

# Measuring the dynamical properties of self-gravitating systems in their outer regions through the caustic technique

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Where does a galaxy cluster come from?



Abell 1689 observed by the Hubble Space Telescope

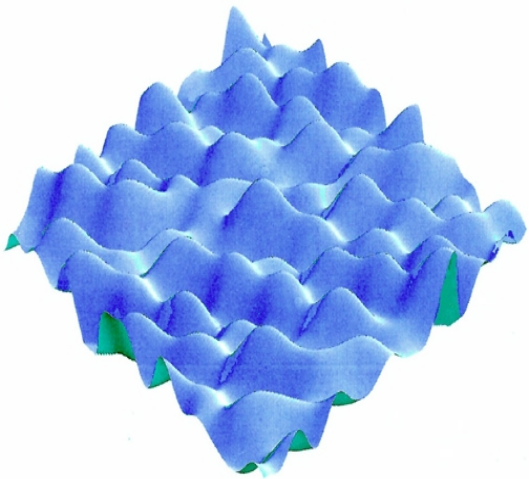
## Formation of large-scale structure in the Universe

**Early Universe**  
(small perturbations)

At first the ripples evolve independently

Then they interact with others in non-linear ways

the small over-density fluctuations attract additional mass as the Universe expands

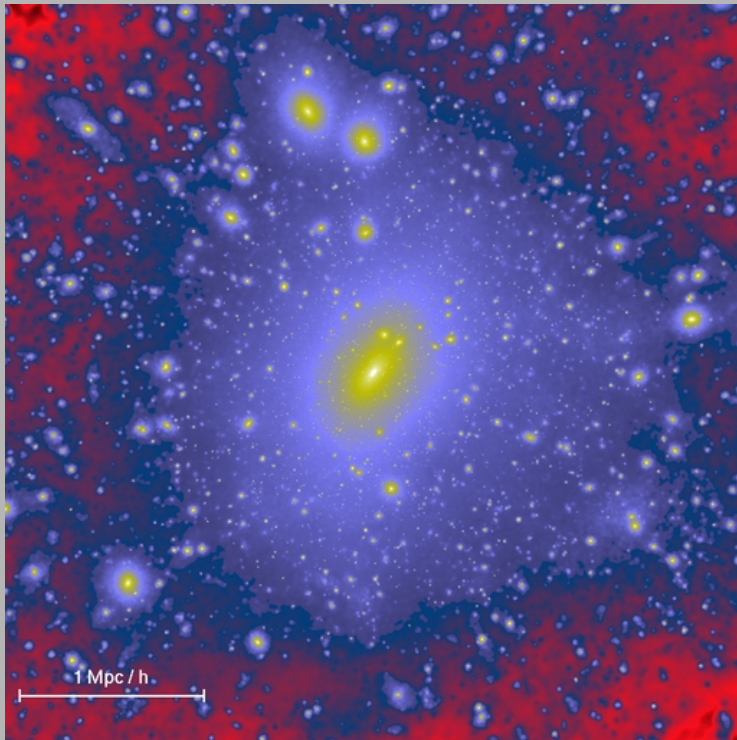


## Formation of large-scale structure in the Universe

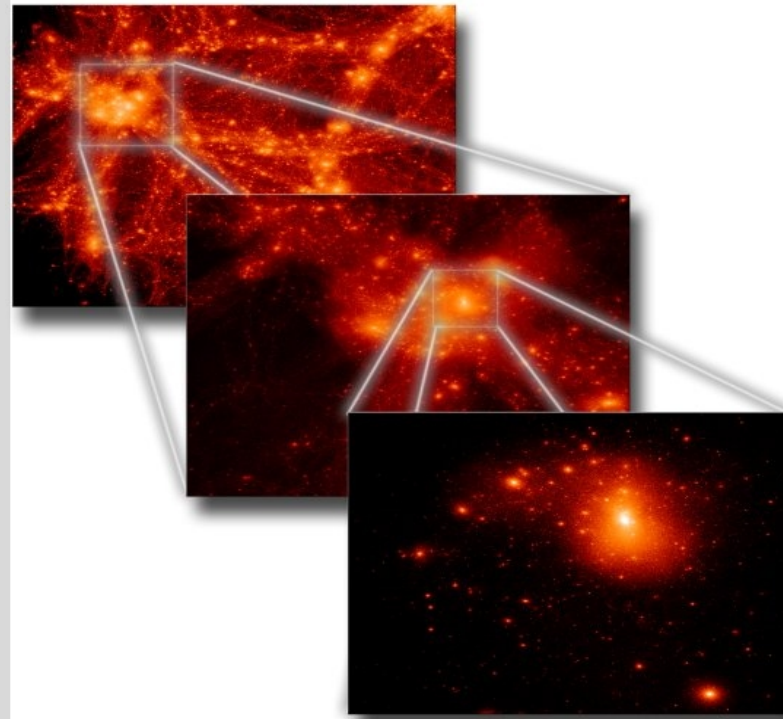
Gravitational instability produces high peaks of the density field



merger of small clumps at the intersection of a filamentary large-scale structure

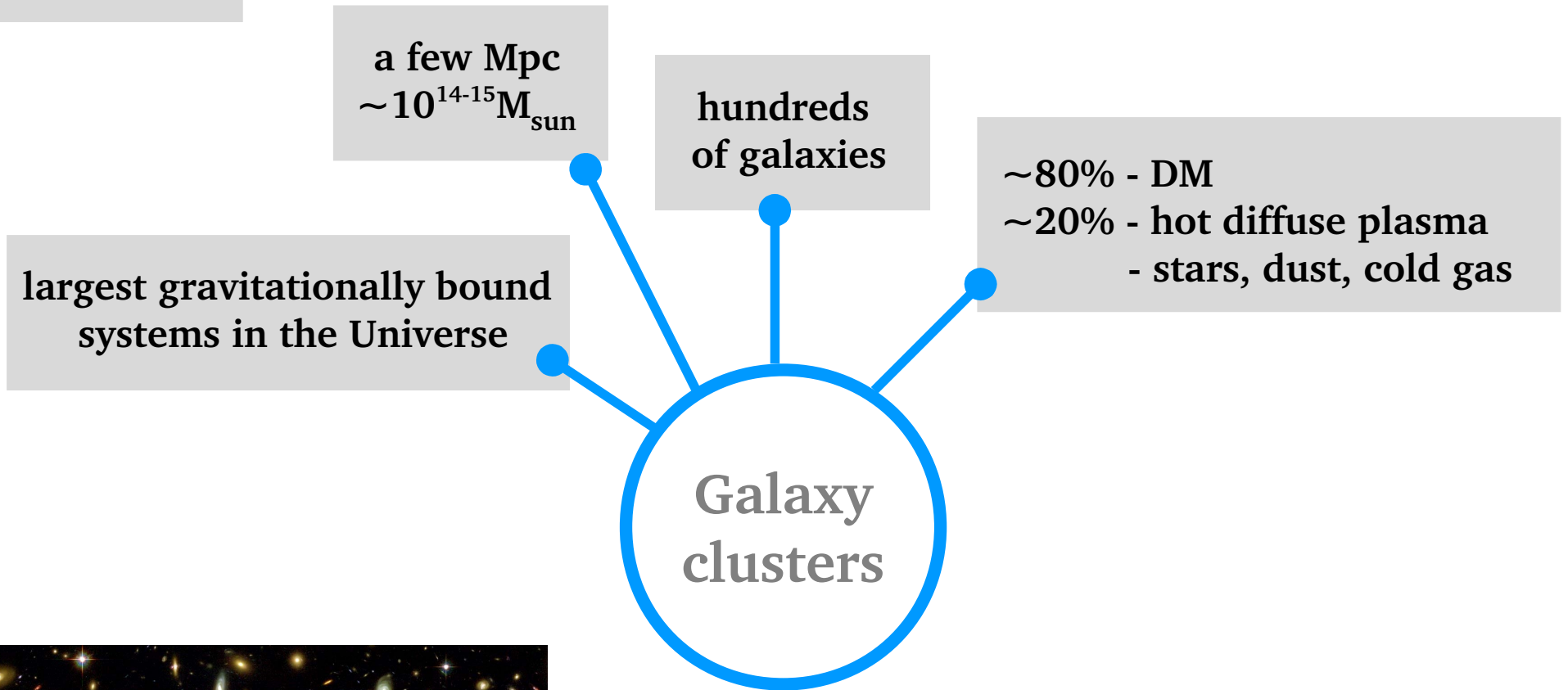


Millennium Simulation, Volker Springel, 2005



**GALAXY  
CLUSTER**

Simulation by Gauss Centre for Supercomputing  
Gottlöber, Khalatyan, Klypin, 2008



large-scale  
cosmological structure  
galaxy formation  
parameters  
*nucleosynthesis*  
galaxy-environment  
matter distribution

a few Mpc  
 $\sim 10^{14-15} M_{\text{sun}}$

hundreds  
of galaxies

$\sim 80\%$  - DM  
 $\sim 20\%$  - hot diffuse plasma  
- stars, dust, cold gas

largest gravitationally bound  
systems in the Universe

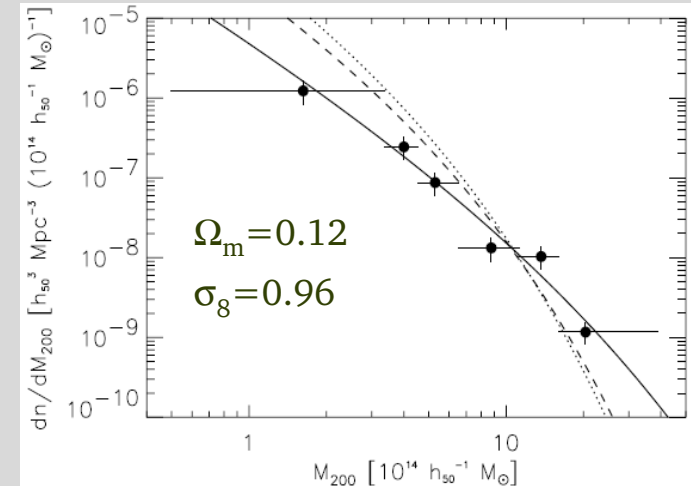
Galaxy  
clusters

High-mass tail of mass function

number density

measurement of  
cosmological parameters

number of systems with a given mass per unit volume



large-scale  
cosmological  
structure  
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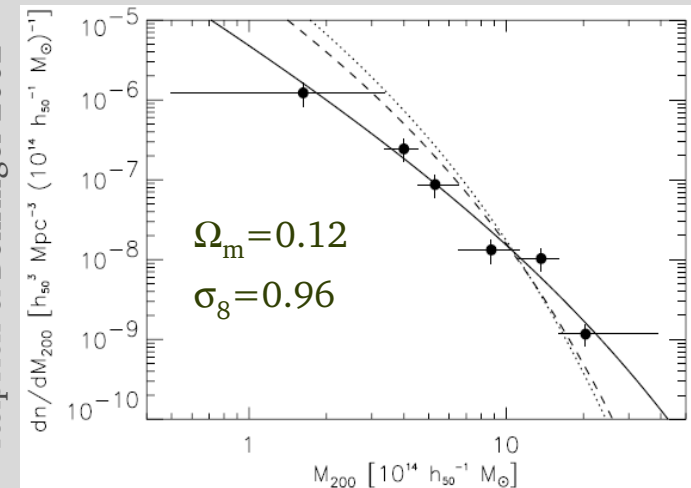
Galaxy  
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large-scale  
cosmological  
structure  
galaxy formation  
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## How to measure the mass of galaxy clusters ?

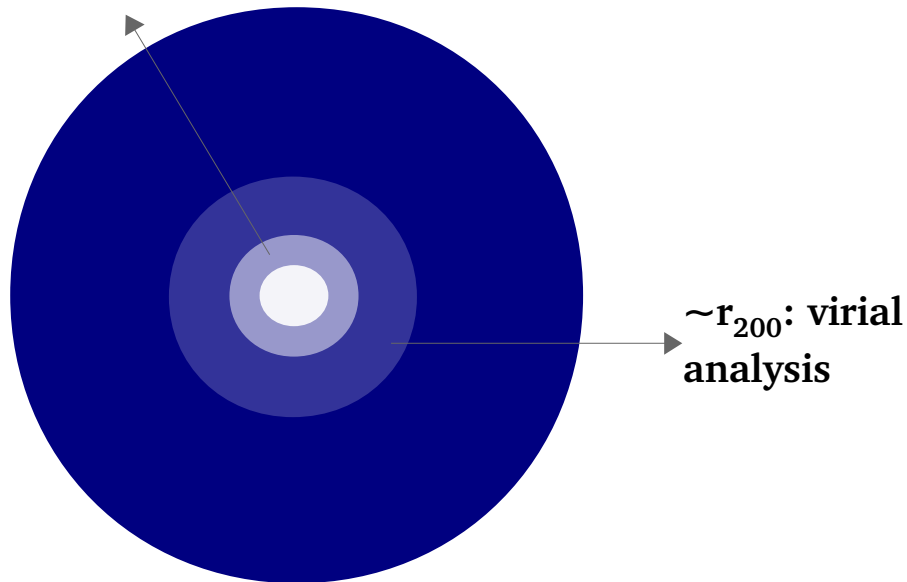
Virial theorem

Jeans Equation

Scaling relations

X-ray temperature

$\sim 0.5 r_{200}$ : X-ray



$r_{200}$ : radius enclosing a matter density 200 times the critical density of the Universe  $\sim 277.5 h^2 M_{\odot} / \text{Mpc}^3$



