics:

$$Y_a^{\ell m} = (\partial_\vartheta Y^{\ell m}, \partial_\varphi Y^{\ell m}) \qquad S_a^{\ell m} = \left(\frac{1}{\sin\vartheta}\partial_\varphi Y^{\ell m}, -\sin\vartheta\partial_\vartheta Y^{\ell m}\right)$$

$$\delta A_\mu(t,r,\vartheta,\varphi) = \underbrace{\sum_{\ell,m}\begin{bmatrix} 0 \\ 0 \\ u_{(4)}^{\ell m}(t,r) S_a^{\ell m}\end{bmatrix}}_{\text{Axial parity}} + \underbrace{\sum_{\ell,m}\begin{bmatrix} u_{(1)}^{\ell m}(t,r) Y^{\ell m} \\ u_{(2)}^{\ell m}(t,r) Y^{\ell m} \\ u_{(3)}^{\ell m}(t,r) Y_a^{\ell m}\end{bmatrix}}_{\text{Polar parity}}$$

- One spurious degree of freedom → three physical perturbation functions

Proca in slowly-rotating Kerr

- The angular part can be eliminated using the orthogonality properties of the spherical harmonics. E.g. :

$$\delta\Pi_t \equiv (A^{(0)}_{\ell m} + \tilde{A}^{(0)}_{\ell m}\cos\vartheta)Y^{\ell m} + B^{(0)}_{\ell m}\sin\vartheta\partial_\vartheta Y^{\ell m} = 0$$

- We compute the following integral:

$$\int \delta\Pi_I Y^{*\,\ell m} d\Omega\,, \quad (I = t, r, L)$$

- Useful properties of spherical harmonics:

$$\cos\vartheta Y^{\ell m} = \mathcal{Q}_{\ell+1m} Y^{\ell+1m} + \mathcal{Q}_{\ell m} Y^{\ell-1m}$$

$$\mathcal{Q}_{\ell m} = \sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}}$$

$$\sin\vartheta\partial_\vartheta Y^{\ell m} = \mathcal{Q}_{\ell+1m}\ell Y^{\ell+1m} - \mathcal{Q}_{\ell m}(\ell+1)Y^{\ell-1m}$$

Proca in slowly-rotating Kerr

- **From nonseparated equations:**

$$\delta\Pi_t \equiv (A^{(0)}_{\ell m} + \tilde{A}^{(0)}_{\ell m}\cos\vartheta)Y^{\ell m} + B^{(0)}_{\ell m}\sin\vartheta\partial_\vartheta Y^{\ell m} = 0$$

- **To radial ODEs:**

$$A^{(I)}_{\ell m} + \mathcal{Q}_{\ell m}\left[\tilde{A}^{(I)}_{\ell-1m} + (\ell-1)B^{(I)}_{\ell-1m}\right] + \mathcal{Q}_{\ell+1m}\left[\tilde{A}^{(I)}_{\ell+1m} - (\ell+2)B^{(I)}_{\ell+1m}\right] = 0$$

- **The system of ODEs has the general form:**

$$\mathcal{A}_{\ell m} + \tilde{a}m\bar{\mathcal{A}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{P}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{P}}_{\ell+1m}) = 0$$

$$\mathcal{P}_{\ell m} + \tilde{a}m\bar{\mathcal{P}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{A}}_{\ell-1m} + \mathcal{Q}_{\ell+1m}\tilde{\mathcal{A}}_{\ell+1m}) = 0$$

Proca in SR Kerr. Field equations

- **Polar** and **axial** sector are **coupled**:

$$\begin{cases} \hat{\mathcal{D}}_2 u^\ell_{(2)} - \dfrac{2F}{r^2}\left(1-\dfrac{3M}{r}\right)\left[u^\ell_{(2)} - u^\ell_{(3)}\right] = \\ \quad = \dfrac{2\tilde{a}M^2 m}{\Lambda r^5 \omega}\left[\Lambda\left(2r^2\omega^2+3F^2\right)u^\ell_{(2)} + 3F\left(r\Lambda F {u'}^\ell_{(2)} - \left(r^2\omega^2+\Lambda F\right)u^\ell_{(3)}\right)\right] \\ \quad - \dfrac{6i\tilde{a}M^2F\omega}{\Lambda r^3}\left[(\ell+1)\mathcal{Q}_{\ell m}u^{\ell-1}_{(4)} - \ell\mathcal{Q}_{\ell+1m}u^{\ell+1}_{(4)}\right] \\ \hat{\mathcal{D}}_2 u^\ell_{(3)} + \dfrac{2F\Lambda}{r^2}u^\ell_{(2)} = \dfrac{2\tilde{a}M^2m}{r^5\omega}\left[2r^2\omega^2u^\ell_{(3)} + 3rF^2{u'}^\ell_{(3)} - 3\left(\Lambda+r^2\mu^2\right)Fu^\ell_{(2)}\right] \\ \hat{\mathcal{D}}_2 u^\ell_{(4)} - \dfrac{4\tilde{a}M^2m\omega}{r^3}u^\ell_{(4)} = -\dfrac{6i\tilde{a}M^2F}{r^5\omega}\left[(\ell+1)\mathcal{Q}_{\ell m}\,\psi^{\ell-1} - \ell\mathcal{Q}_{\ell+1m}\,\psi^{\ell+1}\right] \end{cases}$$

- Where we have used the Lorenz condition and defined:

$$\hat{\mathcal{D}}_2 = \frac{d^2}{dr_*^2} + \omega^2 - F\left[\frac{\ell(\ell+1)}{r^2}+\mu^2\right], \qquad \psi^\ell = \left(\Lambda + r^2\mu^2\right)u^\ell_{(2)} - (r-2M){u'}^\ell_{(3)}$$

Proca in SR Kerr. *Field eqs. at second order*

- **System of second order ODEs:**

$$\mathcal{D}_A\Psi^\ell_A + V_A\Psi^\ell_A = 0 \qquad \mathcal{D}_P\Psi^\ell_P + V_P\Psi^\ell_P = 0$$

