Interaction between dark energy and dark matter

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Outline

- Why do we need the interaction between DE&DM?
- Is the interaction between DE&DM allowed by observations?
- Perturbation theory when DE&DM are in interaction
- How to understand the interaction between DE&DM?

Known? *Unknown*!

5% 95%

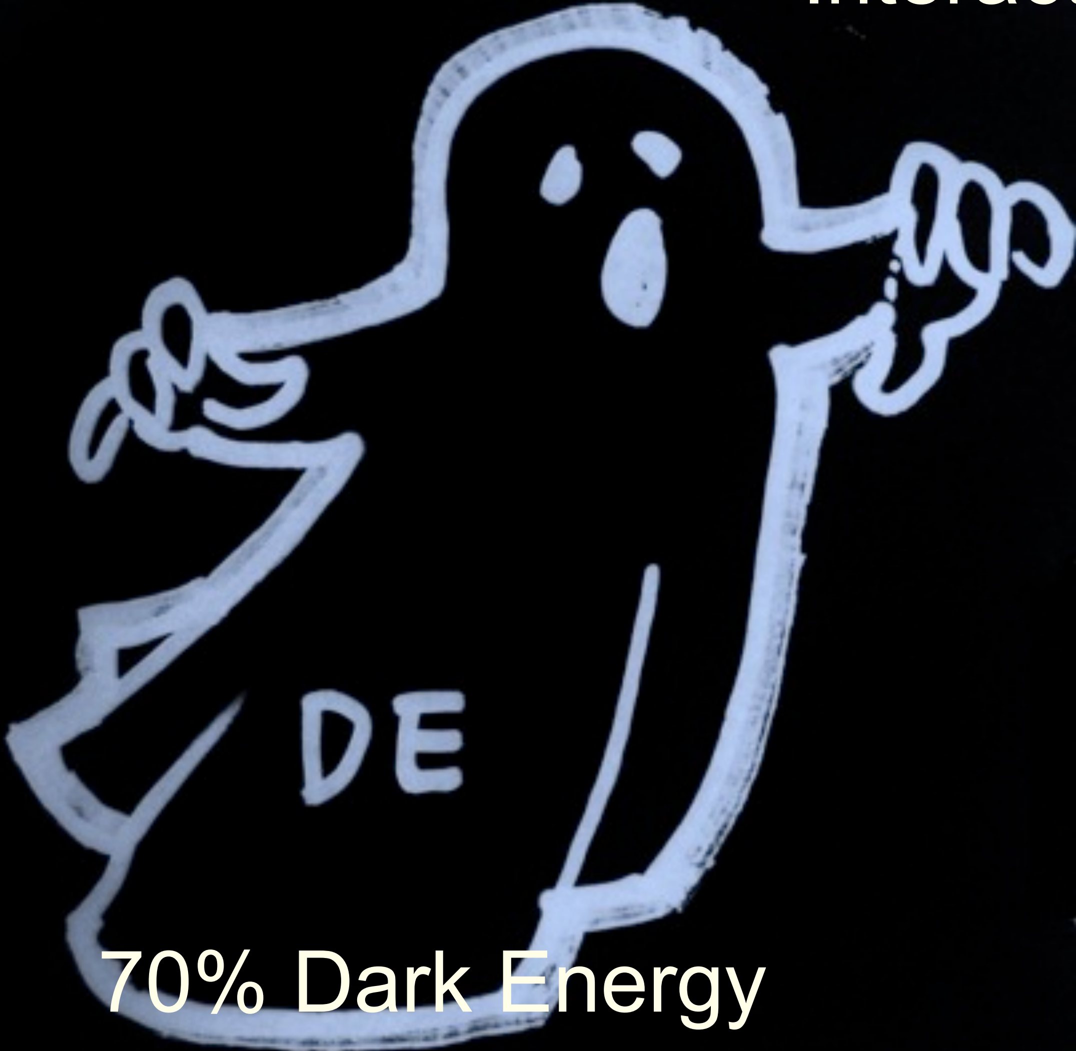
知之為知之，	5%	Present your understanding when you understand;
不知為不知，	95%	recognize your not understanding when you don't understand;
是知也。		that's the true meaning of understanding.

—論語為政篇

— By Confucius

(Analects of Confucius)

Interaction



70% Dark Energy

25% Dark Matter

Why do we want to introduce the

- DE--Λ?
  1. QFT value 123 orders larger than the observed
  2. Coincidence problem:

     Why the universe is accelerating just now?

     In Einstein GR: Why are the densities of DM and DE of precisely the same order today?
- Reason for proposing Quintessence, tachyon field, Chaplygin gas models etc.

  No clear winner in sight

  Suffer fine-tuning

Scaling behavior of energy densities

- A phenomenological generalization of the LCDM model is

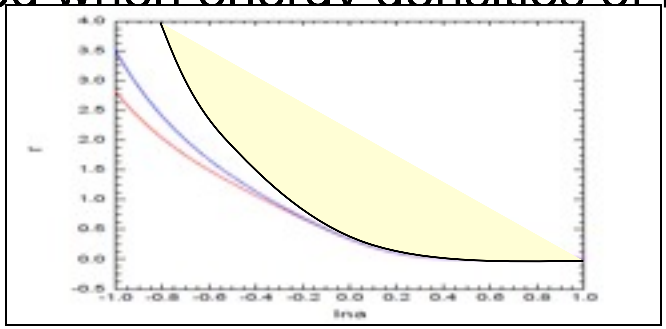
$$\frac{\rho_M}{\rho_X} = r_0 \left(\frac{a_0}{a}\right)^{\xi}$$

$\xi = 3$ LCDM model,

$\xi = 0$ Stationary ratio of energy densities

$\xi < 3$ Coincidence problem less severe than LCDM

The period when energy densities of DE and DM are comparable is longer



The coincidence problem is less acute

$\xi < 3$

can be achieved by a suitable interaction between DE & DM

$$\dot{\rho}_M + 3H\rho_M = Q\ , \quad \dot{\rho}_X + 3H\left(1 + w_X\right)\rho_X = -Q\ .$$

Do we need to live with Phantom?

- Degeneracy in the data.

SNe alone however are consistent with w in the range, roughly

$-1.5 \leq w_{eff} \leq -0.7$ Hannestad et al, Melchiorri et al, Carroll et al

WMAP 3Y(06) w=-1.06{+0.13,-0.08}

w<-1 from data is strong!