## How to measure the mass of galaxy clusters ?

Virial theorem

Jeans Equation

Scaling relations

X-ray temperature

Strong and Weak  
Lensing

CAUSTIC TECHNIQUE

**No assumption of  
dynamical equilibrium**

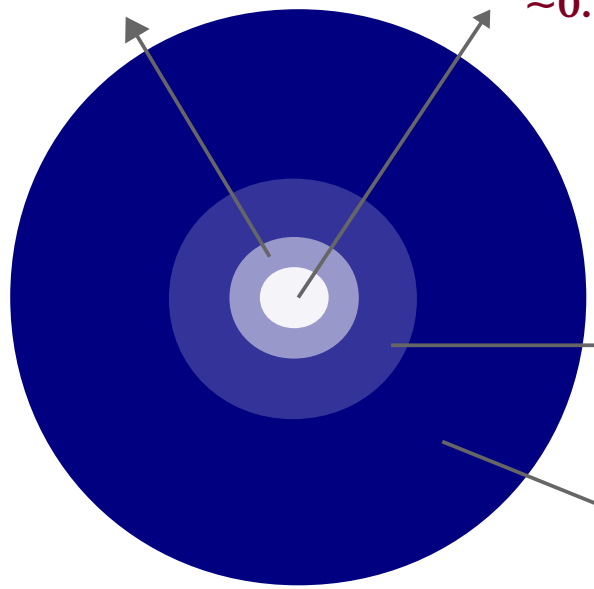
$\sim 0.5 r_{200}$ : X-ray

$\sim 0.1 r_{200}$ : SL

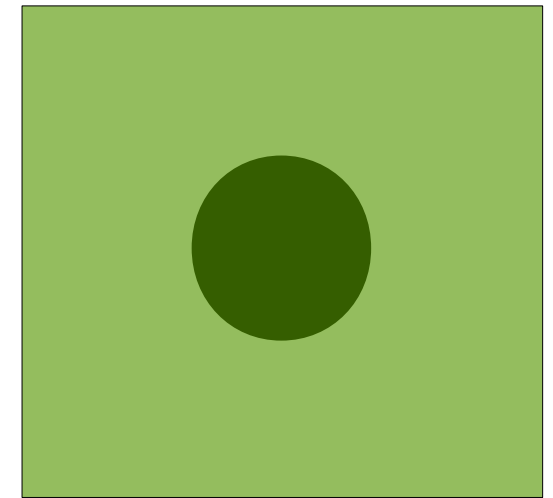
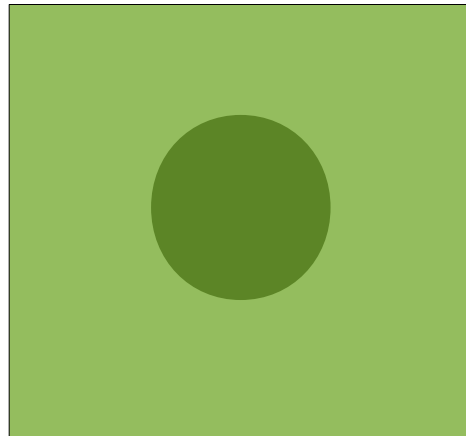
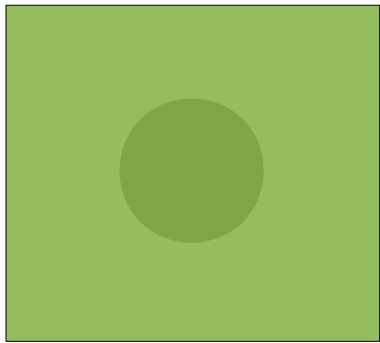
$\sim r_{200}$ : virial  
analysis

$\sim [0-3] r_{200}$   
WL & CT

$r_{200}$ : radius enclosing a matter density 200 times the  
critical density of the Universe  $\sim 277.5 h^2 M_{\odot} / \text{Mpc}^3$



## caustic technique – spherical infall model



evolves like a Friedmann model (expanding medium)

for any small density perturbation there will be a competition between its self-gravity (which is attempting to increase the density) and the general expansion of the universe (which decreases the density)

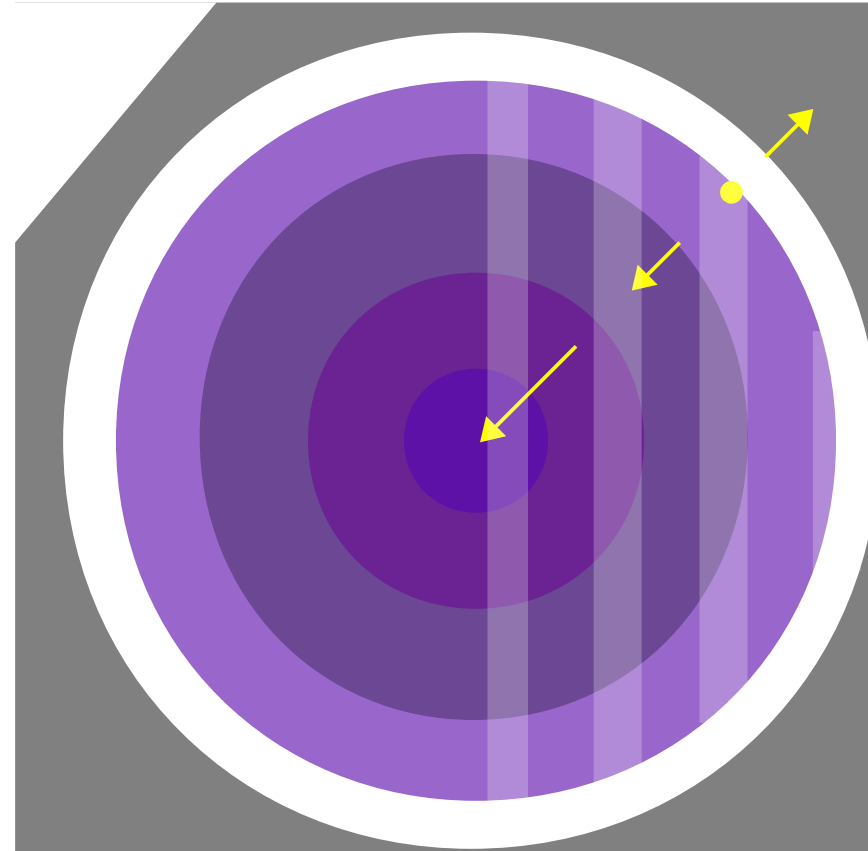
structures will be formed if, at some time, the spherical region ceases to expand with the background universe and begins to collapse

When observed in redshift space, the infall pattern around a rich cluster appears as a “trumpet” whose amplitude  $\mathcal{A}(\theta)$  decreases with  $\theta$  (Kaiser, 1987)

from spherical infall model

(Regös & Geller, 1989)

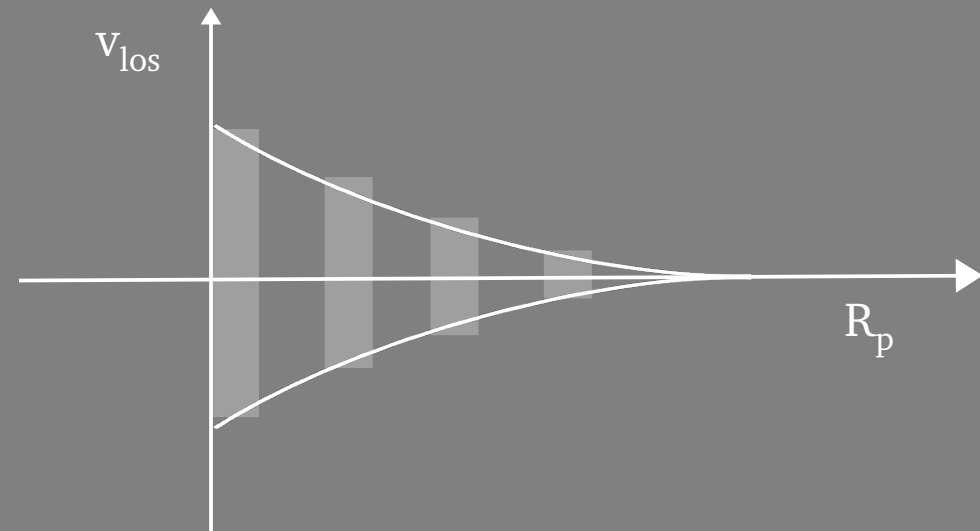
$$\mathcal{A}(\theta) \sim \Omega_0^{0.6} r f(\delta) \sqrt{-\frac{d \ln f(\delta)}{d \ln r}}$$



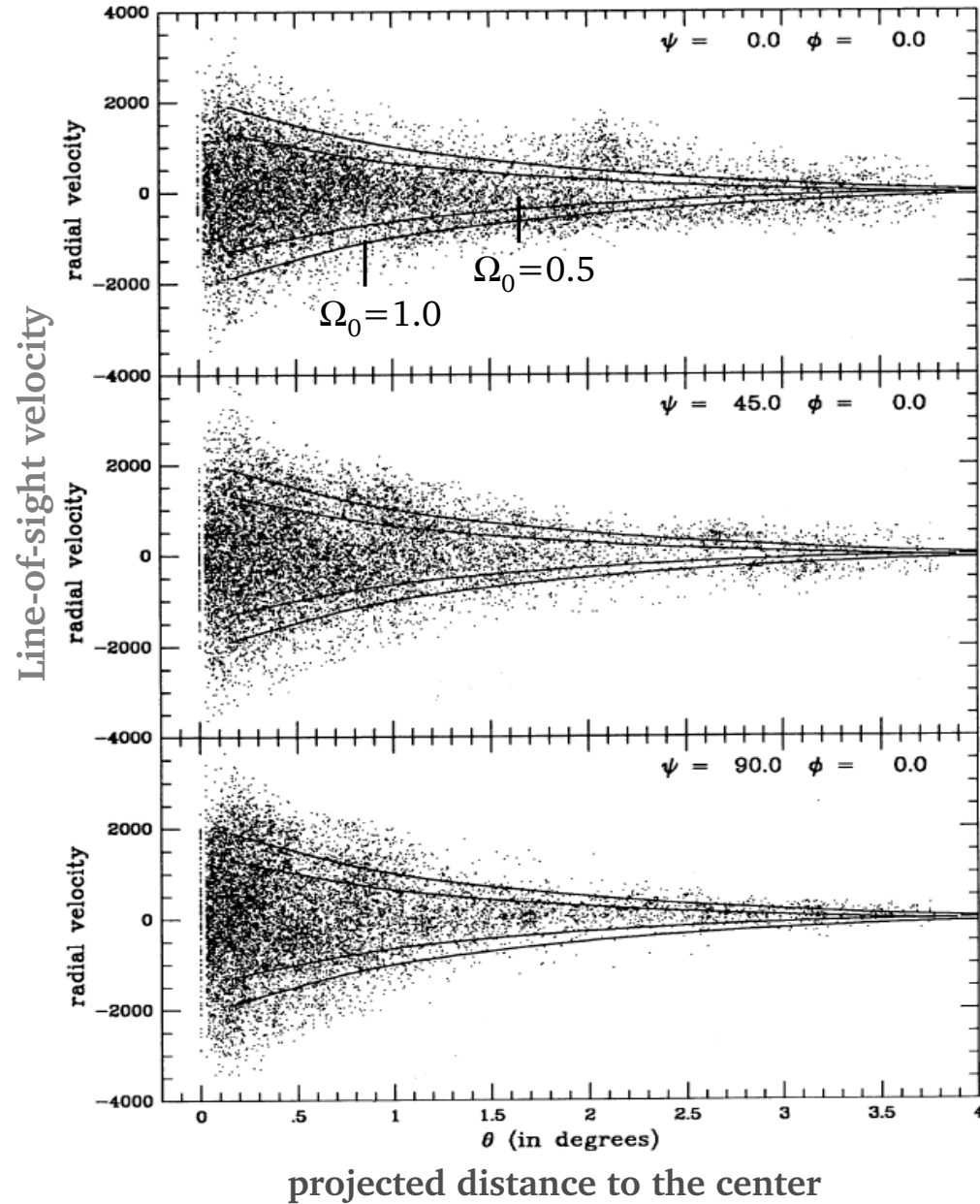
$$\mathbf{v} = H_o \mathbf{r} + \mathbf{v}_{\text{pec}}$$