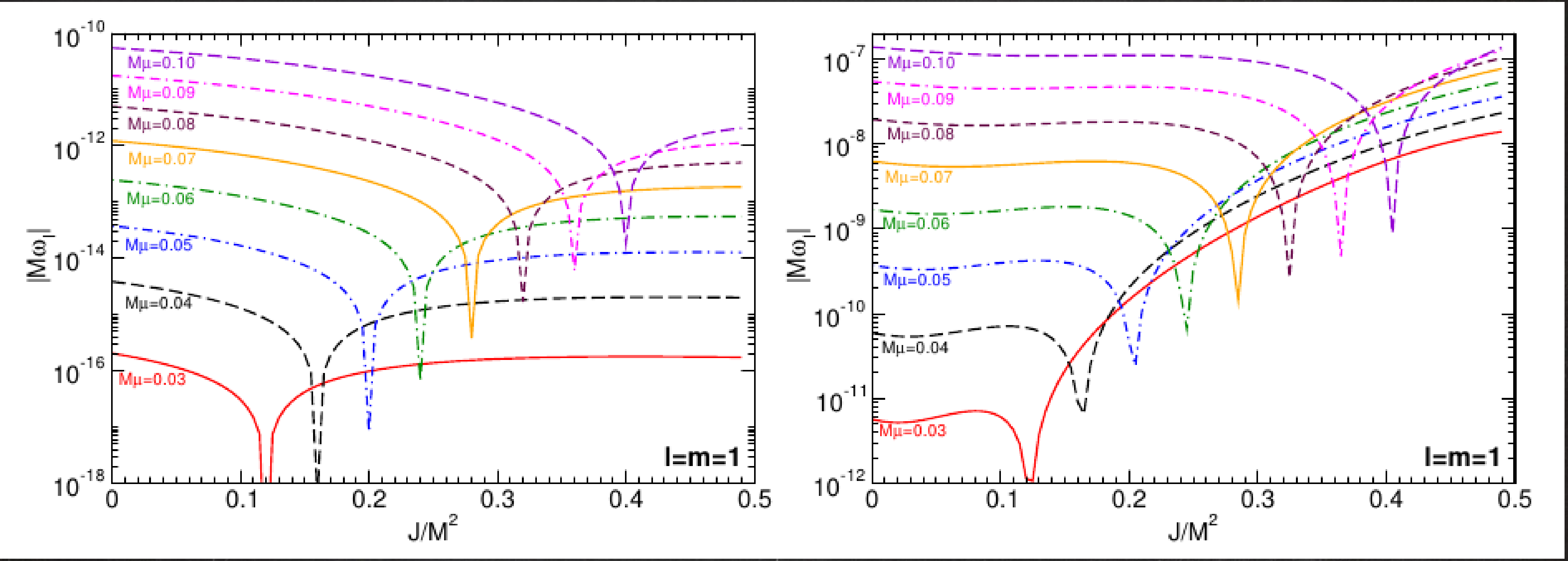
$$\Psi_A = (u^\ell_{(4)}, u^{\ell\pm1}_{(2)}, u^{\ell\pm1}_{(3)}, u^{\ell\pm2}_{(4)}) \qquad \Psi_P = (u^\ell_{(2)}, u^\ell_{(3)}, u^{\ell\pm1}_{(4)}, u^{\ell\pm2}_{(2)}, u^{\ell\pm2}_{(3)})$$

- **Near-horizon behavior:** $u_{(i)} \sim e^{-ik_H r_*}$

$$k_H \sim \underbrace{\omega - m\Omega_H}_{\text{Superradiance}} \simeq \omega - \frac{m\tilde{a}}{4M} + \mathcal{O}(\tilde{a}^3)$$

Proca in SR Kerr. Results

Axial modes (S=0) **Polar modes (S=+1,-1)**



- **Small mass limit:**

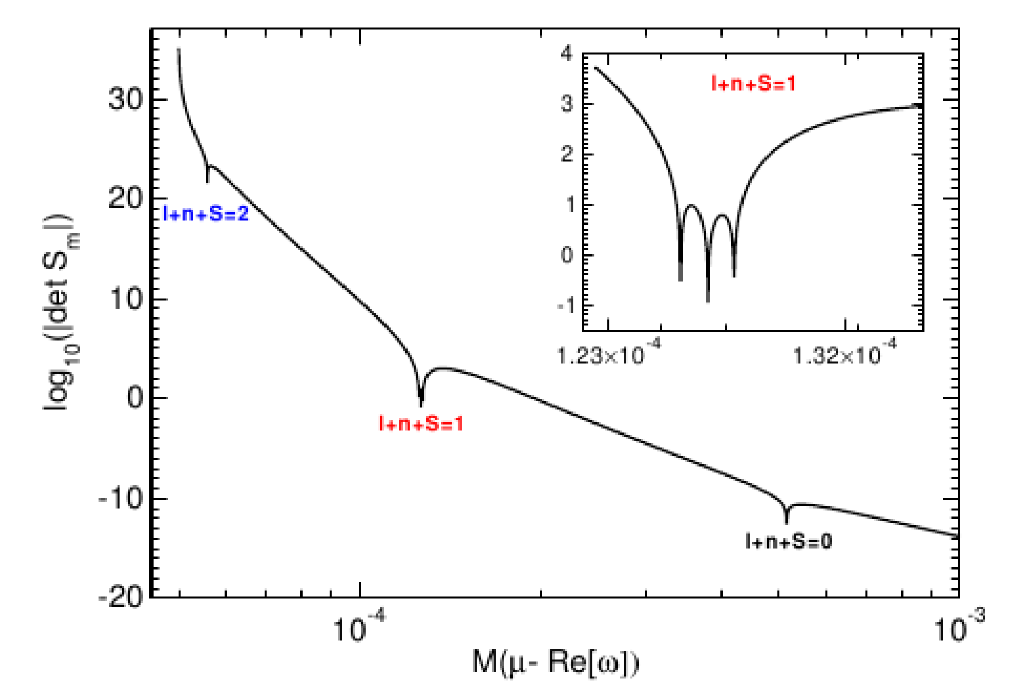
$$\omega_R \sim \mu - \frac{\mu(M\mu)^2}{2(\ell+n+S+1)}$$

$$M\omega_I \sim \gamma_{S\ell}\left(\tilde{a}m - 2r_+\mu\right)(M\mu)^{4\ell+5+2S}$$