- One can try to model w<-1 with scalar fields like quintessence. But that requires **GHOSTS**: fields with negative kinetic energy, and so with a Hamiltonian not bounded from below:

$$3\, M_4^2\, H^2 = -\, (\phi')^2/2 + V(\phi)$$

`Phantom field', Caldwell, 2002

- **Phantoms an**

Theoretical prejudice against w<-1 is strong!

Exorcising w<-1

MAYBE NOT!!

- ***Conspiracies are more convincing if they DO NOT rely on supernatural elements!***

Ghostless explanations:

1) Modified gravity affects **EVERYTHING,** with the effect to make w<-1.

S. Yin, B. Wang, E.Abdalla, C.Y.Lin, arXiv:0708.0992, PRD (2007)

A. Sheykhi, B. Wang, N. Riazi, Phys. Rev. D 75 (2007) 123513

R.G. Cai, Y.G. Gong, B. Wang, JCAP 0603 (2006) 006

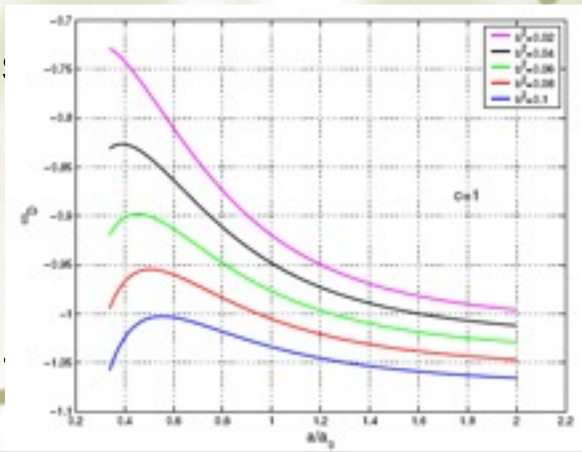
2) Another option: Interaction between DE and DM

Super-acceleration (w<-1) as signature of dark

B. Wang, Y.G.Gong and E. Abdalla, Phys.Lett.B624(2005)141

B. Wang, C.Y.Lin and E. Abdalla, Phys.Lett.B637(2006)357.

S. Das, P. S. Corasaniti and J. Khoury, Phys.Rev. D73 (2006) 083509.



The Interaction Between DE & DM

Phenomenological interaction forms:

$$\dot{\rho}_M + 3H\rho_M = Q\ , \quad \dot{\rho}_X + 3H\,(1+w_X)\,\rho_X = -Q\ .$$

For Q > 0 the energy proceeds from DE to DM

Phenomenological forms of Q

(1) $Q = \delta H(\rho_{DM} + \rho_{DE})$, (2) $Q = \delta H \rho_{DM}$ and (3) $Q = \delta H \rho_{DE}$

Is the interaction between DE & DM allowed by observations?

Universe expansion history observations:

SNIa+CMB+BAO+Age constraints

B. Wang, Y.G.Gong and E. Abdalla, Phys.Lett.B(05),

B. Wang, C. Lin, E. Abdalla, PLB (06)

B.Wang, J.Zang, C.Y.Lin, E.Abdalla, S.Micheletti, Nucl.Phys.B(07)

C.Feng, B.Wang, Y.G.Gong, R.K.Su, JCAP (07);

C.Feng, B.Wang, E.Abdalla, R.K.Su, PLB(08),

J.He, B.Wang, JCAP(08),

J.H. He, B.Wang, P.J.Zhang, PRD(09)

J.H.He, B.Wang,E.Abdalla, PRD(11),   X.D.Xu, J.H.He, B.Wang, PLB (11)

Galaxy cluster scale test

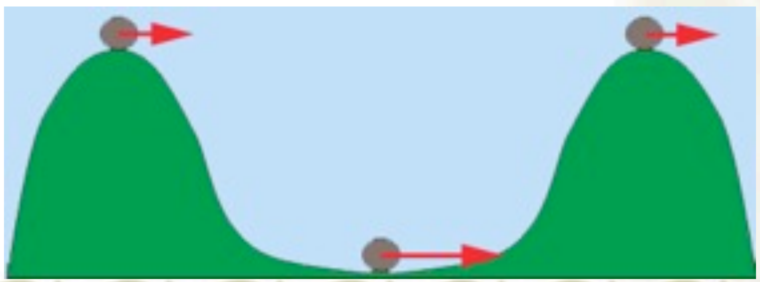
E. Abdalla, L.Abramo, L.Sodre, B.Wang, PLB(09)
J.H.He, B.Wang, Y.P.Jing, JCAP(09)
J.H.He, B.Wang, E.Abdalla, D.Pavon, JCAP(10)

Signature of the interaction in the CMB

Sachs-Wolfe effects:

- non-integrated — photons' initial conditions
- integrated
  - Early ISW
  - Late ISW — has the unique ability to probe the **"size"** of DE: **EOS,** the speed of sound

Signature of the interaction between DE and DM?



Perturbation theory when DE&DM are in interaction

Choose the perturbed spacetime

$$ds^2 = a^2\Big\{ -(1+2\phi)d\tau^2 + 2\partial_i B\, d\tau dx^i + \Big[(1-2\psi)\delta_{ij} + 2\partial_i\partial_j E\Big] dx^i dx^j \Big\}.$$

DE and DM, each with energy-momentum tensor $T^{\mu\nu}_{(\lambda);\mu} = Q^{\nu}_{(\lambda)}$

$Q^{\nu}_{(\lambda)}$ denotes the interaction between different components.