$$v_{\text{turn}} = H_o r$$

$$v_{\text{inf}} = H_o r - v_{\text{pec}}$$

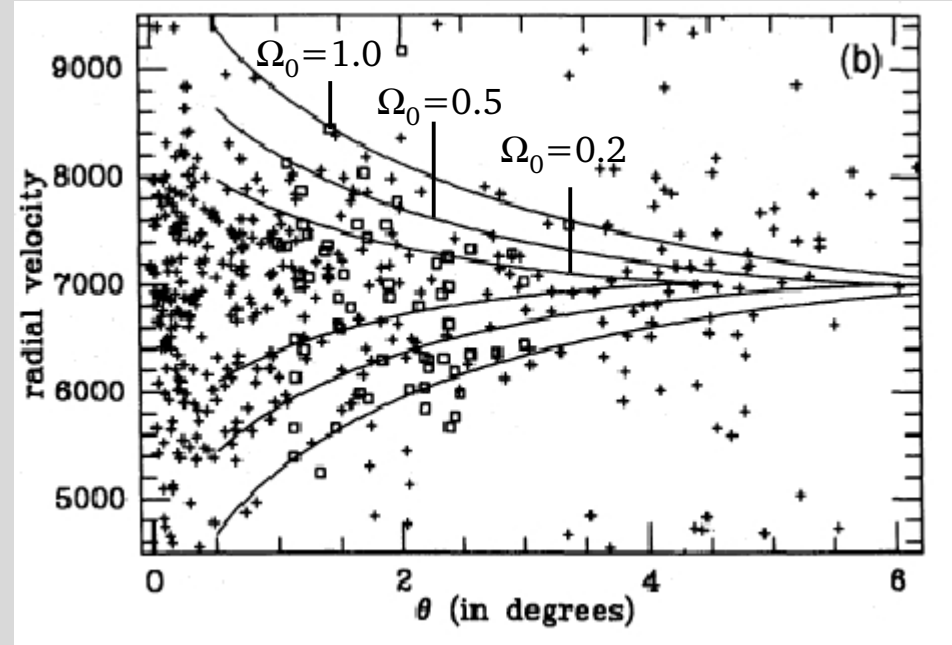


Simulated data



Real data

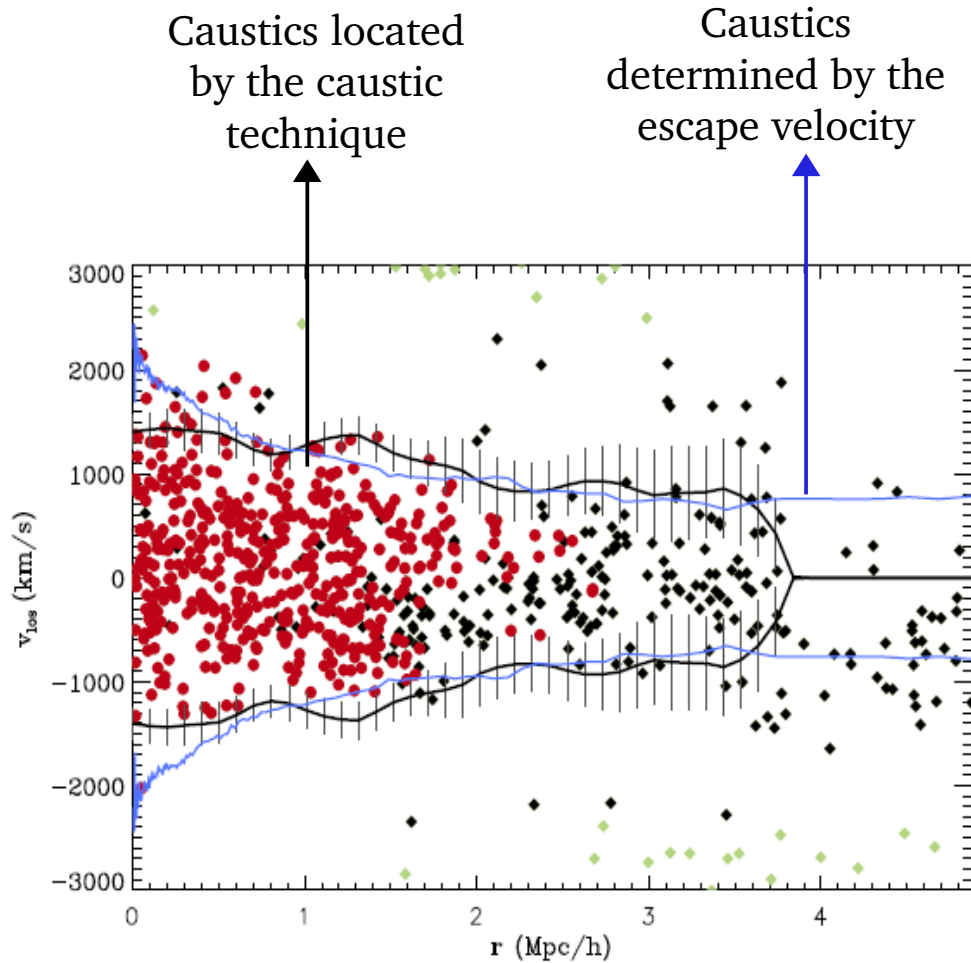
Coma cluster



van Haarlem et al. 1993

but clusters accrete mass anisotropically  
 → the velocity field can have a substantial non-radial random component

# $A(r) \Leftrightarrow$ escape velocity



The random components increase the caustic amplitude when compared to the spherical model

$$A_{\text{infall model}} < A_{\text{non-radial}}$$

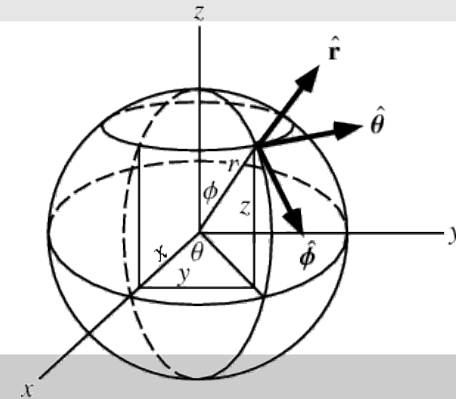
but clusters accrete mass anisotropically  
 $\rightarrow$  the velocity field can have a substantial non-radial random component

**Interpretation:**  $\mathcal{A}(\theta)$  is the average over a volume  $d^3\mathbf{r}$  of the square of the l.o.s. component of the escape velocity

$$\mathcal{A}^2(r) = \langle v_{esc,los}^2 \rangle$$

$$\langle v_{esc,los}^2 \rangle = -2\phi(r)g^{-1}(\beta)$$

$$\beta(r) = 1 - \frac{\langle v_{\theta}^2 \rangle + \langle v_{\phi}^2 \rangle}{2\langle v_r^2 \rangle}$$



$\mathcal{A}_{infall\ model} < \mathcal{A}_{non-radial}$

but clusters accrete mass anisotropically  
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**HOLDS INDEPENDENTLY OF THE DYNAMICAL STATE OF THE CLUSTER**

$$A_{infall\ model} < A_{non-radial}$$

but clusters accrete mass anisotropically  
 → the velocity field can have a substantial non-radial random component

## Mass estimate

$$\mathcal{A}^2(r) = \langle v_{esc,los}^2 \rangle$$

$$\langle v_{esc,los}^2 \rangle = -2\phi(r)g^{-1}(\beta)$$

mass of an infinitesimal shell

$$G dm = -2\phi(r)\mathcal{F}(r) dr = \mathcal{A}^2(r)g(\beta)\mathcal{F}(r) dr$$

where  $\mathcal{F}(r) = \frac{-2\pi G\rho(r)r^2}{\phi(r)}$  and

$\mathcal{F}_\beta(r) = \mathcal{F}(r)g(\beta)$  is a slowly changing function of  $r$

theoretical framework of the

**CAUSTIC TECHNIQUE**

$$GM(< r) = \mathcal{F}_\beta \int_0^r \mathcal{A}^2(r) dr$$



Developed in the '90s

(Diaferio & Geller, 1997;  
Diaferio, 1999)

Problem: it requires  
hundreds of galaxy  
redshifts

nowadays the  
required data are  
easily collectable

## CAUSTIC TECHNIQUE

Can be applied for

- MASS/POTENTIAL ESTIMATES
- IDENTIFICATION OF MEMBERS
- [ IDENTIFICATION OF SUBSTRUCTURES ]

to **simulated** and **real data**

We have applied the caustic technique to 3000 mock catalogs, built from 100 simulated clusters with  $M(<r_{200}) \geq 10^{14}h^{-1}M_{\odot}$  (Borgani et al. 2004)

1

arrange the galaxies in a binary tree according to a hierarchical method

2

select two thresholds to cut the tree: the largest group obtained from the upper-level threshold identifies the cluster candidate members

3

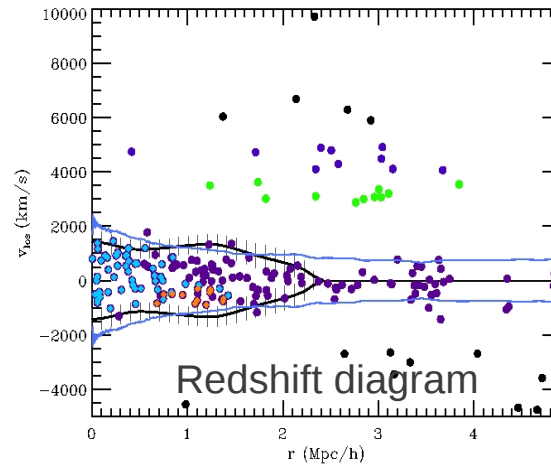
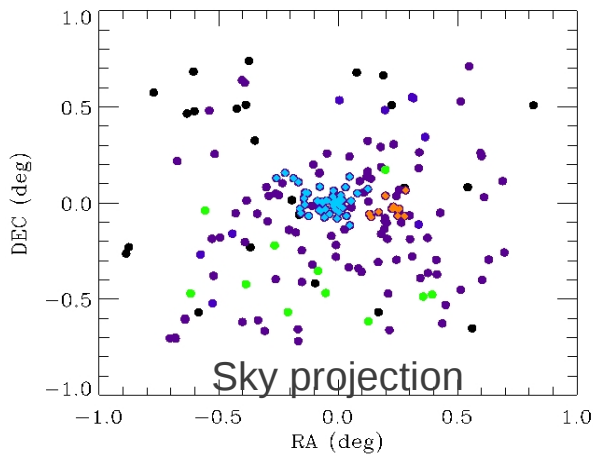
build the redshift diagram of all the galaxies in the field; locate the caustics, and identify the final cluster members

4

the caustic amplitude determines the escape velocity and mass profiles

four steps

# 1 Binary tree & $\sigma$ -plateau



binding energy

$$E_{ij} = -G \frac{m_i m_j}{R_p} + \frac{1}{2} \frac{m_i m_j}{m_i + m_j} \Pi^2$$