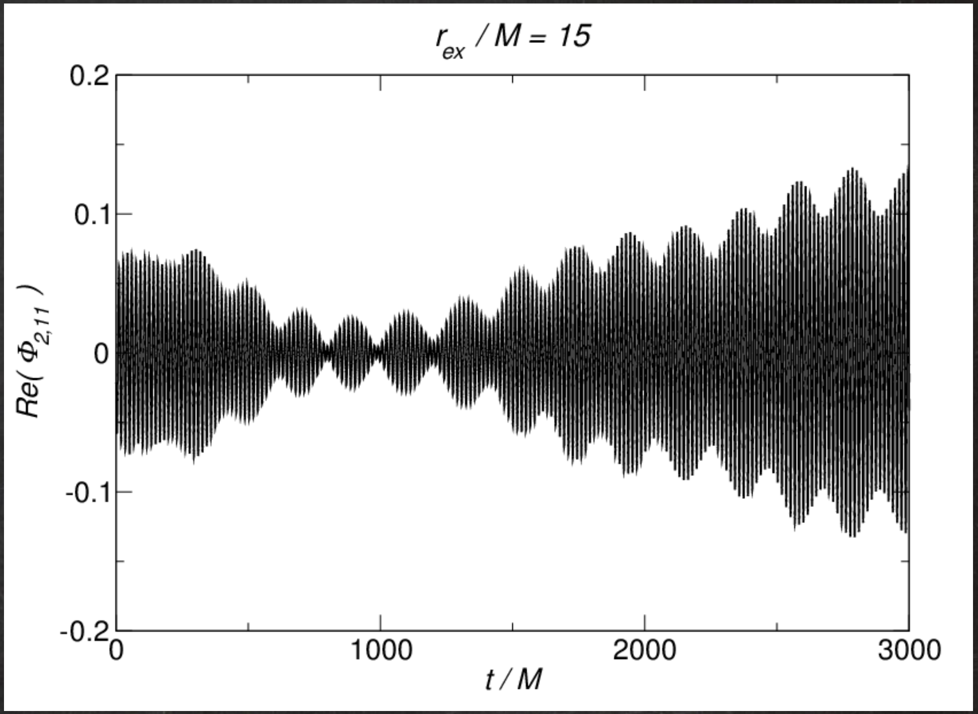
[Rosa & Dolan 2011]

Proca in SR Kerr. *Fully coupled system*

$$\omega_R \sim \mu - \frac{\mu(M\mu)^2}{2(\ell+n+S+1)} \qquad M\omega_I \sim \gamma_{S\ell}\left(\tilde{a}m - 2r_+\mu\right)(M\mu)^{4\ell+5+2S}$$



Breit-Wigner resonances



Confirmed by numerical simulations

[Witek et al., in preparation]

Proca in SR Kerr. *Analytical results*

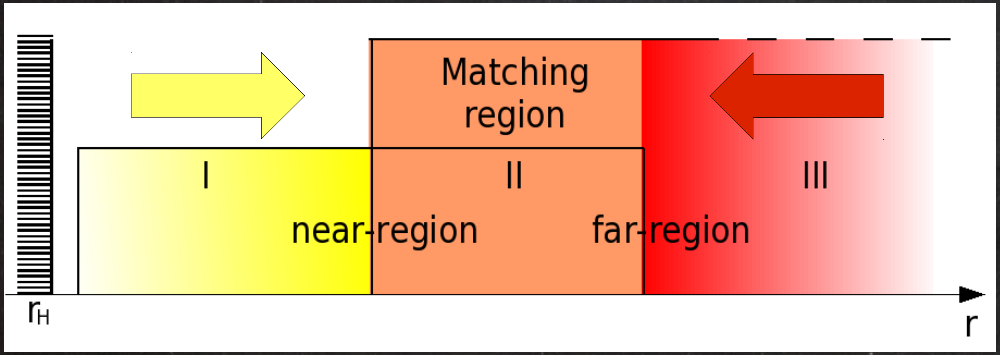
- In the axial case → **master equation** (scalar → **s=0** , axial vector → **s=1**)