The perturbed energy-monentum tenser reads

$$\begin{aligned}
\delta T^{00} &= \frac{1}{a^2}(\delta\rho - 2\psi\rho) \\
\delta T^{i0} &= \frac{1}{a^2}[(p+\rho)V^i + p\partial^i B] \\
\delta T^{ij} &= \frac{1}{a^2}[\delta p \delta^{ij} - p(2\phi\delta^{ij} + D^{ij}E)] \\
\delta T^{0i} &= \delta T^{i0}
\end{aligned}$$

,

Perturbation Theory

The perturbed Einstein equations

$$\delta g^{\nu}_{\mu} \Rightarrow \delta R^{\nu}_{\mu} \Rightarrow \delta G^{\nu}_{\mu} \qquad\qquad \delta T^{\nu}_{\mu}$$

$$\nabla^2\phi + 3\mathcal{H}(\mathcal{H}\psi - \phi') + \mathcal{H}\nabla^2 B - \frac{1}{6}[\nabla^2]^2 E = -4\pi G a^2 \delta\rho$$

$$\mathcal{H}\nabla^2\psi - \nabla^2\phi' + 2\mathcal{H}^2\nabla^2 B - \frac{a''}{a}\nabla^2 B + \frac{1}{6}[\nabla^2]^2 E' = -4\pi G a^2(\rho + p)\theta$$

$$-\partial^i\partial_j\psi - \partial^i\partial_j\phi + \frac{1}{2}\partial^i\partial_j E'' + \mathcal{H}\partial^i\partial_j E' + \frac{1}{6}\partial^i\partial_j\nabla^2 E - 2\mathcal{H}\partial^i\partial_j B - \partial^i\partial_j B' = 8\pi G a^2 \Pi^i_j$$

$$2\mathcal{H}\psi' + 4\frac{a''}{a}\psi - 2\mathcal{H}^2\psi + \frac{2}{3}\nabla^2\psi + \frac{2}{3}\nabla^2\phi - 4\mathcal{H}\phi' - 2\phi'' + \frac{4}{3}\mathcal{H}\nabla^2 B + \frac{2}{3}\nabla^2 B' - \frac{1}{9}[\nabla^2]^2 E = 8\pi G a^2 \delta p$$

The perturbed pressure of DE:

$$\delta p_d = C_e^2 \delta_d \rho_d + (C_e^2 - C_a^2)\left[\frac{3\mathcal{H}(1+w)V_d\rho_d}{k} - a^2 Q_d^0 \frac{V_d}{k}\right]$$

C_e^2 is the sound speed in the rest frame, C_a^2 is the adiabatic sound speed,

Perturbation Theory

$$\delta\nabla_\mu T^\mu_\nu = \delta Q_\nu$$

DM:

$$D'_{gc} + \left\{\left(\frac{a^2Q^0_c}{\rho_c\mathcal{H}}\right)' + \frac{\rho'_c}{\rho_c\mathcal{H}}\frac{a^2Q^0_c}{\rho_c}\right\}\Phi + \frac{a^2Q^0_c}{\rho_c}D_{gc} + \frac{a^2Q^0_c}{\rho_c\mathcal{H}}\Phi'$$
$$= -kV_c + 2\Psi\frac{a^2Q^0_c}{\rho_c} + \frac{a^2\delta Q^{0I}_c}{\rho_c} + \frac{a^2Q^{0\prime}_c}{\rho_c\mathcal{H}}\Phi - \frac{a^2Q^0_c}{\rho_c}\left(\frac{\Phi}{\mathcal{H}}\right)'$$
$$V'_c + \mathcal{H}V_c = k\Psi - \frac{a^2Q^0_c}{\rho_c}V_c + \frac{a^2\delta Q^I_{pc}}{\rho_c}$$

DE:

$$D'_{gd} + \left\{\left(\frac{a^2Q^0_d}{\rho_d\mathcal{H}}\right)' - 3w' + 3(C_e^2 - w)\frac{\rho'_d}{\rho_d} + \frac{\rho'_d}{\rho_d\mathcal{H}}\frac{a^2Q^0_d}{\rho_d}\right\}\Phi + \left\{3\mathcal{H}(C_e^2 - w) + \frac{a^2Q^0_d}{\rho_d}\right\}D_{gd} + \frac{a^2Q^0_d}{\rho_d\mathcal{H}}\Phi'$$
$$= -(1+w)kV_d + 3\mathcal{H}(C_e^2 - C_a^2)\frac{\rho'_d}{\rho_d}\frac{V_d}{k} + 2\Psi\frac{a^2Q^0_d}{\rho_d} + \frac{a^2\delta Q^{0I}_d}{\rho_d} + \frac{a^2Q^{0\prime}_d}{\rho_d\mathcal{H}}\Phi - \frac{a^2Q^0_d}{\rho_d}\left(\frac{\Phi}{\mathcal{H}}\right)'$$
$$V'_d + \mathcal{H}(1-3w)V_d = \frac{kC_e^2}{1+w}D_{gd} + \frac{kC_e^2}{1+w}\frac{\rho'_d}{\rho_d\mathcal{H}}\Phi - (C_e^2 - C_a^2)\frac{V_d}{1+w}\frac{\rho'_d}{\rho_d} - \frac{w'}{1+w}V_d + k\Psi - \frac{a^2Q^0_d}{\rho_d}V_d + \frac{a^2\delta Q^I_{pd}}{\rho_d}$$

He, Wang, Jing, JCAP(09); He, Wang, Abdalla, PRD(11)

We have not specified the form of the interaction between dark sectors.

Perturbations

Phenomenological interaction forms:

(1) $Q = \delta H(\rho_{DM} + \rho_{DE})$, (2) $Q = \delta H \rho_{DM}$ and (3) $Q = \delta H \rho_{DE}$

$$a^2 Q_m^0 = 3\mathcal{H}(\lambda_1 \rho_m + \lambda_2 \rho_d)$$
$$a^2 Q_d^0 = -3\mathcal{H}(\lambda_1 \rho_m + \lambda_2 \rho_d)$$