Binary Tree

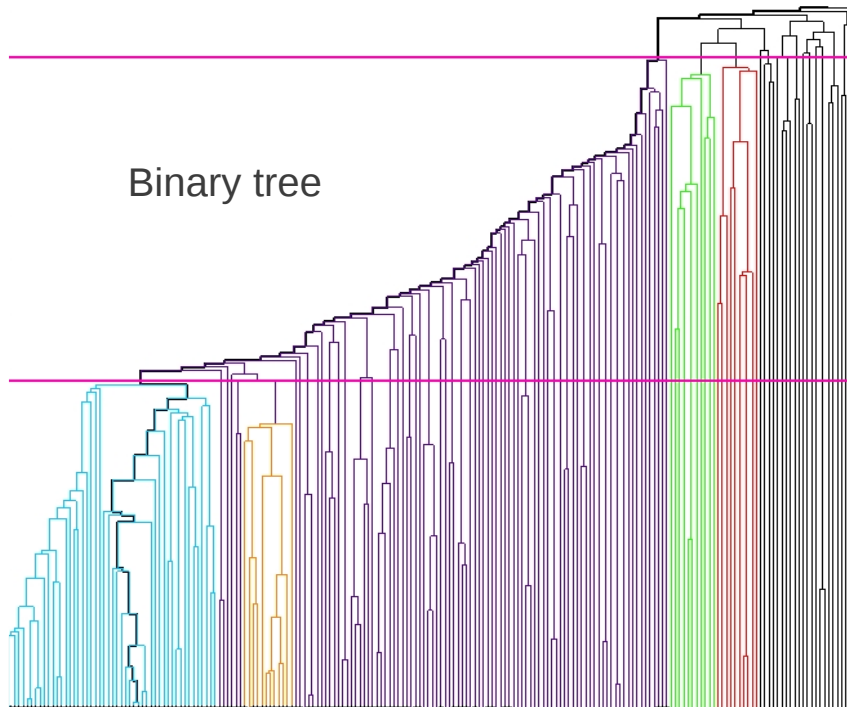
→  $\sigma$  plateau

→ main group members

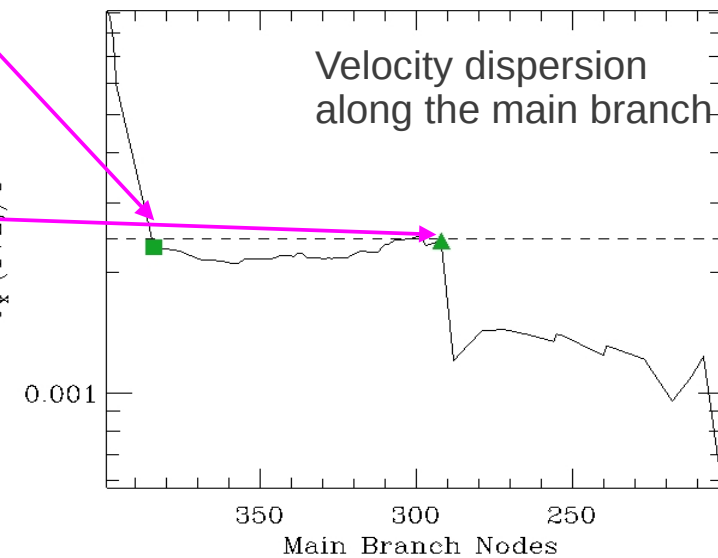
→ size  $R$ , center

Increasing main branch node

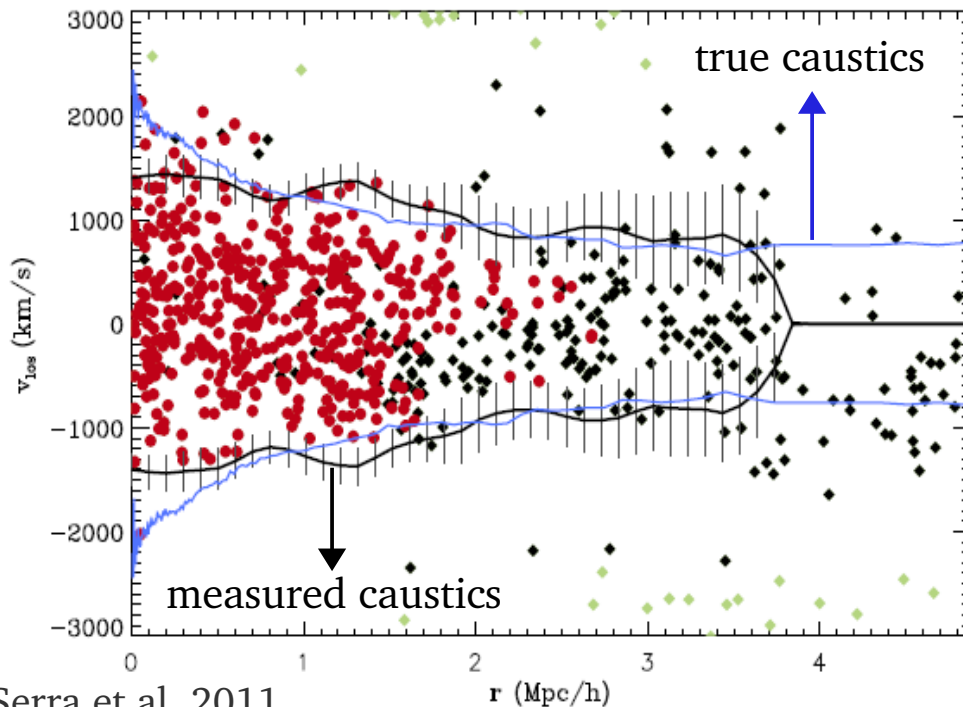
Binary tree



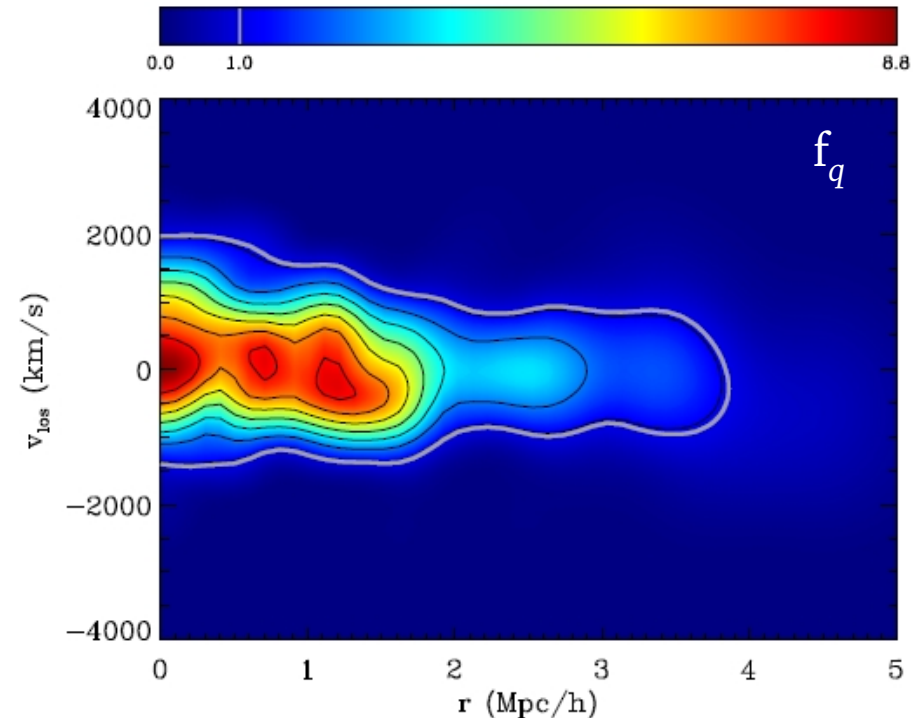
$\sigma_x (1+z)/c$



## 2 Redshift diagram



Serra et al. 2011



## 3 Caustic location

we choose the parameter  $\kappa$  that determines the correct caustic location as the root of the equation

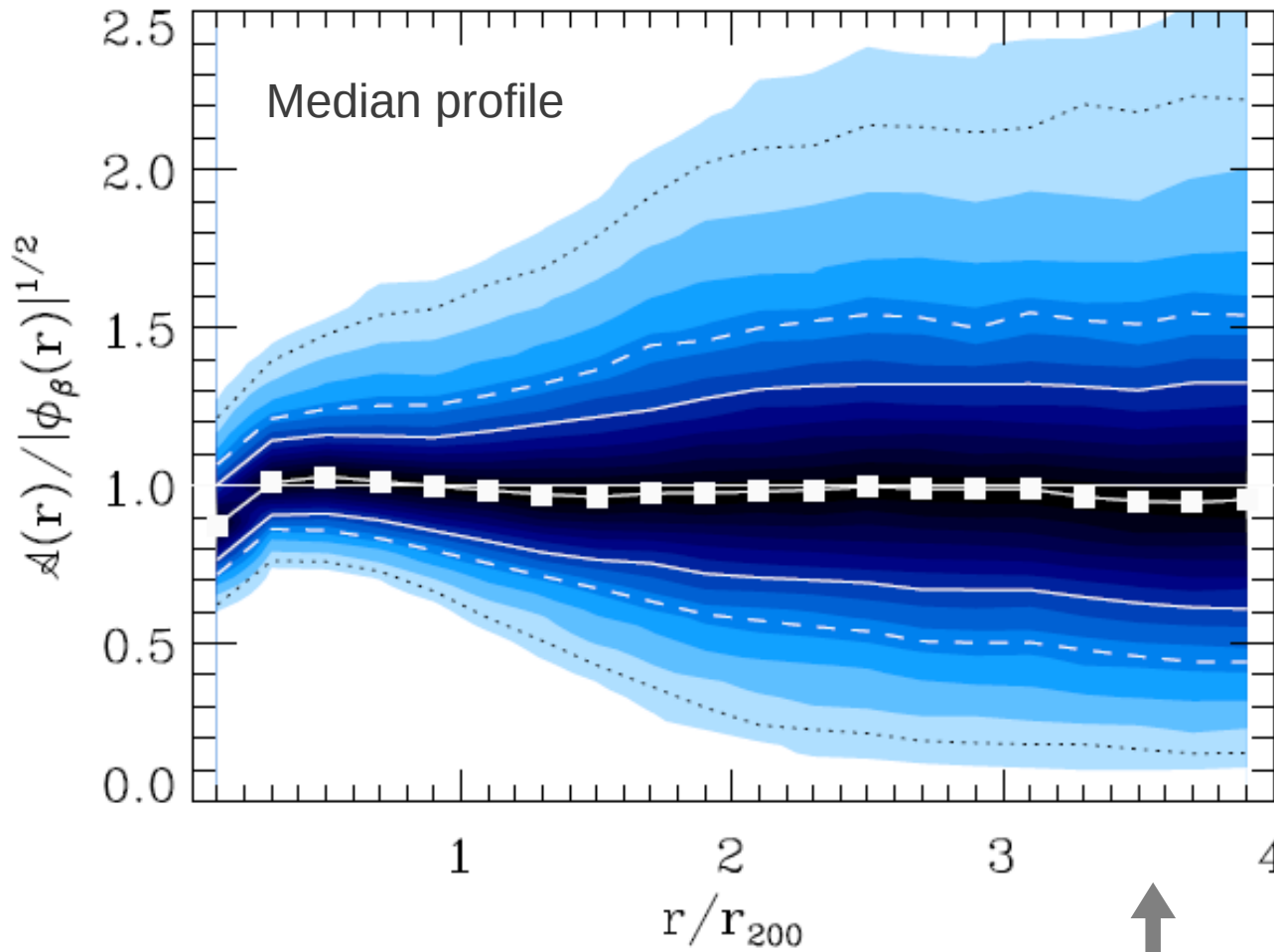
$$S(\kappa) \equiv \langle v_{\text{esc}}^2 \rangle_{\kappa, R} - 4 \langle v^2 \rangle = 0$$

distribution of  $N$  galaxies

$$f_q(\mathbf{x}) = \frac{1}{N} \sum_{i=1}^N \frac{1}{h_i^2} K \left( \frac{\mathbf{x} - \mathbf{x}_i}{h_i} \right)$$

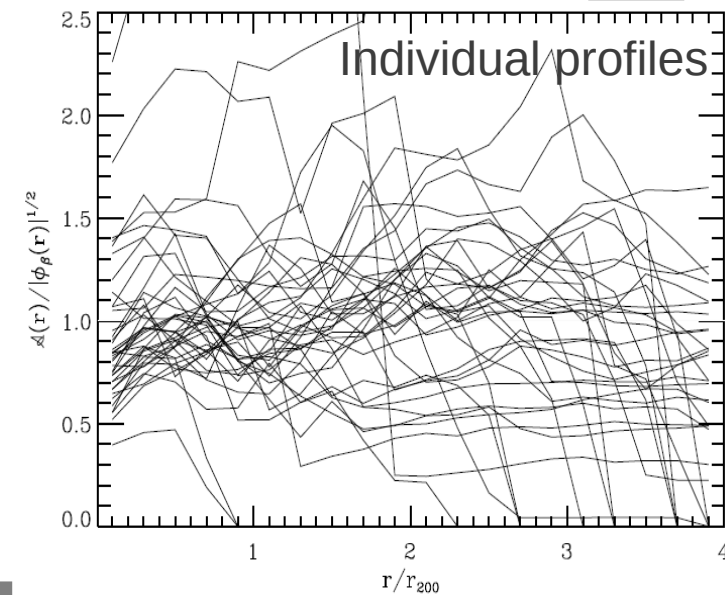
$$\mathbf{x} = (r, v)$$

### 3 Gravitational potential profiles



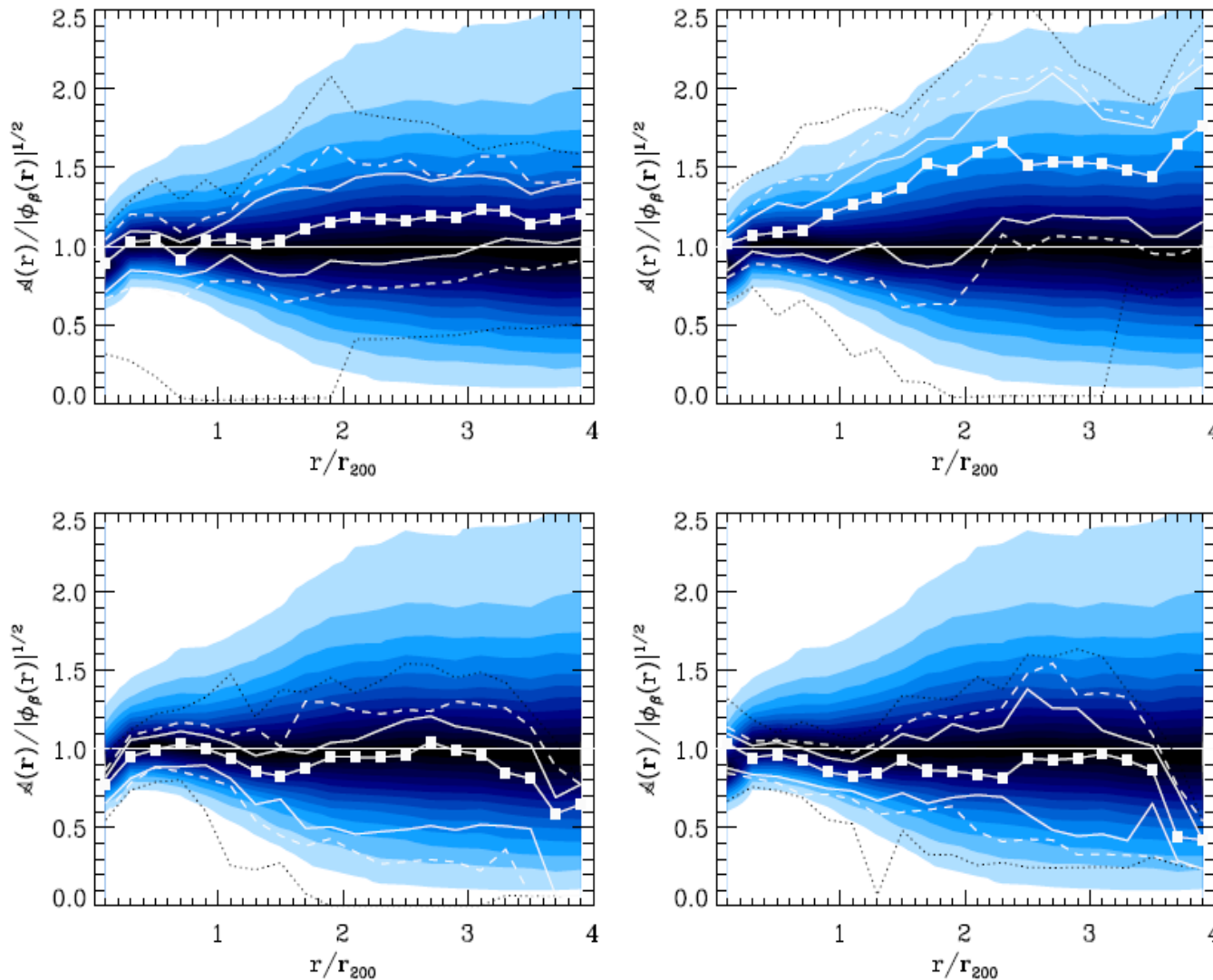
Escape velocity: better than 10% up to  $r \sim r_{200}$