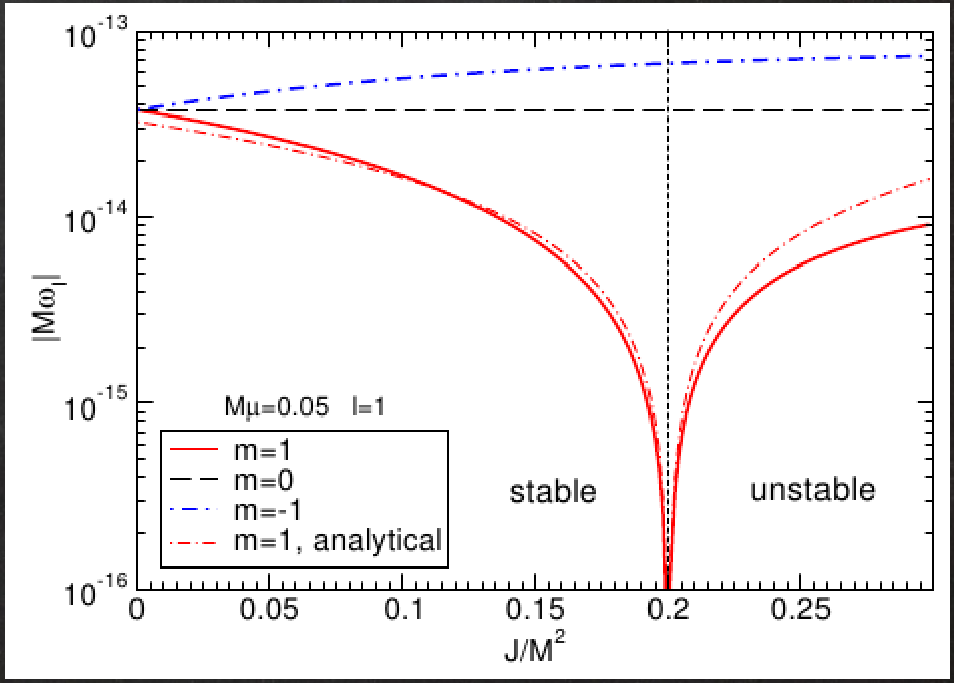
$$\frac{d^2\Psi}{dr_*^2}+\left[\omega^2-\frac{2m\varpi(r)\omega}{r^2}-F\left(\frac{\Lambda}{r^2}+\mu^2+(1-s^2)\left\{\frac{B'}{2r}+\frac{BF'}{2rF}\right\}\right)\right]\Psi=0$$

$$ds_0^2=g^{(0)}_{\mu\nu}dx^\mu dx^\nu=-F(r)dt^2+B(r)^{-1}dr^2+r^2d^2\Omega-2\varpi(r)\sin^2\theta d\varphi dt$$

- Suitable for analytical methods
- Matching asymptotics

[Starobisky 1973]
[Detweiler 1980]





$$M\omega_I\sim\gamma_{s\ell}\left(\tilde{a}m-2r_+\mu\right)(M\mu)^{4\ell+5}$$

Part III

Astrophysical consequences of the Proca instability

Proca instability

- **Can we extrapolate these results to higher rotation?**
- **Scalar case (l=1)** $$M\omega_I \sim \frac{1}{48}(\tilde{a}m - 2r_+\mu)(M\mu)^9$$

[Dolan 2007]

TABLE III. Maximum instability growth rates of the $l = 1$, $m = 1$ state.

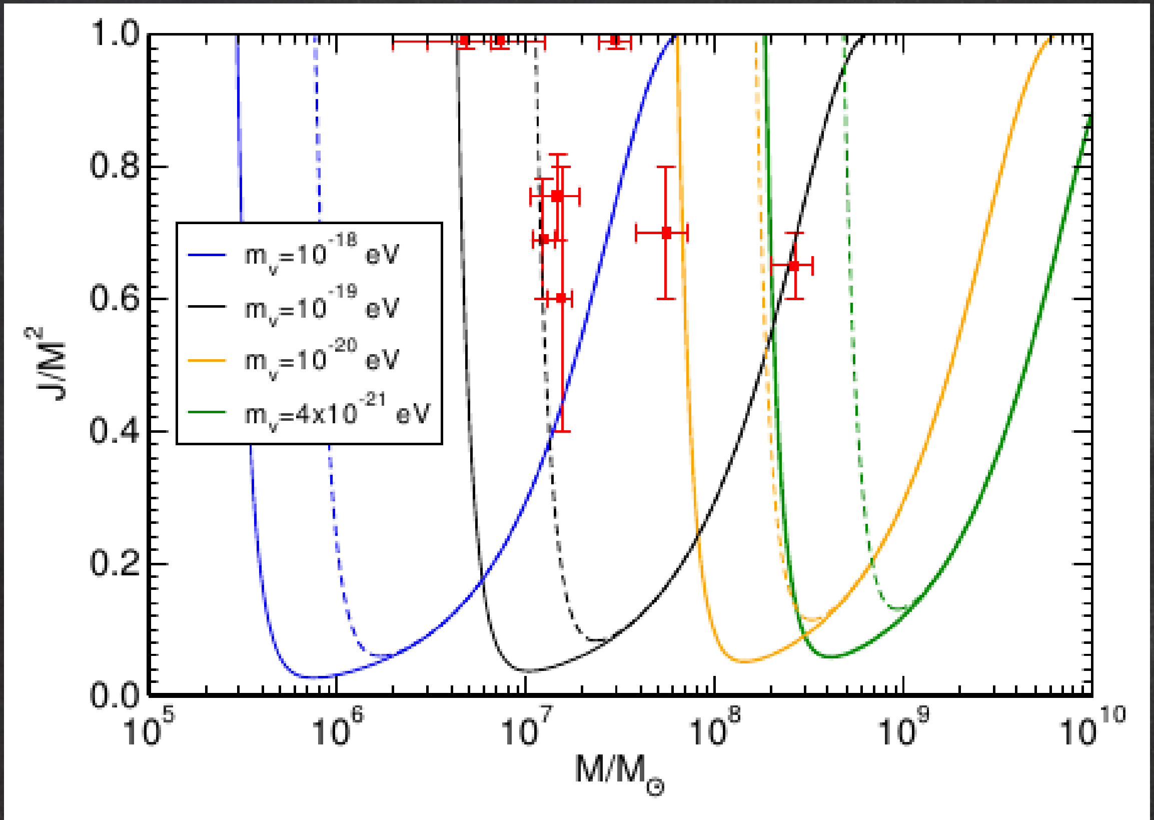
a	0.7	0.8	0.9	0.95	0.98	0.99
μ	0.187	0.231	0.293	0.343	0.393	0.421
τ^{-1}	3.33×10^{-10}	2.16×10^{-9}	1.55×10^{-8}	4.88×10^{-8}	1.11×10^{-7}	1.50×10^{-7}
	3.43×10^{-10}	2.37×10^{-9}	1.94×10^{-8}	6.82×10^{-8}	1.75×10^{-7}	2.53×10^{-7}

- **Extrapolation should provide an order of magnitude for the instability**
- **Proca case:** $$M\omega_I \sim \gamma_{S\ell}(\tilde{a}m - 2r_+\mu)(M\mu)^{4\ell+5+2S}$$
- **Stronger instability when S = -1 and l=1:**

$$\tau_{\text{vector}} = \omega_I^{-1} \sim \frac{M(M\mu)^{-7}}{\gamma_{-11}(\tilde{a} - 2\mu r_+)}$$

Proca instability. *Regge plane*

- Instability is effective roughly for any non-vanishing spin!