Perturbation equations:

$$\begin{aligned}
D'_m &= -kU_m + 6\mathcal{H}\Psi(\lambda_1 + \lambda_2/r) - 3(\lambda_1 + \lambda_2/r)\Phi' + 3\mathcal{H}\lambda_2(D_d - D_m)/r \quad , \\
U'_m &= -\mathcal{H}U_m + k\Psi - 3\mathcal{H}(\lambda_1 + \lambda_2/r)U_m \quad , \\
D'_d &= -3\mathcal{H}C_e^2 \left\{D_d - [3(\lambda_1 r + \lambda_2) + 3(1+w)]\,\Phi\right\} - 3\mathcal{H}(C_e^2 - C_a^2)\left[\frac{3\mathcal{H}U_d}{k} - a^2 Q_d^0 \frac{U_d}{(1+w)\rho_d k}\right] \\
&\quad -3\mathcal{H}w\,[3(\lambda_1 r + \lambda_2) + 3(1+w)]\,\Phi + 3\mathcal{H}wD_d + 3w'\Phi + 3(\lambda_1 r + \lambda_2)\Phi' - kU_d - 6\Psi\mathcal{H}(\lambda_1 r + \lambda_2) \\
&\quad +3\mathcal{H}\lambda_1 r(D_d - D_m) \\
U'_d &= -\mathcal{H}(1-3w)U_d + kC_e^2\left\{D_d - 3[(\lambda_1 r + \lambda_2) + (1+w)]\Phi\right\} \\
&\quad -(C_e^2 - C_a^2)a^2 Q_d^0 \frac{U_d}{(1+w)\rho_d} + 3(C_e^2 - C_a^2)\mathcal{H}U_d + (1+w)k\Psi + 3\mathcal{H}(\lambda_1 r + \lambda_2)U_d.
\end{aligned}$$

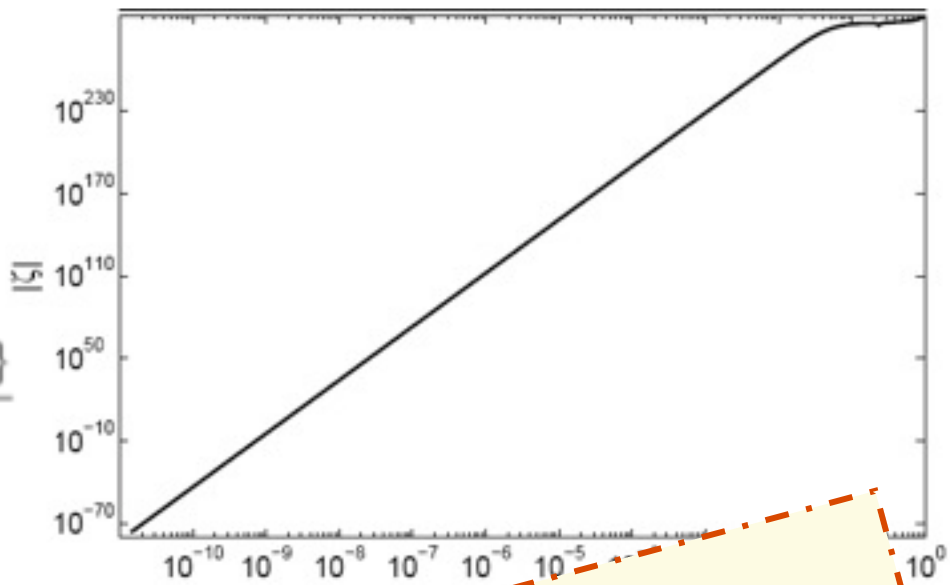
Perturbations

Choosing interactions

$$(1)\ Q = \delta H(\rho_{DM} + \rho_{DE}),\ (2)\ Q = \delta H \rho_{DM}$$

$$\delta G^{\nu}_{\mu} = 8\pi G \delta T^{\nu}_{\mu}$$

$$\Phi = \frac{4\pi G a^2 \sum \rho_i \{D^i_g - \rho'_i U_i/\rho_i(1+w_i)k\}}{k^2 - 4\pi G a^2 \sum \rho'_i/\mathcal{H}}$$



w>-1 **Maartens et al, JCAP(08)**

Curvature perturbation is no

Is this result gene

How about the other forms of the interaction? Stable??

How about the case with w<-1?

Perturbations

$$D'_d = (-1+w+\lambda_1 r)3\mathcal{H}D_d - 9\mathcal{H}^2(1-w)(1+\frac{\lambda_1 r+\lambda_2}{1+w})\frac{U_d}{k} - kU_d + 9\mathcal{H}(1-w)(\lambda_1 r+\lambda_2+1+w)\Phi$$
$$+3(\lambda_1 r+\lambda_2)\Phi' - 6\Psi\mathcal{H}(\lambda_1 r+\lambda_2) - 3\mathcal{H}\lambda_1 r D_m \quad ,$$
$$U'_d = 2\left\{1+\frac{3}{1+w}(\lambda_1 r+\lambda_2)\right\}\mathcal{H}U_d + kD_d - 3k(\lambda_1 r+\lambda_2+1+w)\Phi + (1+w)k\Psi \quad .$$

w>-1 $\lambda_1 \neq 0$ divergence **the interaction proportional to DM density**

$\lambda_1 = 0,\ \lambda_2 \neq 0$ divergence disappears

w>-1

the interaction proportional to DE density

w<-1, always Stable perturbation

couplings	DE	DM	Total
W>-1	Stable	Unstable	Unstable
W<-1	Stable	Stable	Stable

J.He, B.Wang, E.Abdalla, PLB(09)

ISW imprint of the interaction

The analytical descriptions for such effect

$$C_l^{ISW} = 4\pi \int \frac{d^3k}{(2\pi)^3} P_\psi(k) \mid \int_{\tau_i}^{\tau_0} d\tau j_l(k[\tau_0 - \tau]) e^{\kappa(\tau_0)-\kappa(\tau)} [\Psi' - \Phi'] \mid^2$$

where $P_\psi(k)$ is the power spectrum of the primordial coverture perturbation. j_l is the spherical Bessel functions. κ denotes the optical depth for Thompson scattering.From Einstein's equations,we obtain,

$$\Psi' - \Phi' = -2\Phi' - \mathcal{T}' = 2\mathcal{H}\left\{\Phi + 4\pi G a^2 \sum V^i(p^i + \rho^i)/(\mathcal{H}k) + \mathcal{T}\right\} - \mathcal{T}'$$

$$\Phi' = -\mathcal{H}\Phi - \mathcal{H}\mathcal{T} - 4\pi G a^2 \sum V^i(p^i + \rho^i)/k$$