3000 simulated clusters with  $M_{200} \geq 10^{14} h^{-1} M_{\odot}$



# Projection effects

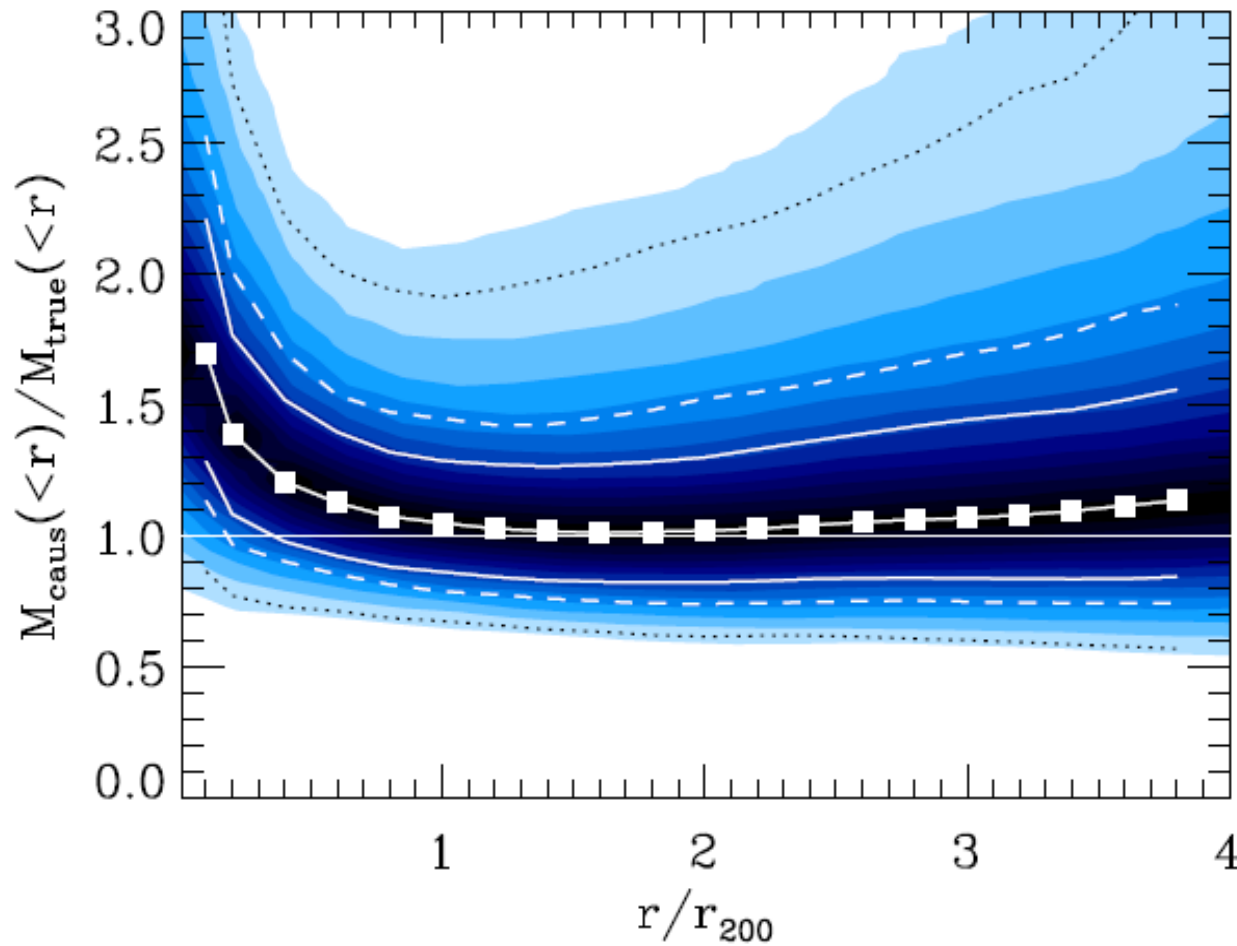
Examples of individual clusters



Escape velocity: better than 10% up to  $r \sim r_{200}$

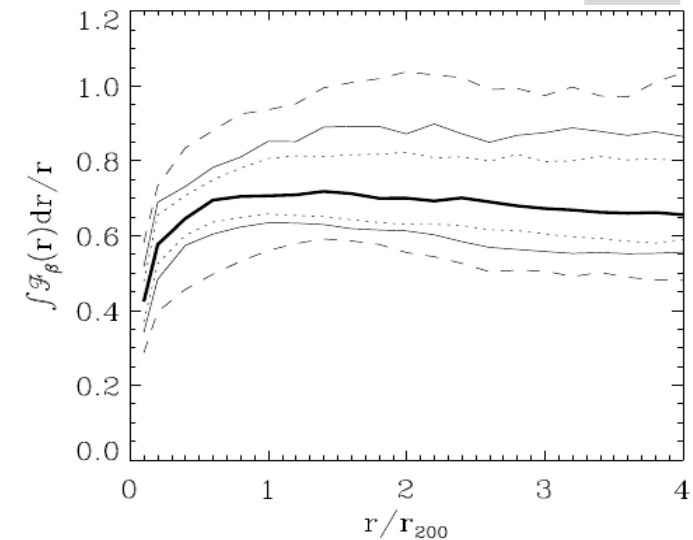
3000 simulated clusters with  $M_{200} \geq 10^{14} h^{-1} M_{\odot}$

## 4 Mass profiles



- $(0.6-4) r_{200} \rightarrow$  better than 15%
- $r < 0.6 r_{200} \rightarrow$  overestimation of the mass up to 70%

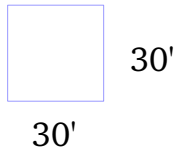
3000 simulated clusters with  
 $M_{200} \geq 10^{14} h^{-1} M_{\odot}$



$$GM(< r) = \mathcal{F}_{\beta} \int_0^r \mathcal{A}^2(r) dr$$

# Stacked cluster

$$|v_{\text{los}}^x - v_{\text{los}}^{\text{clus}}| = 2000 \text{ km/s}$$

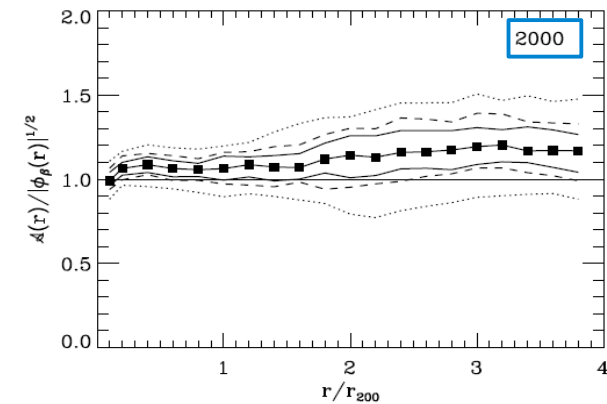
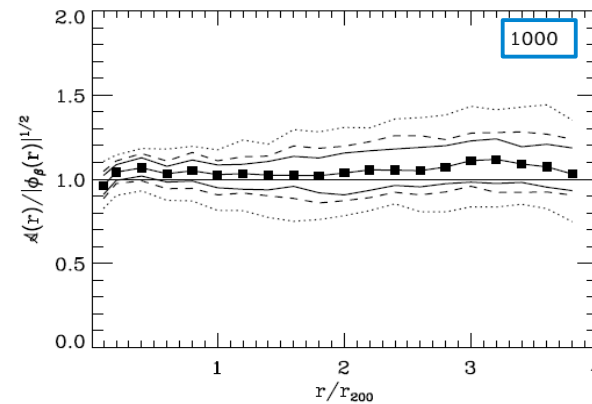
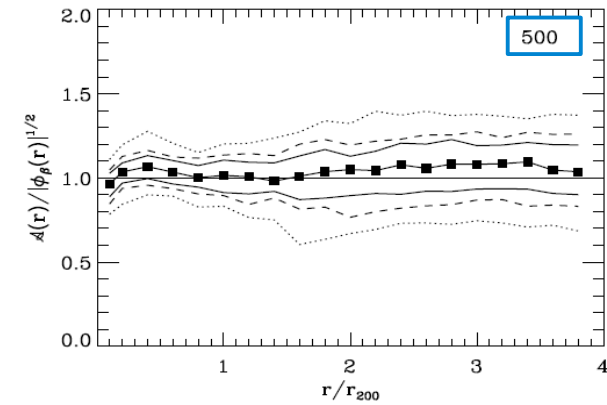
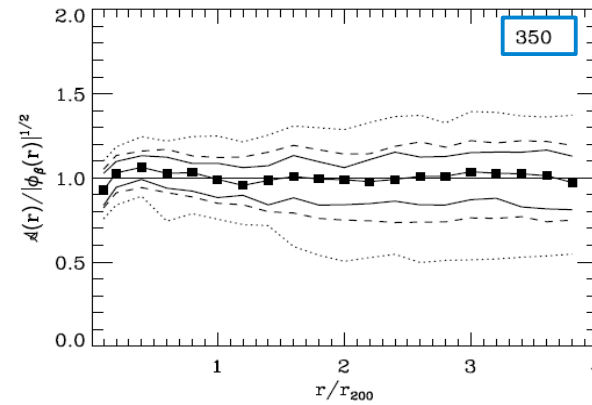
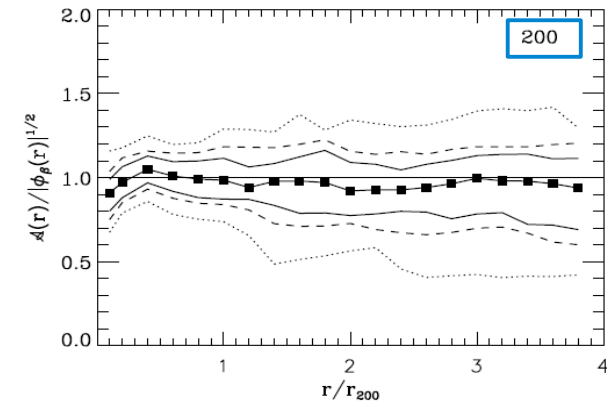
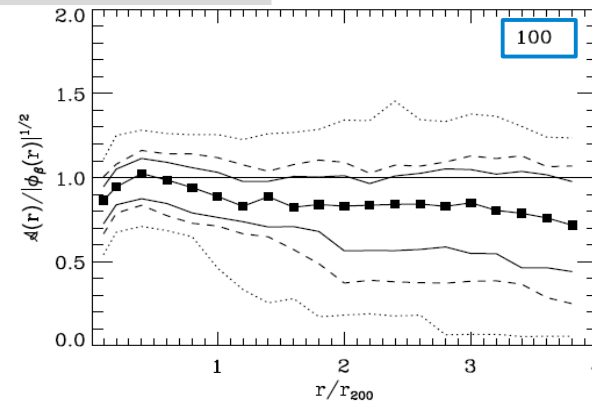


$$z^{\text{clus}} = 0.1$$

$$30' \rightarrow 2.46 \text{ Mpc/h}$$

spread decreases with  
increasing number of galaxies

measured caustic amplitude/true caustic amplitude



projected distance to the center



Can be applied for

- MASS/POTENTIAL ESTIMATES
- IDENTIFICATION OF MEMBERS

CAUSTIC TECHNIQUE

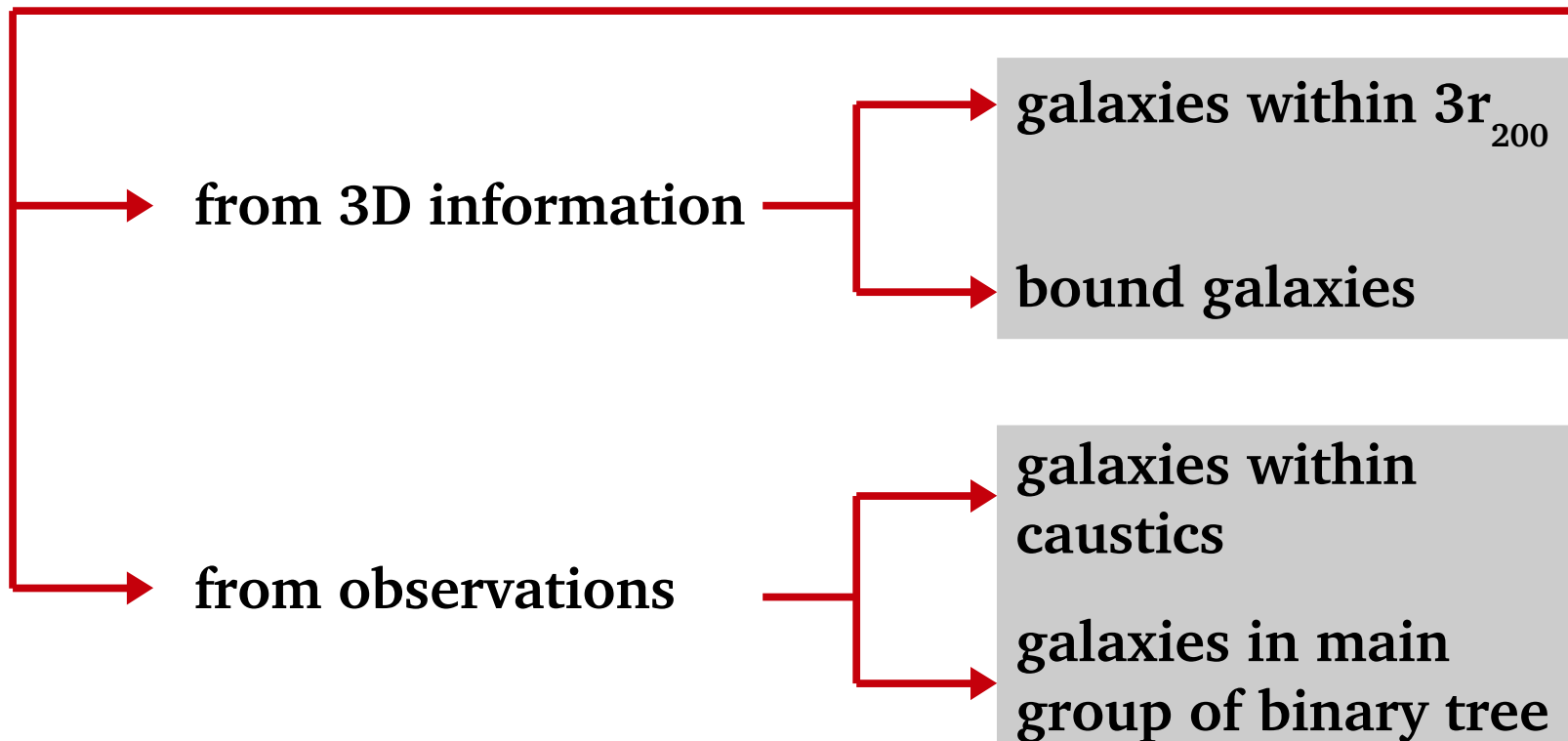
to **simulated** and **real data**

we have applied the caustic technique to 3000 mock catalogs, built from 100 simulated clusters with  $M(<r_{200}) \geq 10^{14}h^{-1}M_{\odot}$  (Borgani et al. 2004)