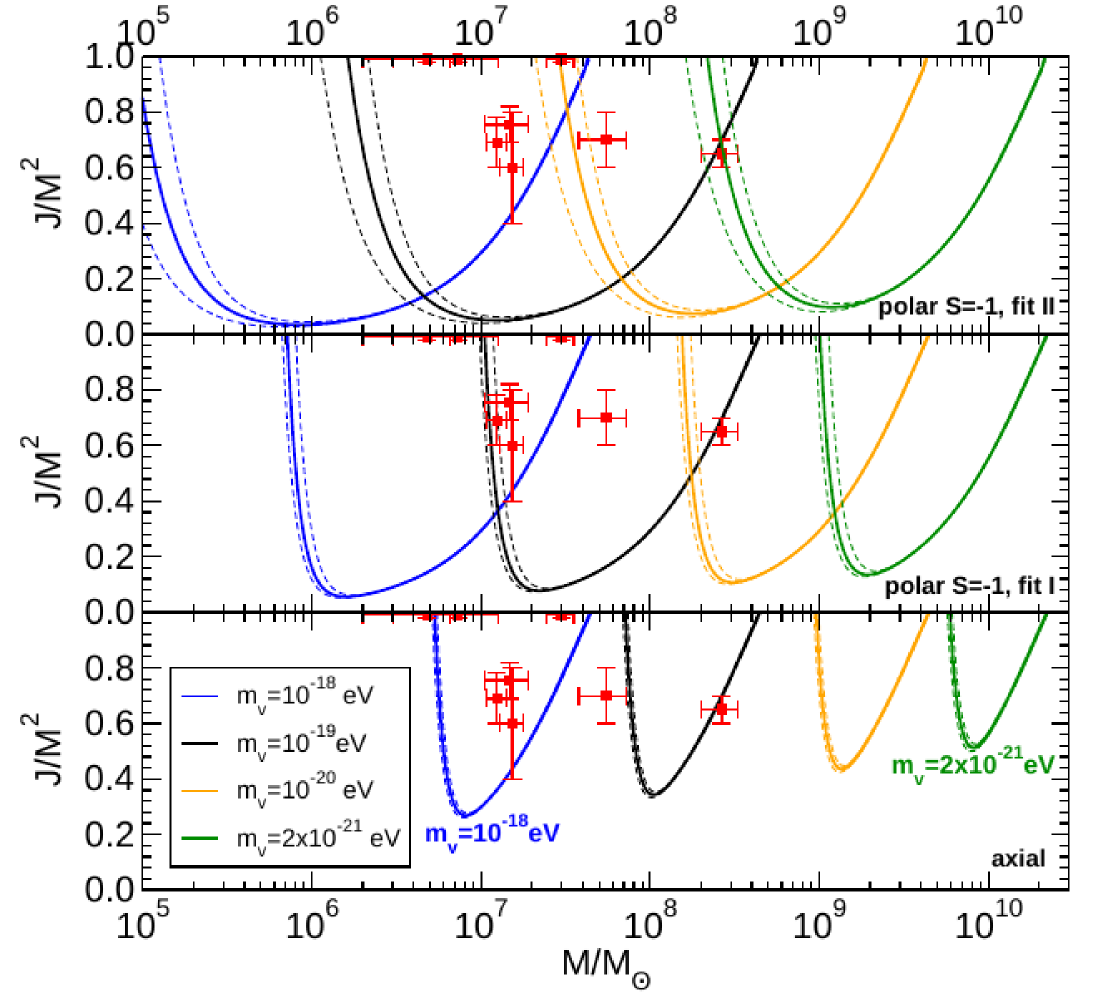


[Data taken from Brenneman et. al 2011]

- Current bound on the photon mass [from PDG] → $m_\gamma < 10^{-18}\mathrm{eV}$
- Depend very mildly on the fit coefficient and on the threshold

Proca instability

- Not strongly dependent on the timescale nor on type of mode



- From the existence of spinning BHs → $10^{-21}\mathrm{eV} \lesssim m_\gamma \lesssim 10^{-17}\mathrm{eV}$

Proca instability. Limitations

- Nonlinear effects:
  - Photon self-interaction is very weak → gradual slow down
  - Might be important for exotic fields
- Accretion disk
  - Hidden U(1) fields are weakly coupled to matter
  - Might be relevant for massive photons, but
    - Superradiant mode are coherent and $\lambda \sim$ BH size
    - Disks are charge neutral and matter coupling incoherent
    - Equatorial disks can at most quench some unstable modes

Part IV

QNMs of Kerr-Newman BHs

Kerr-Newman BHs

- Most general rotating solution in GR
- Gravitational and EM perturbations are coupled → not separable?
  [Berti & Kokkotas 2004]
- Apply the method to slowly-rotating Reissner-Nordstrom:

Kerr-Newman BHs

- **Most general rotating solution in GR**
- **Gravitational and EM perturbations are coupled → not separable?**
  [Berti & Kokkotas 2004]
- **Apply the method to slowly-rotating Reissner-Nordstrom:**

$$\hat{\mathcal{D}} Z_i = V_0^{(i)} Z_i$$

Zeroth order (i=1,2)

$$\hat{\mathcal{D}} = \frac{d^2}{dr_*^2} + \omega^2 - F\frac{\ell(\ell+1)}{r^2} \qquad F(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

Kerr-Newman BHs

- Most general rotating solution in GR
- Gravitational and EM perturbations are coupled → not separable?