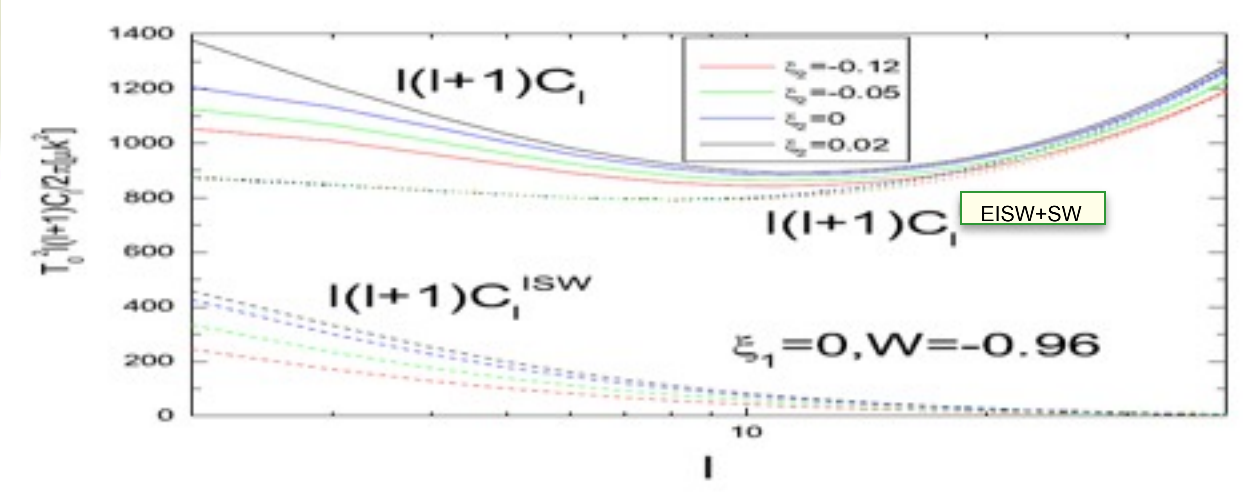
$$\Phi = \frac{4\pi G a^2 \sum \rho_i \{D_g^i + 3\mathcal{H}U^i/k\}}{k^2 - 4\pi G a^2 \sum \rho_i'/\mathcal{H}}$$

ISW effect is not simply due to the change of the CDM perturbation. The interaction enters each part of gravitational potential.

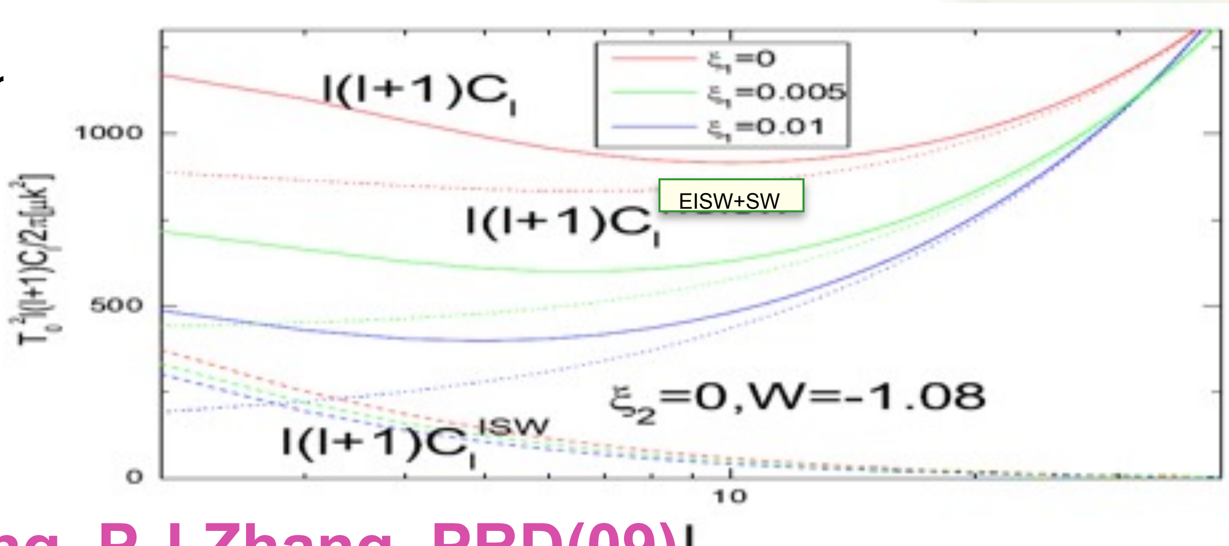
J.H. He, B.Wang, P.J.Zhang, PRD(09)
J.H.He, B.Wang, E.Abdalla, PRD(11)

Imprint of the interaction in CMB

- Interaction proportional to the energy density of DE



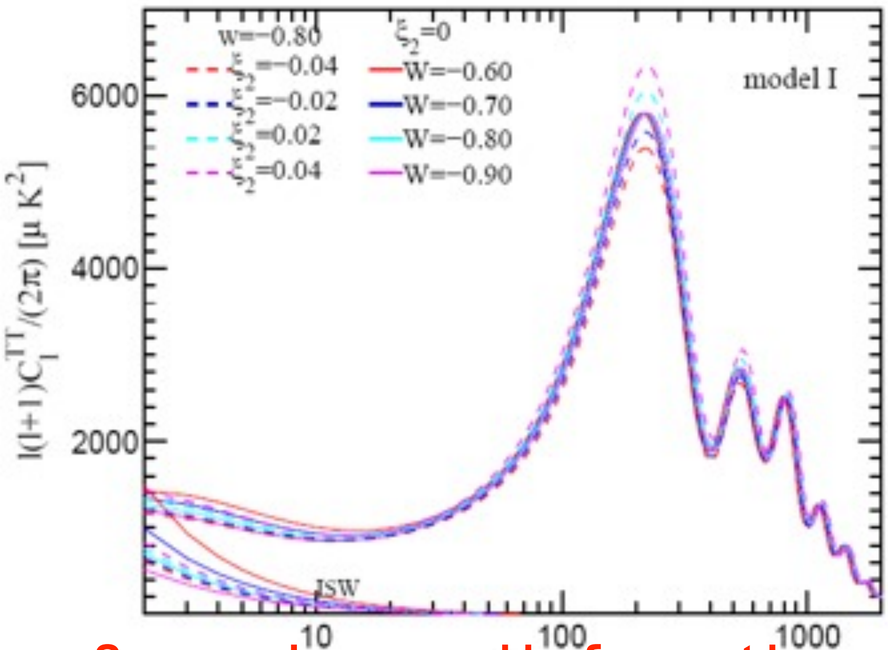
- Interaction pr



J.H.He, B.Wang, P.J.Zhang, PRD(09)