# Membership

To study the dependence of properties on the environment we need to know whether a galaxy is member of a cluster

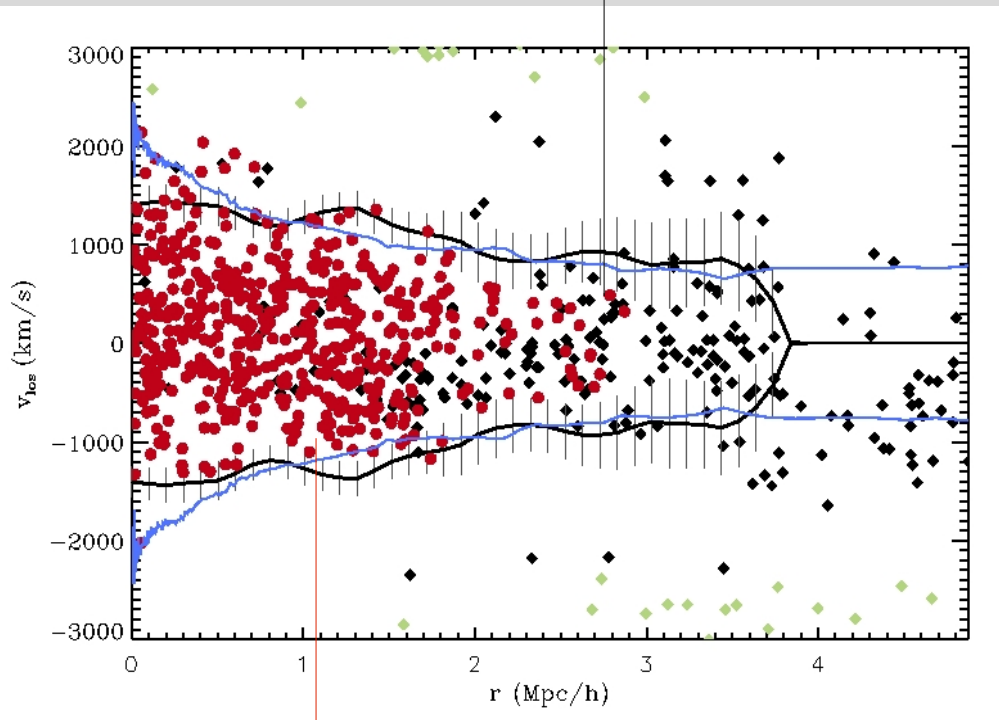
## How to define members?



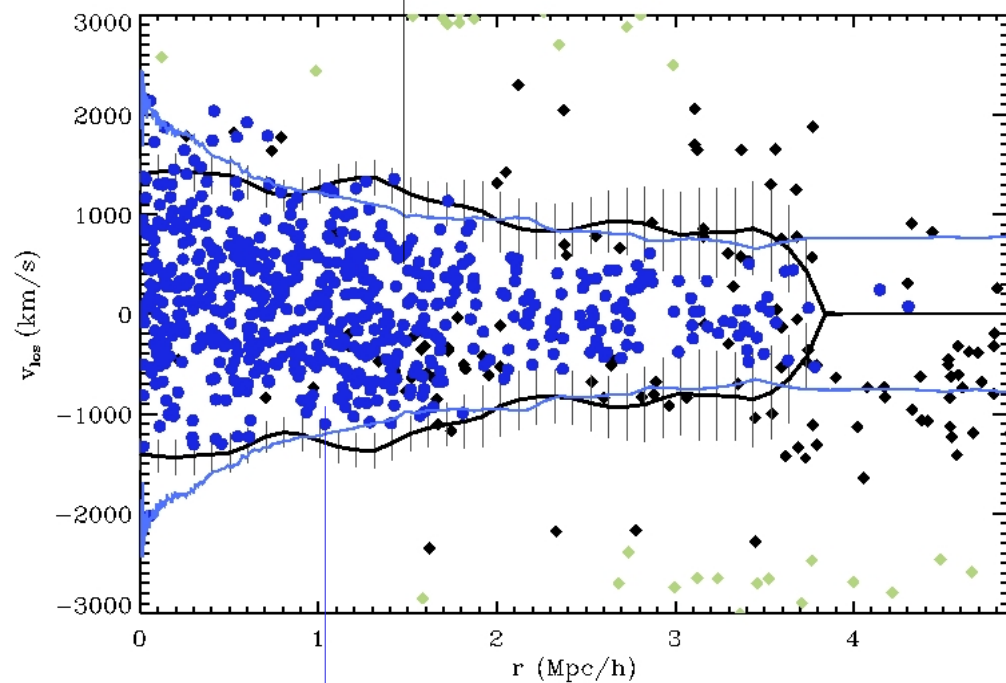
# Membership

mock catalogs → identified members

→ galaxies within the caustics ←



galaxies with  $r \leq 3r_{200}$



bound galaxies

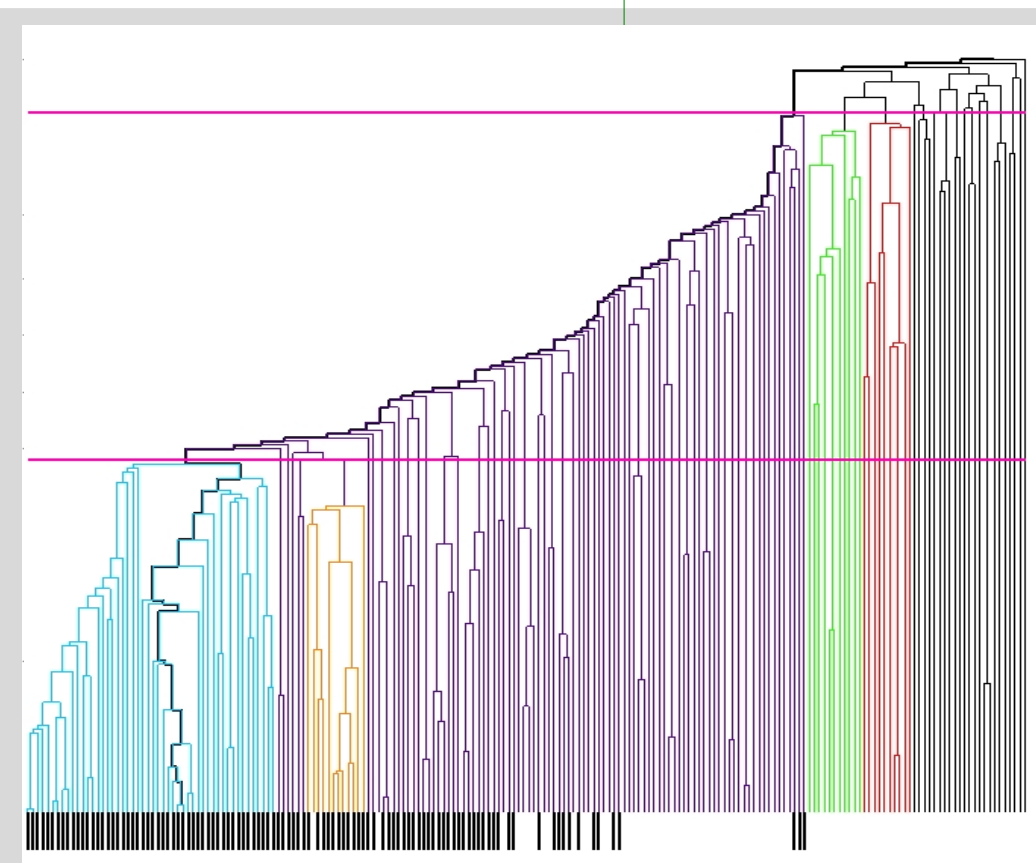
3D data → true members

Serra et al., in prep

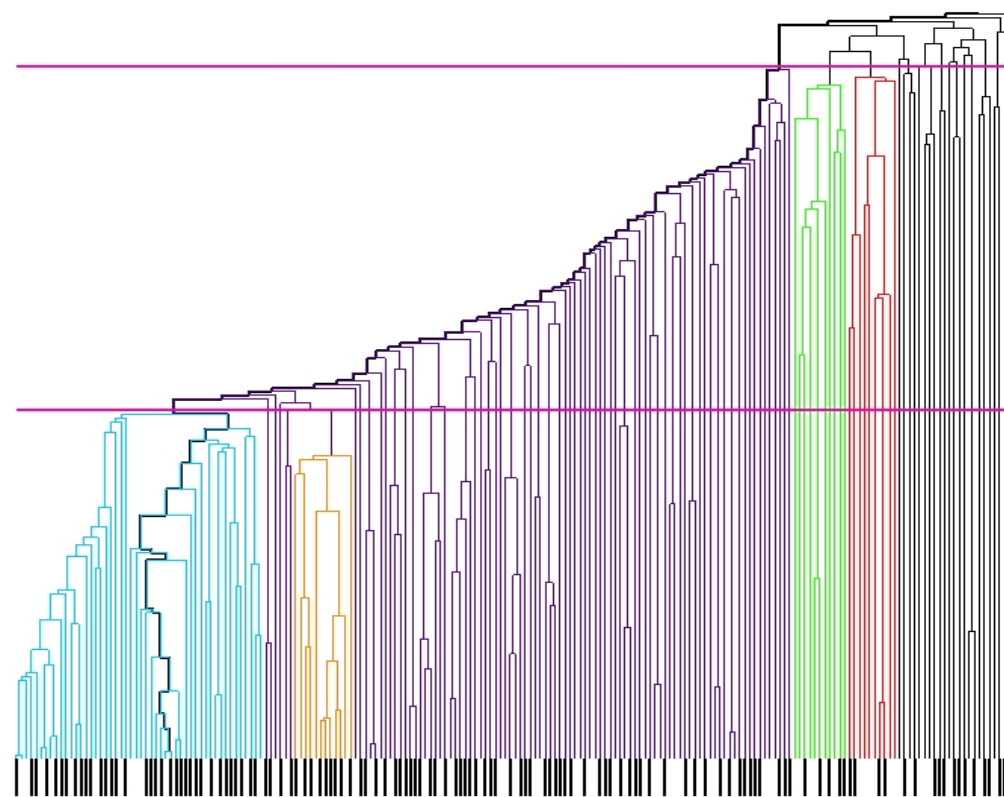
# Membership

mock catalogs → identified members

galaxies in the main group



galaxies with  $r \leq 3r_{200}$

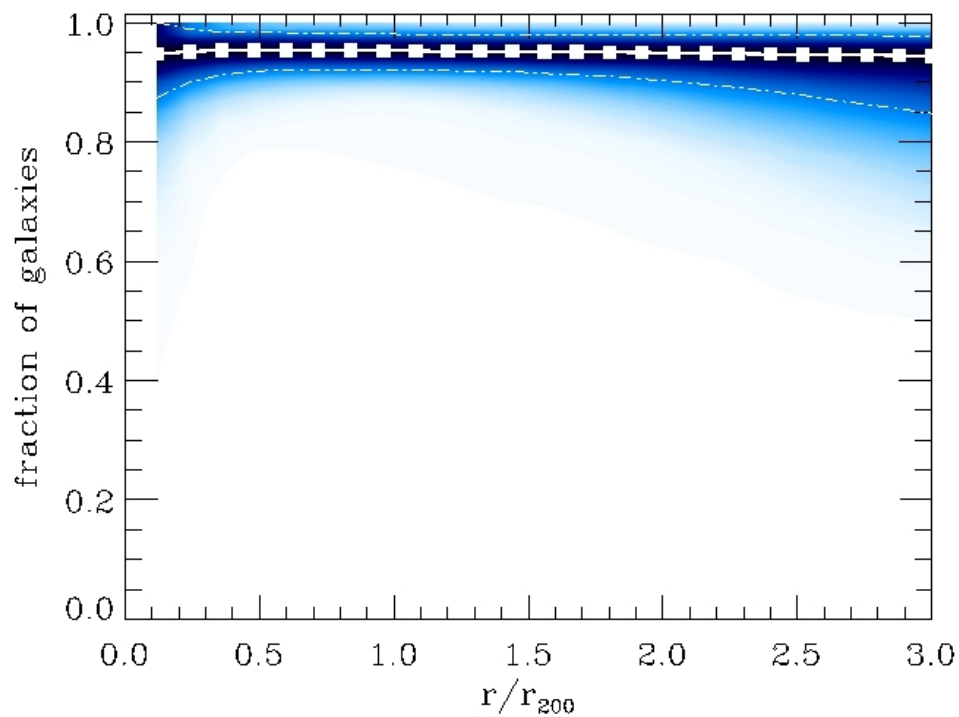


bound galaxies

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Serra et al., in prep

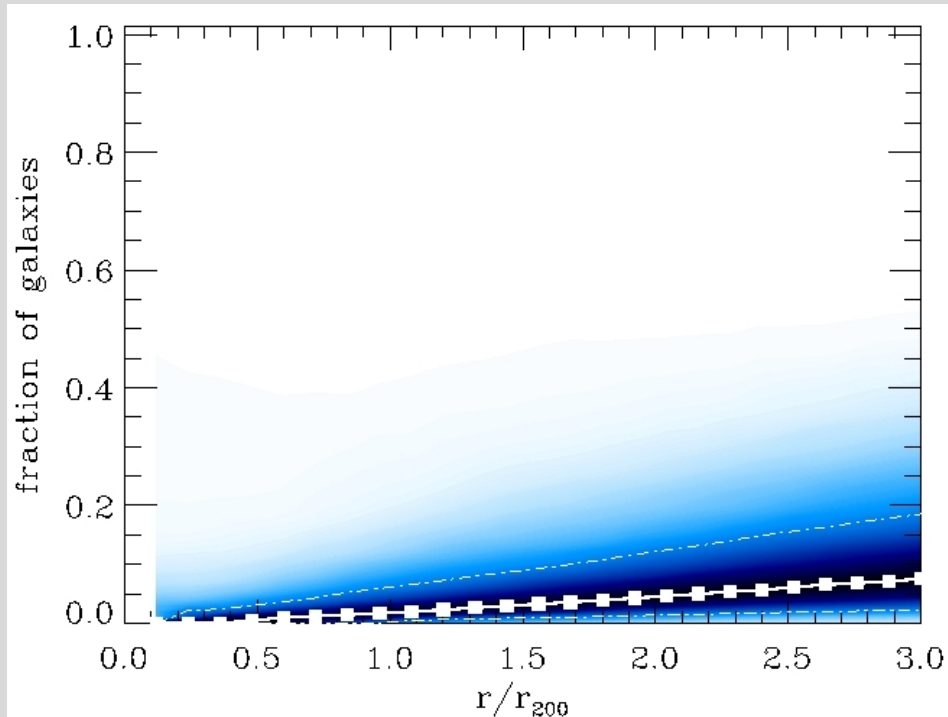
galaxies within caustics compared with bound galaxies