  [Berti & Kokkotas 2004]
- Apply the method to slowly-rotating Reissner-Nordstrom:
  - Axial sector and polar sector (isospectrality?)

$$\hat{\mathcal{D}} Z_i = V_0^{(i)} Z_i + m\tilde{a}\left[V_1^{(i)} Z_i + V_2^{(i)} Z_i'\right] + m\tilde{a}Q^2\left[W_1^{(i)} Z_j + W_2^{(i)} Z_j'\right]$$

Zeroth order (i=1,2) — 1st order: Zeeman effect — 1st order: coupliing between i and j

$$\hat{\mathcal{D}} = \frac{d^2}{dr_*^2} + \omega^2 - F\frac{\ell(\ell+1)}{r^2} \qquad F(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

Conclusion & Extensions

- Perturbation theory of rotating objects is challenging
- Slowly-rotating approximation: general method
- Spinning BHs as labs for exotic particles and modified gravity
- Proca perturbations of Kerr BHs in GR
  - Strong (est?) instability
  - Bounds on the photon mass, Hidden U(1) sector
- Extensions
  - BHs in alternative theories (Chern-Simons, Gauss-Bonnet)
  - Kerr-Newman, higher dimensions, stellar r-modes, …

[Yunes & Pretorius 2009]
[Pani et al. 2011]
[Yagi, Yunes, Tanaka 2012]

$T^{\text{he}}_{\mu\nu}\, G^{\text{ravity}}_{\mu\nu}\, R^{\text{oom}}_{\mu\nu}$

Calls for bloggers now open!

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MARIE CURIE ACTIONS

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

thegravityroom.blogspot.com

Thanks!

Backup slides

“Nothing is More Necessary than the Unnecessary”

- **Curiosity: similar bounds for the graviton?** → **probably not! (S= -2, l=2)**

$$M\omega_I \sim \gamma_{S\ell}\,(\tilde{a}m - 2r_+\mu)\,(M\mu)^{4\ell+5+2S}$$

Proca in SR Kerr. *Field equations*

- In Proca theory, the monopole (l=0,m=0) is dynamical:

$$\left[\frac{d^2}{dr_*^2}+\omega^2-F\left(\frac{2(r-3M)}{r^3}+\mu^2\right)\right]u^{00}_{(2)}=\underbrace{\frac{2i\sqrt{3}\tilde{a}M^2\omega F}{r^3}u^{10}_{(4)}}$$

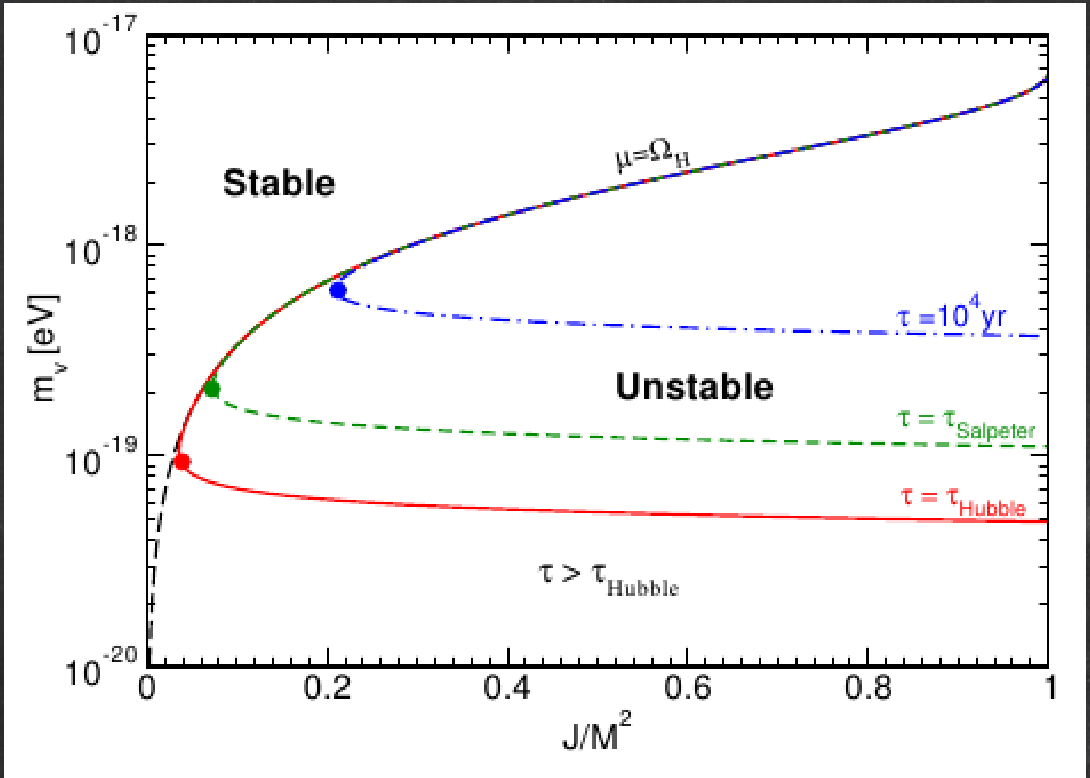
Propensity rule $(\mathcal{Q}_{00}=0)$

- m=0 → no corrections at first order! Same modes as in Schwarzschild

[Rosa & Dolan 2011]

- Modes can be labelled by the total angular momentum → j=l+S
  - Axial → S=0
  - Polar → S=+1 , S= -1
  - Monopole → S=+1

Proca instability



$$m_v^{(c)} = \hbar\mu^{(c)} \sim \frac{7.055 \times 10^{-20}}{\gamma_{-11}^{1/7}} \left[\frac{10^7 M_\odot}{M}\right]^{6/7} \mathrm{eV}$$

- Depend **very mildly** on the fit coefficient and on the threshold
- $\tau_{Salpeter}$ → timescale for accretion at the Eddington limit