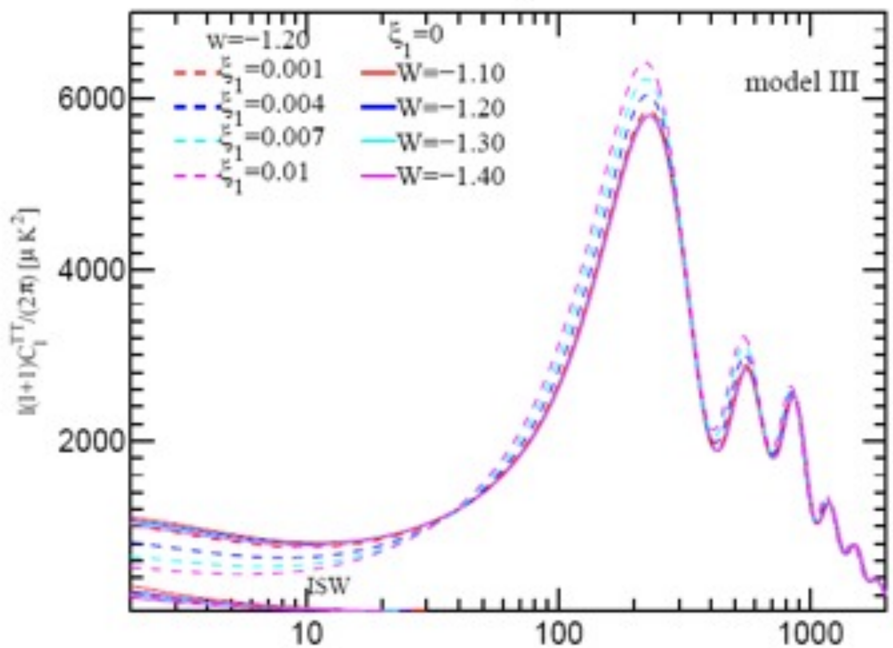
Degeneracy between the ξ and w in CMB

- The small l suppression caused by changing ξ can also be produced by changing w
- ξ can cause the change of acoustic peaks but w cannot



Suppression caused by ξ cannot be distinguished from that by w

ξ ~DE



Suppression caused by ξ is more than that by w

ξ ~DM, DE+DM

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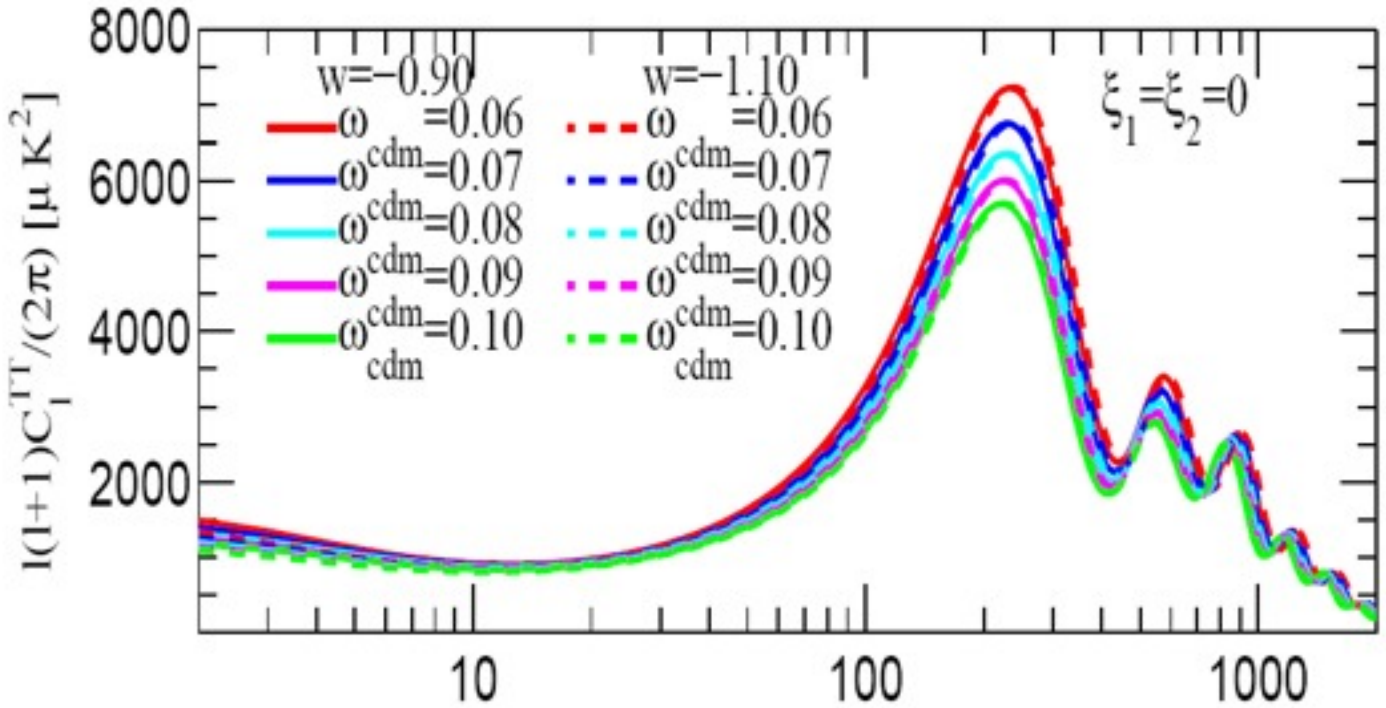
Degeneracy between the ξ and ω_c

- The change of the acoustics peaks caused by ξ can also be produced by the abundance of the cold DM $\omega_c = \Omega_c h^2$