**completeness**

	$-1\sigma$	median	$+1\sigma$
$r_{200}$	0.919	0.953	0.981
$2r_{200}$	0.902	0.949	0.979
$3r_{200}$	0.848	0.944	0.978

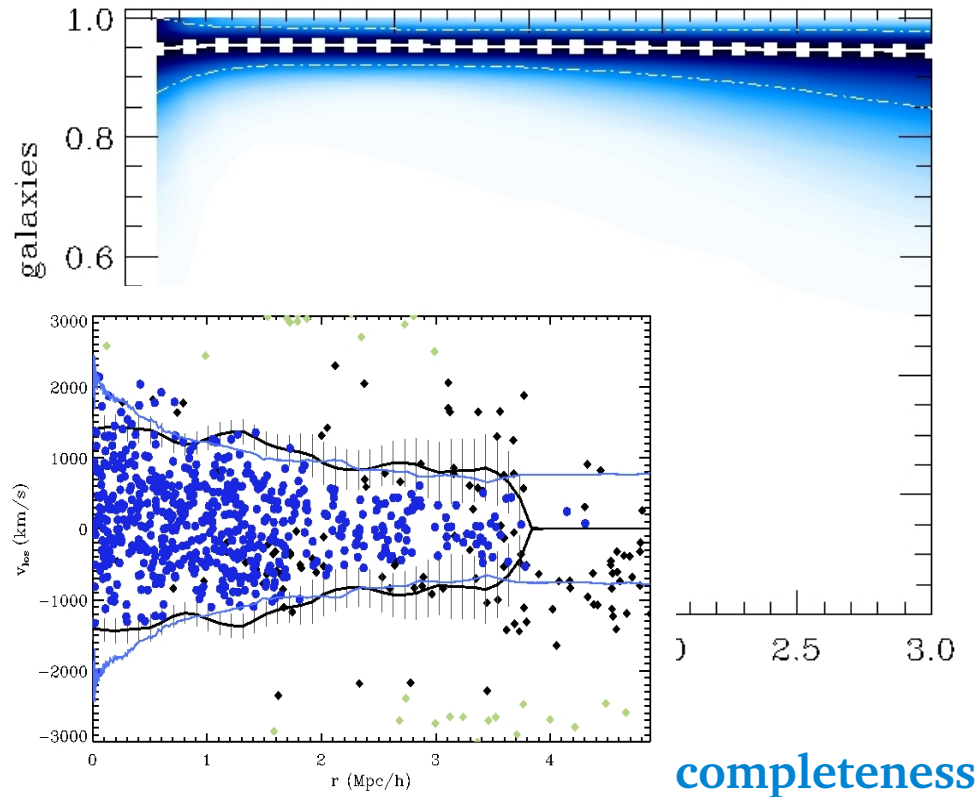
Serra et al., in prep



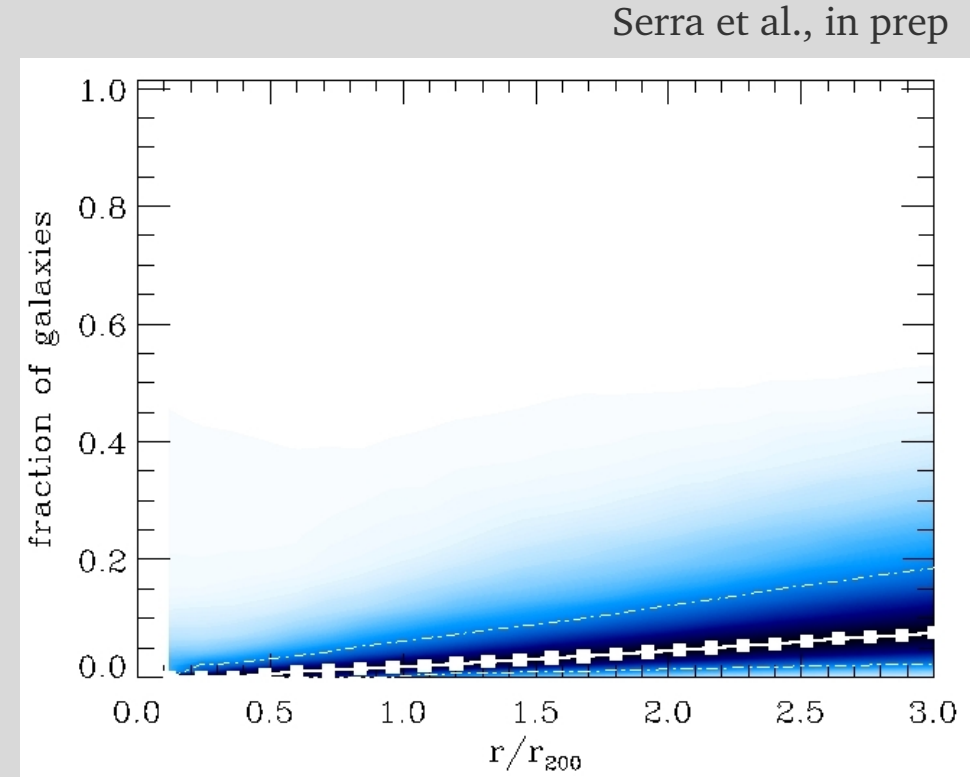
**contamination**

	$-1\sigma$	median	$+1\sigma$
$r_{200}$	0.005	0.020	0.066
$2r_{200}$	0.014	0.046	0.124
$3r_{200}$	0.072	0.075	0.186

galaxies within caustics compared with bound galaxies



completeness

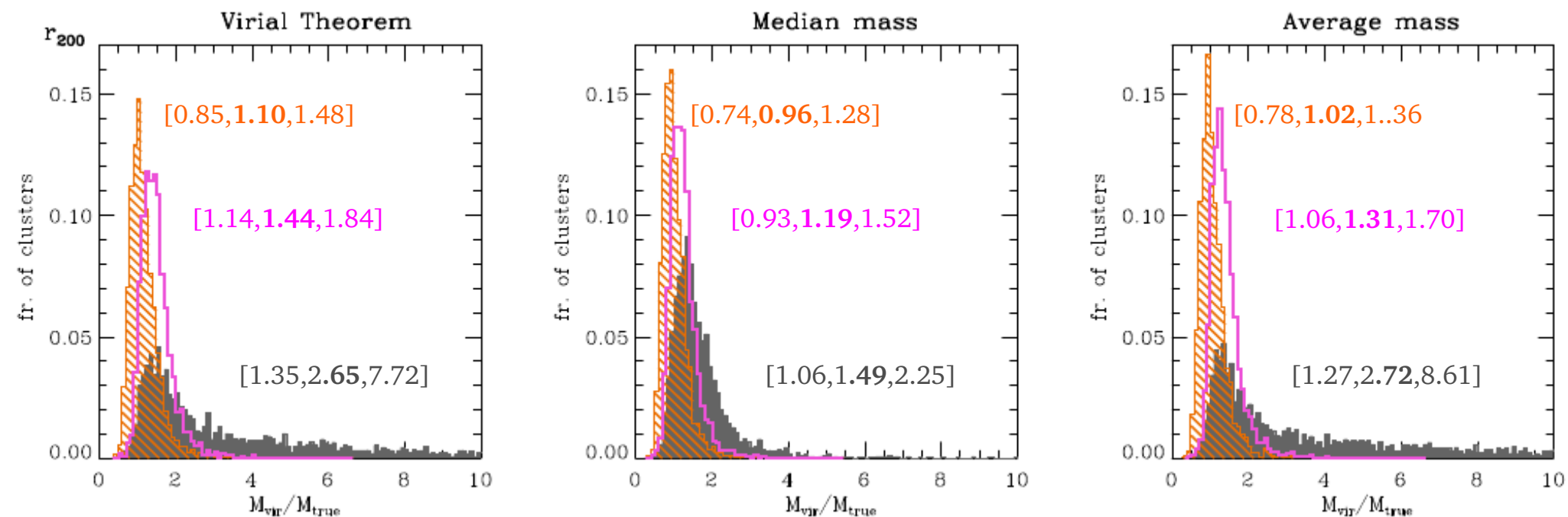


Serra et al., in prep

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## Cluster mass estimates with interlopers removed



- caustic location
- binary tree algorithm
- $3\sigma$  clipping

The caustic location performs systematically better in removing interlopers and, on average, the bias in the mass estimate is minimized

Can be applied for

- MASS/POTENTIAL ESTIMATES
- IDENTIFICATION OF MEMBERS

**CAUSTIC TECHNIQUE**

to simulated and **real data**



## Equation of state of dark matter

if we do not assume that the pressure profiles are small compared to the mass-energy density

$$\nabla^2 \Phi \approx \underbrace{\frac{4\pi G}{c^2} (c^2 \rho + p_r + 2p_t)}_{\nabla^2 \Phi_N}$$

kinematic mass profile  $\rightarrow \Phi$

lensing mass profile  $\rightarrow$

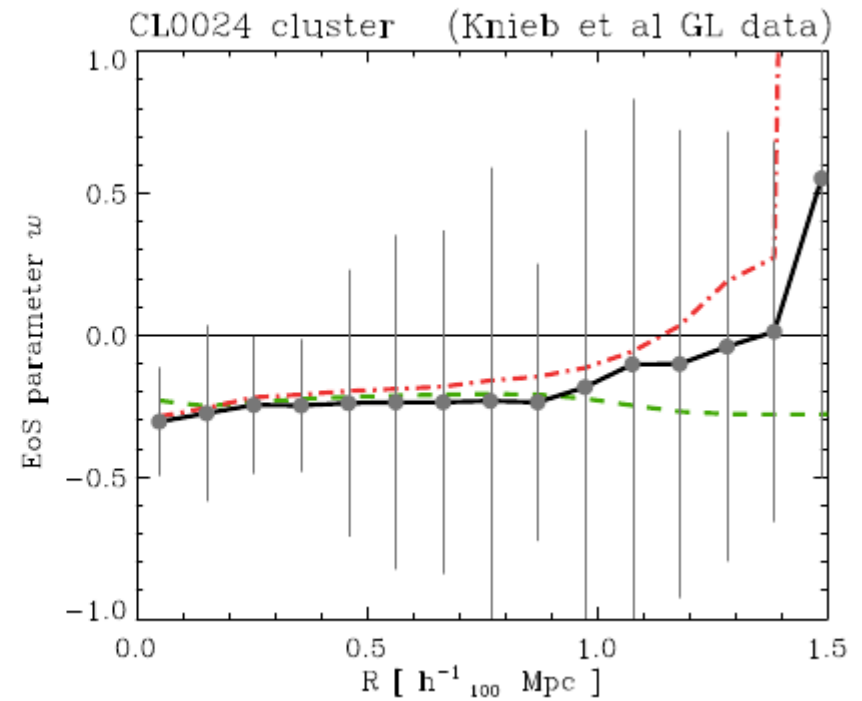
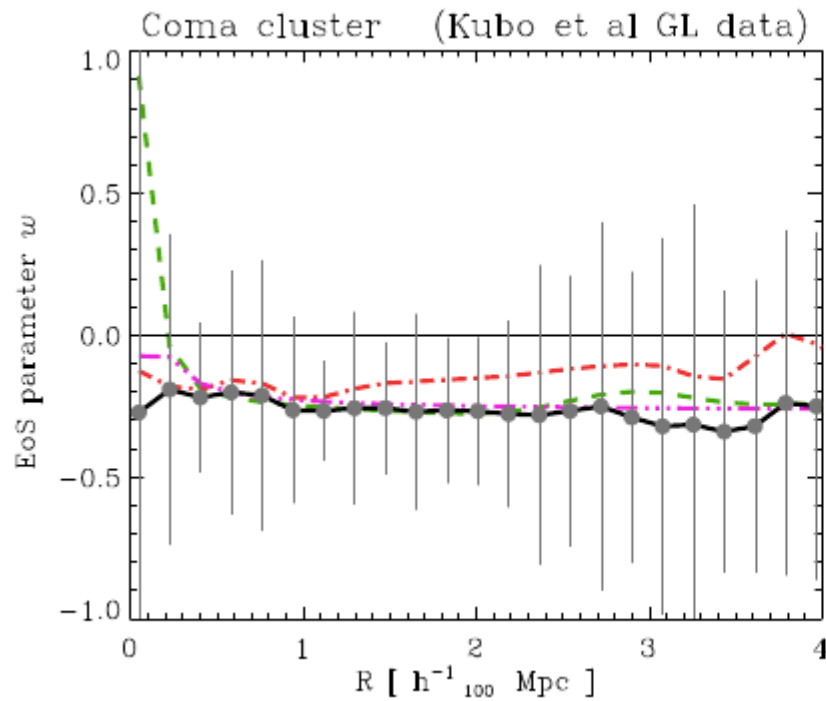
$$\Phi_L = \frac{1}{2}\Phi_N + \frac{1}{2}\Phi$$

density and pressure profiles of dark matter

$$w = \frac{1}{c^2} \frac{p_r + 2p_t}{3\rho}$$

$w_{DM}$

## Equation of state of dark matter



- · — · — mass from Jeans analysis,  $\beta=0$
- mass from Jeans analysis,  $\beta$ =linear fit from sim. clusters
- - - mass by the caustic technique
- · · mass from Łokas & Mamon, 2003

Escape velocity → members



gravitational potential

M/L up to 40



CDM dominated systems

Interlopers: do they bias the velocity dispersion estimate?