$\mathcal{O}(\nu^2)$ $\mathcal{O}(\nu^3)$

AAA AAA

Method. *Perturbations of slowly rotating BHs*

- **At first order in the rotation, the couplings can be neglected:**

$$\mathcal{A}_{\ell m} + \tilde{a} m \bar{\mathcal{A}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{P}}_{\ell-1 m} + \mathcal{Q}_{\ell+1 m}\tilde{\mathcal{P}}_{\ell+1 m}) = 0$$

$$\mathcal{Q}_{\ell m} = \sqrt{\frac{\ell^2 - m^2}{4\ell^2 - 1}}$$

$$\mathcal{P}_{\ell m} + \tilde{a} m \bar{\mathcal{P}}_{\ell m} + \tilde{a}(\mathcal{Q}_{\ell m}\tilde{\mathcal{A}}_{\ell-1 m} + \mathcal{Q}_{\ell+1 m}\tilde{\mathcal{A}}_{\ell+1 m}) = 0$$

- **Symmetry of the equations**

$$a_{\ell m} \to \mp a_{\ell -m}\,, \qquad p_{\ell m} \to \pm p_{\ell -m}\,, \qquad \tilde{a} \to -\tilde{a}\,, \qquad m \to -m$$

- **Eigenfrequency**

$$\omega = \omega_0 + \tilde{a} m\, \omega_1 + \mathcal{O}(\tilde{a}^2)$$

- **"Decoupled" equations:**

$$\mathcal{A}_{\ell m} + \tilde{a} m \bar{\mathcal{A}}_{\ell m} = 0 \qquad \mathcal{P}_{\ell m} + \tilde{a} m \bar{\mathcal{P}}_{\ell m} = 0$$

EMRIs: imprints of light scalars

$$\left[\Box - \mu_s^2\right]\varphi = \alpha\mathcal{T} \Longrightarrow \frac{d^2\Psi_{\ell m}(\omega,r)}{dr_*^2} + V(\omega,r)\Psi_{\ell m}(\omega,r) = \mathcal{T}_{\ell m}(\omega,r)$$

Suitable for analytical computation in the small mass limit

$$\dot{E}_S^{\rm resonance} \sim -\mu_s^{1-4l/3} \sim -v^{-4l+3}$$

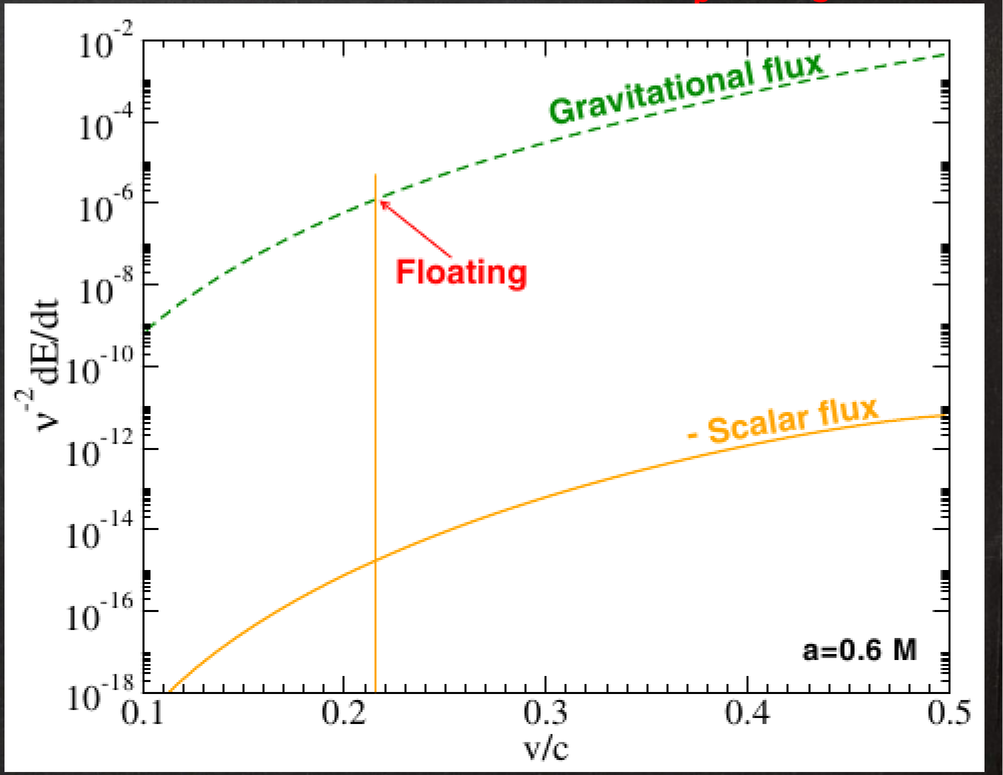
$$dE_p/dt = -\dot{E}_{\rm total} = -(\dot{E}_S + \dot{E}_G)$$

$$\text{if } \dot{E}_S = -\dot{E}_G \Longrightarrow \dot{E}_p = 0$$

$$\Omega_{\rm res} = \mu_s\left[1 - \left(\frac{\mu_s M}{l+1+n}\right)^2\right]^{1/2}$$