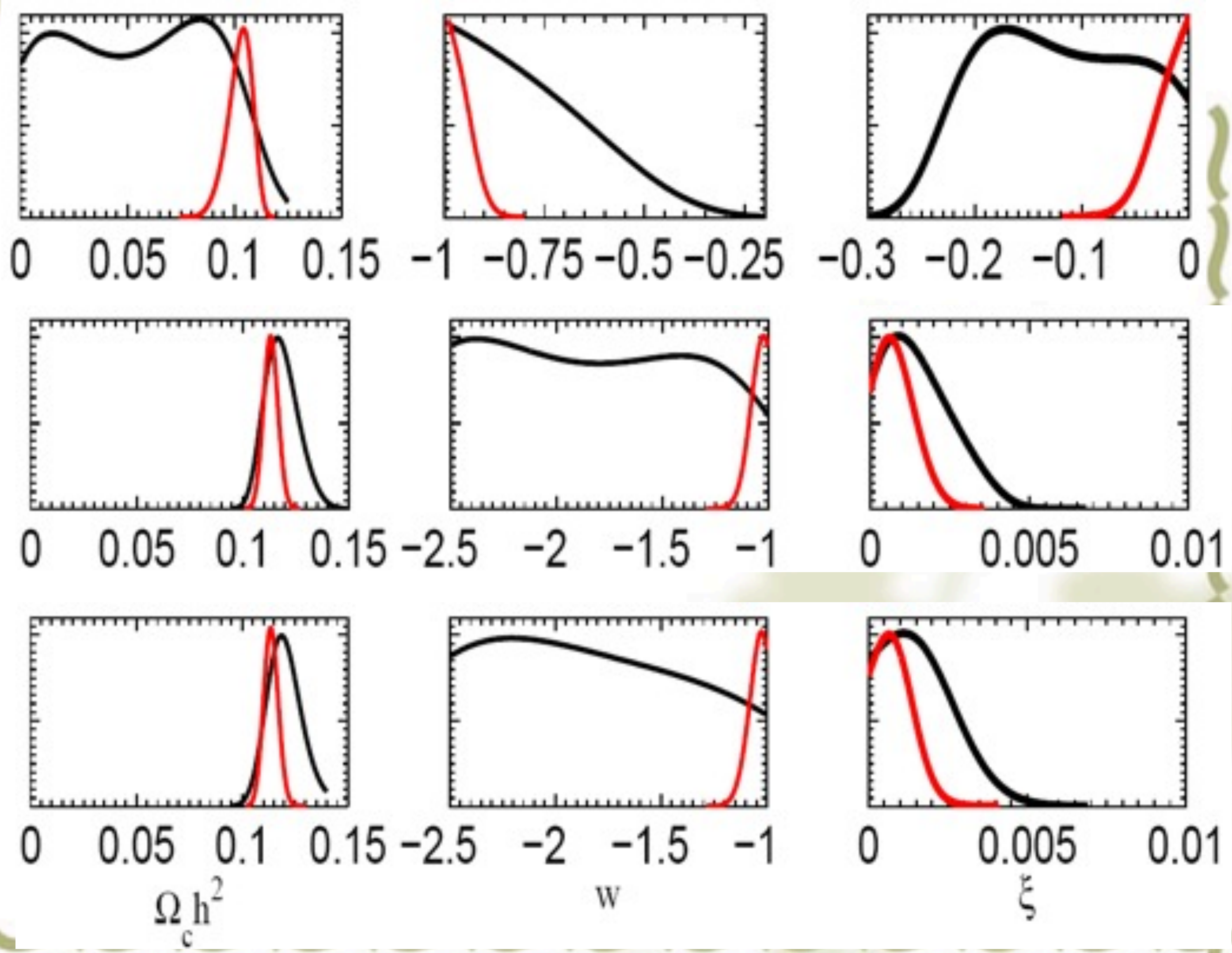
To break the degeneracy between ξ and ω_c, we can look at small l spectrum. ξ can bring clear suppression when ξ~DM or DM+DE, but not for ξ~DE

Likelihoods of $\Omega_c h^2$, w and ξ

ξ ~DE

ξ ~DM

ξ ~DE+DM



Fitting results

- WMAP7-Y

Parameters	Model I	Model II	Model III	Model IV
h	$0.678^{+0.061}_{-0.075}$	$1.09^{+0.23}_{-0.26}$	$0.80^{+0.21}_{-0.13}$	$0.83^{+0.36}_{-0.15}$
$\Omega_b h^2$	$0.0224^{+0.0006}_{-0.0006}$	$0.0221^{+0.0005}_{-0.0005}$	$0.0219^{+0.0006}_{-0.0006}$	$0.0219^{+0.0006}_{-0.0006}$
$\Omega_c h^2$	$< 0.111(68\%CL)$	$< 0.151(68\%CL)$	$0.117^{+0.009}_{-0.007}$	$0.119^{+0.008}_{-0.007}$
τ	$0.084^{+0.015}_{-0.014}$	$0.085^{+0.015}_{-0.014}$	$0.087^{+0.016}_{-0.015}$	$0.085^{+0.016}_{-0.015}$
n_s	$0.966^{+0.014}_{-0.015}$	$0.957^{+0.014}_{-0.014}$	$0.944^{+0.016}_{-0.016}$	$0.943^{+0.017}_{-0.018}$
$\ln[10^{10}As]$	$3.071^{+0.037}_{-0.036}$	$3.072^{+0.036}_{-0.035}$	$3.079^{+0.039}_{-0.038}$	$3.077^{+0.037}_{-0.035}$
w	$< -0.694(68\%CL)$	unconstrained	unconstrained	unconstrained
ξ	$-0.17^{+0.17}_{-0.05}$	$-0.13^{+0.20}_{-0.05}$	$0.0010^{+0.0012}_{-0.0010}$	$0.0011^{+0.0010}_{-0.0011}$