ANY

SYSTEM

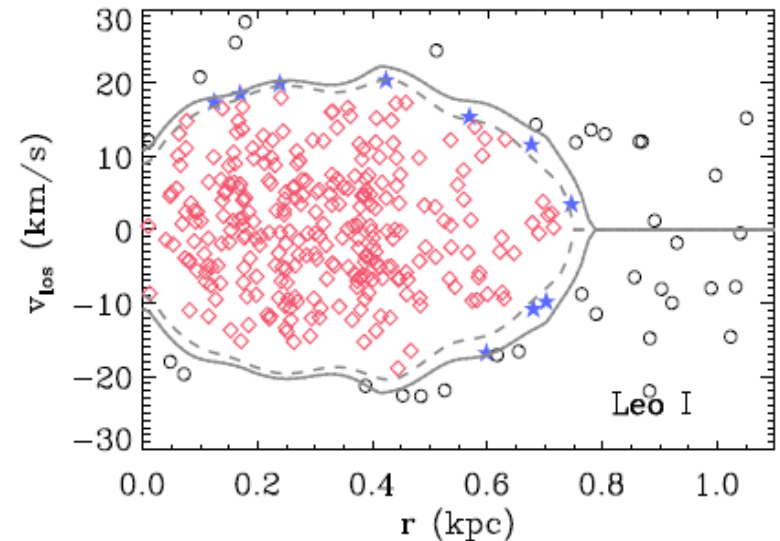
Galaxy clusters

**Dwarf spheroidal galaxies**

check the constant  $M(<300\text{pc})$ -Luminosity relation found by Strigari et al. 2008 in LCDM

analysis whether we can solve the issue of high M/L in MOND

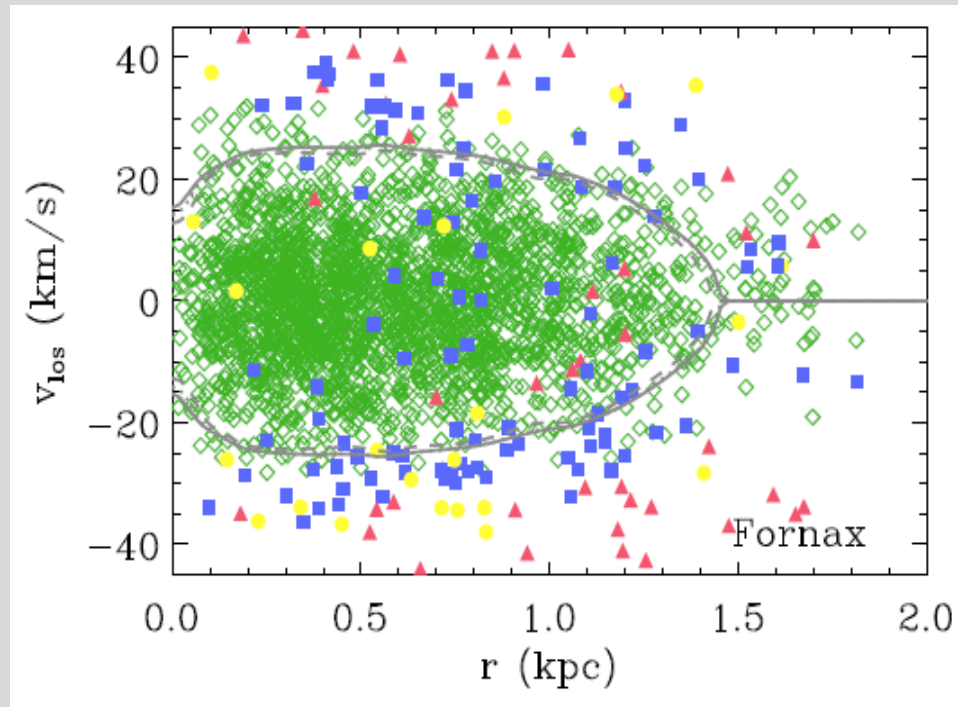
The technique is highly versatile and allows for the detection of interlopers also at small scales



## Dwarf Spheroidal Galaxies

(Fornax, Carina, Leo I, Sculptor, Sextans)

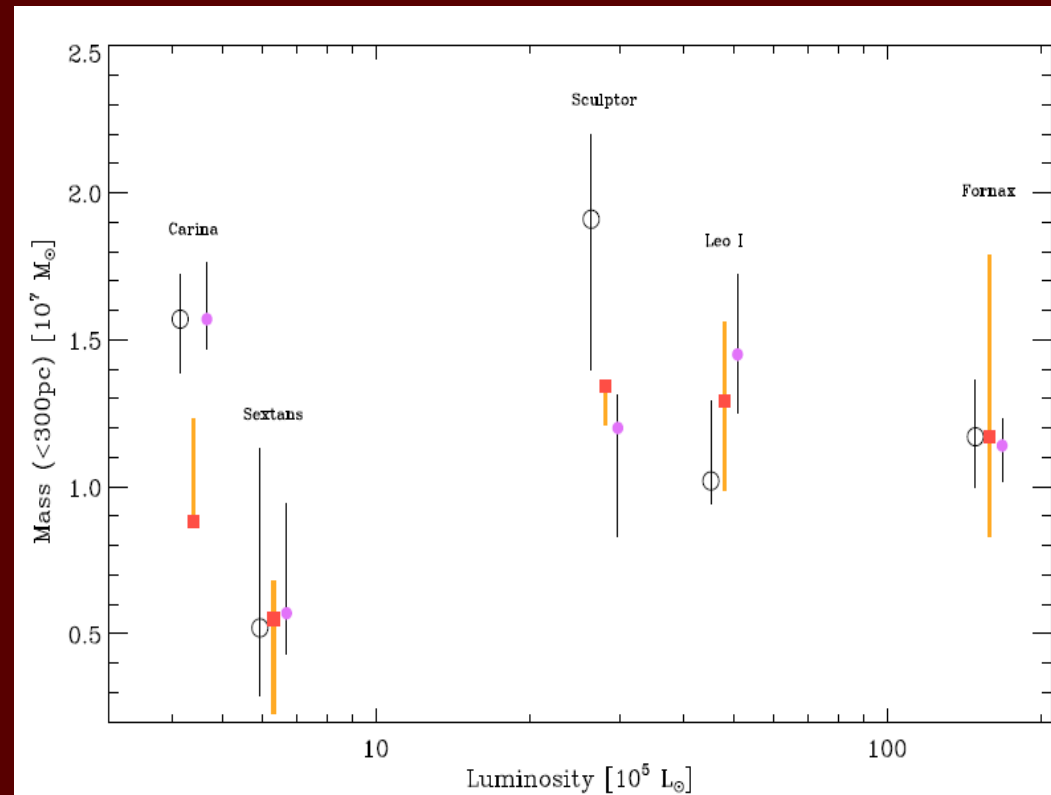
Escape velocity  $\rightarrow$  members



Walker et al. 2007

- ▲  $p \leq 0.5$
- $0.5 \leq p \leq 0.7$
- $0.7 \leq p \leq 0.9$
- ◇  $0.9 \leq p$

Mass ( $< 300\text{pc}$ ) in  $\Lambda\text{CDM}$



- Strigari et al. 2008
- Serra, Angus & Diaferio, 2010
- Angus 2008

## Dwarf spheroidal galaxies

 $\Lambda$ CDMlarge amount  
of dark matterHigh M/L  
(up to 40)

MOND

no dark matter?  
M/L=1-3Carina, Sextans and Sculptor have  
M/L too largeidentification of members with  
caustic location

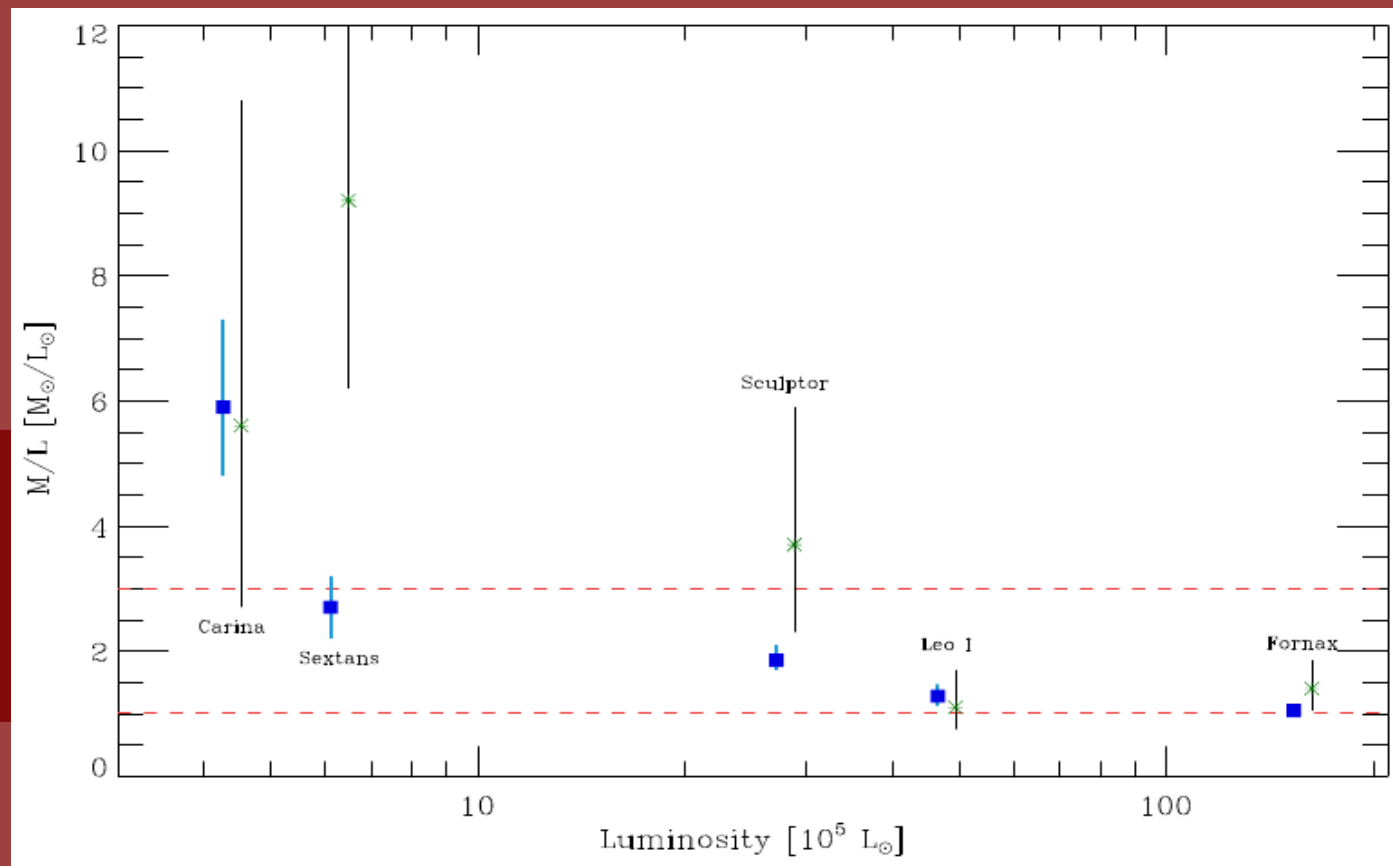
M/L in [1-3]

still a problem

## Dwarf Spheroidal Galaxies

(Fornax, Carina, Leo I, Sculptor, Sextans)

### Mass-to-light ratios in MOND



## Real data

The mass profiles calculated with the caustic technique for two real clusters (CL0024 and Coma), combined with gravitational lensing data, yields slightly negative profiles for the dark matter equation of state, that might be interpreted in the framework of alternative theories of gravity (Serra & Dominguez, 2010).

The membership analysis applied to 5 dSphs has yielded results compatible with previous mass estimates and decreases the mass-to-light ratios in MOND for two dwarfs, which turn out to be consistent with stellar population synthesis models (Serra, Angus & Diaferio, 2010).

## Systematics: mass/potential profiles, membership

The caustic technique and gravitational lensing are the only two methods available to measure the mass profile of clusters beyond the virial radius without assuming dynamical equilibrium

~200 gxs in a field of 2.46 Mpc/h x 2.46 Mpc/h are enough to have an accurate escape velocity profile

The applications of the caustic technique to a large sample of simulated clusters demonstrated that the escape velocity is recovered with ~25% 1- $\sigma$  uncertainty and the mass profile with ~50% 1- $\sigma$  uncertainty up to  $4r_{200}$  (Serra et al. 2011).

The spread mostly originates from the assumption of spherical symmetry the same cluster when looked from another l.o.s. gives different caustics, but the errors account for that

The technique is able to detect true members with a completeness of ~94% and a contamination of ~8% at  $3r_{200}$  (Serra & Diaferio in prep.)



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**THANK YOU!!!**