$$\Delta\Omega \sim \frac{1}{12M}(\mu_s M)^9 (q - 2r_+\mu_s)$$

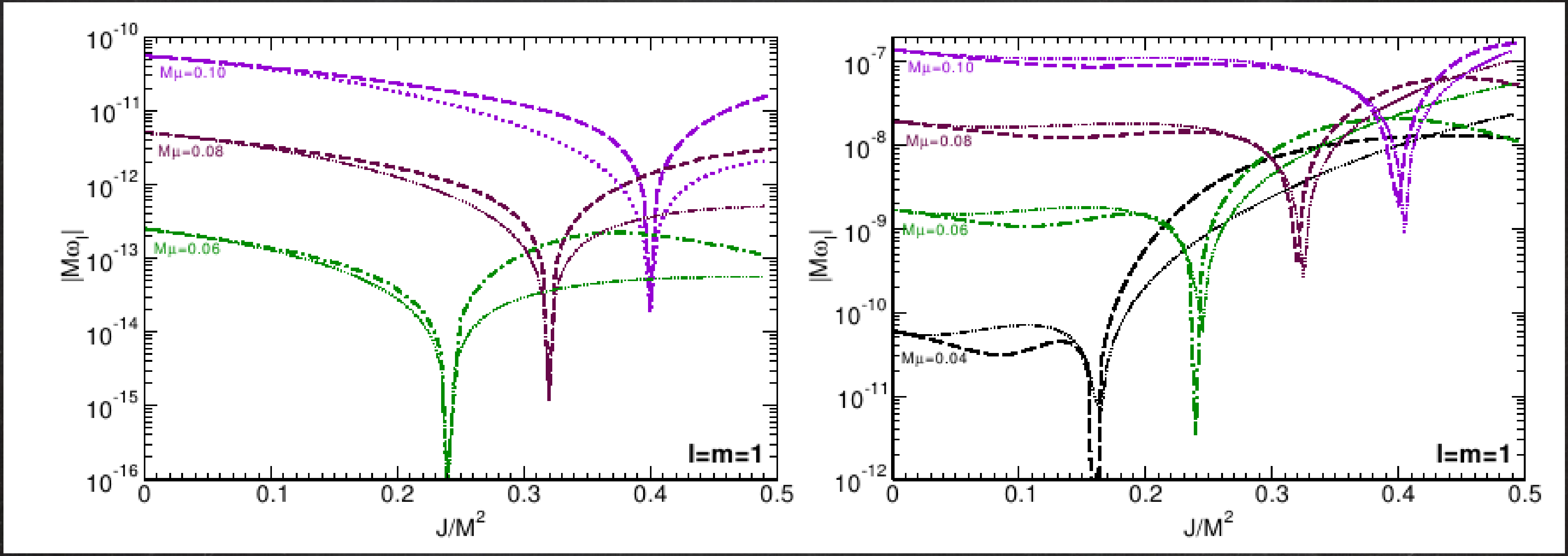
Floating orbit at $\Omega_p \sim \mu_s$



Quite generic effect → only needs a rotating BH and a light scalar

Proca in SR Kerr. Results (second order)

Axial modes (S=0) **Polar modes (S=+1,-1)**



- **Small mass limit:**

$$\omega_R \sim \mu - \frac{\mu(M\mu)^2}{2(\ell+n+S+1)}$$

$$M\omega_I \sim \gamma_{S\ell}\left(\tilde{a}m - 2r_+\mu\right)(M\mu)^{4\ell+5+2S}$$

Proca in slowly-rotating Kerr

- **Proca equations can be written as**

$$\delta\Pi_t \equiv (A^{(0)}_{\ell m} + \tilde{A}^{(0)}_{\ell m}\cos\vartheta)Y^{\ell m} + B^{(0)}_{\ell m}\sin\vartheta\partial_\vartheta Y^{\ell m} = 0$$

$$\delta\Pi_r \equiv (A^{(1)}_{\ell m} + \tilde{A}^{(1)\,\ell m}\cos\vartheta)Y^{\ell m} + B^{(1)}_{\ell m}\sin\vartheta\partial_\vartheta Y^{\ell m} = 0$$

$$\delta\Pi_\vartheta \equiv \alpha_{\ell m}\partial_\vartheta Y^{\ell m} - im\beta_{\ell m}\frac{Y^{\ell m}}{\sin\vartheta} + \eta_{\ell m}\sin\vartheta Y^{\ell m} = 0$$

$$\frac{\delta\Pi_\varphi}{\sin\vartheta} \equiv \beta_{\ell m}\partial_\vartheta Y^{\ell m} + im\alpha_{\ell m}\frac{Y^{\ell m}}{\sin\vartheta} + \zeta_{\ell m}\sin\vartheta Y^{\ell m} = 0$$

- **Lorenz condition can be written in the same form as {t} or {r} components**
- **All coefficients can be divided in two sets:**

$$A^{(I)}_{\ell m}\,,\quad \alpha_{\ell m}\,,\quad \zeta_{\ell m} \qquad\qquad \tilde{A}^{(I)}_{\ell m}\,,\quad B^{(I)}_{\ell m}\,,\quad \beta_{\ell m}\,,\quad \eta_{\ell m}$$

Axial coefficients **Polar coefficients**

BH perturbations. *Symmetries matter*

- In spherically symmetry the field eqs. can be always separated
- If the background is rotating, separability is not guaranteed!
- Teukolsky formalism
  - Newman-Penrose tetrad formalism, Weyl scalars
  - Separability in Kerr is almost a miracle! (Petrov Type D)

[Teukolsky ~ 1973]
[Teukolsky and Press]
[Chandra's book]

- Perturbations of generic rotating BHs are important:
  - Astrophysical BHs are spinning
  - Coupling to matter
  - Stability (e.g. superradiance, r-modes in stars, no-hair theorem)

Second order formalism

- **Particularly advantageous:**
  - **Cauchy horizon, even horizons, ergosphere**

$$r_+ = 2M\left(1 - \frac{\tilde{a}^2}{4}\right) \qquad r_- = \frac{M\tilde{a}^2}{2} \qquad r_{\mathrm{ER}} = 2M\left(1 - \cos^2\vartheta\,\frac{\tilde{a}^2}{4}\right)$$

  - **The superradiance regime is now consistent**

$$\omega = \omega_0 + \tilde{a}m\omega_1 + \tilde{a}^2\omega_2 + \mathcal{O}(\tilde{a}^3)$$

Outline

- BH superradiant instabilities
- Perturbations of slowrly-rotating Bhs: general framework
- Proca instability of Kerr Bhs
- Extensions