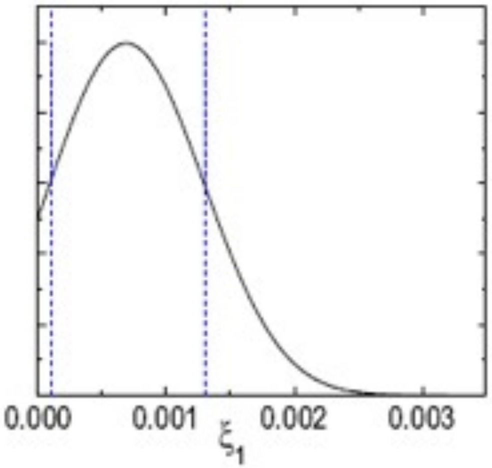
- WMAP+SN+BAO+H

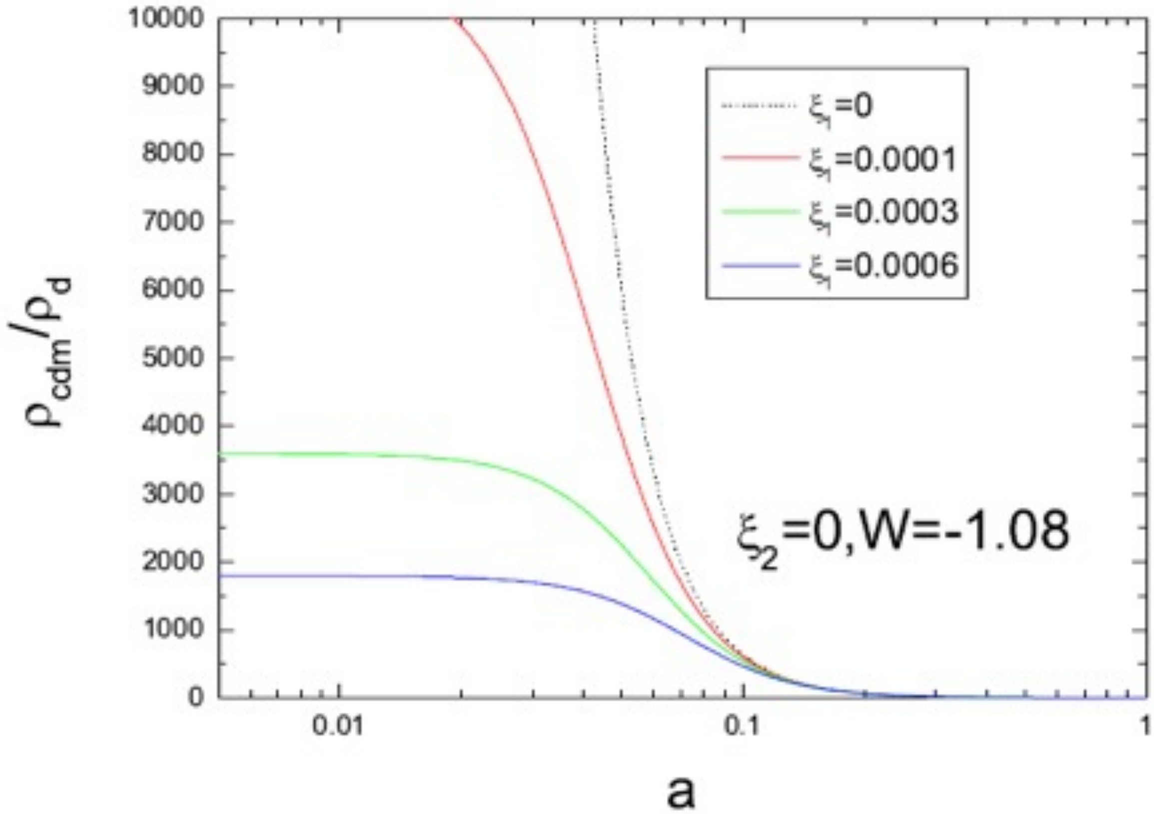
Parameters	Model I	Model II	Model III	Model IV
h	$0.699^{+0.012}_{-0.012}$	$0.709^{+0.013}_{-0.012}$	$0.700^{+0.013}_{-0.013}$	$0.699^{+0.013}_{-0.013}$
$\Omega_b h^2$	$0.0224^{+0.0006}_{-0.0006}$	$0.0222^{+0.0005}_{-0.0005}$	$0.0222^{+0.0006}_{-0.0006}$	$0.0222^{+0.0006}_{-0.0006}$
$\Omega_c h^2$	$0.107^{+0.006}_{-0.007}$	$0.120^{+0.010}_{-0.008}$	$0.113^{+0.003}_{-0.003}$	$0.114^{+0.003}_{-0.003}$
τ	$0.086^{+0.016}_{-0.015}$	$0.083^{+0.016}_{-0.014}$	$0.087^{+0.017}_{-0.015}$	$0.087^{+0.016}_{-0.015}$
n_s	$0.967^{+0.013}_{-0.013}$	$0.961^{+0.013}_{-0.013}$	$0.956^{+0.014}_{-0.014}$	$0.956^{+0.014}_{-0.014}$
$\ln[10^{10}As]$	$3.070^{+0.036}_{-0.034}$	$3.069^{+0.035}_{-0.033}$	$3.074^{+0.038}_{-0.036}$	$3.074^{+0.036}_{-0.034}$
w	$< -0.938(68\%CL)$	$-1.03^{+0.03}_{-0.04}$	$-1.02^{+0.02}_{-0.05}$	$-1.03^{+0.03}_{-0.05}$
ξ	$-0.003^{+0.017}_{-0.024}$	$0.024^{+0.034}_{-0.027}$	$0.0006^{+0.0006}_{-0.0005}$	$0.0006^{+0.0005}_{-0.0006}$

He, Wang, Abdalla, PRD(11)

Alleviate the coincidence problem

Interaction proportional to the energy density of DM





J.H. He, B.Wang, P.J.Zhang, PRD(09)

To reduce the uncertainty and put tighter constraint on the value of the coupling between DE and DM, new observables should be added.

Galaxy cluster scale test

E. Abdalla, L.Abramo, L.Sodre, B.Wang, PLB(09)

Growth factor of the structure formation

J.He, B.Wang, Y.P.Jing, JCAP(09)

Number of galaxy counting

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Growth of structures

In the subhorizon approximation k>>aH, DM perturbation

$$\Delta_m'' = -(\mathcal{H}+\frac{2a^2Q_m^0}{\rho_m})\Delta_m' + (-\Delta_m\frac{a^2Q_m^0}{\rho_m}+\frac{a^2\delta Q_m^{0(VI)}}{\rho_m})(\mathcal{H}+\frac{a^2Q_m^0}{\rho_m}) - \Delta_m(\frac{a^2Q_m^0}{\rho_m})' + (\frac{a^2\delta Q_m^{0(VI)}}{\rho_m})' - \frac{a^2k\delta Q_{pm}^{(VI)}}{\rho_m} - k^2\Psi.$$

DE perturbation

$$\Delta_d'' = -3\mathcal{H}'C_e^2\Delta_d - (\frac{a^2Q_d^0}{\rho_d})'\Delta_d + \left\{\mathcal{H}(1-3w) - \frac{w}{1+w}\frac{\rho_d'}{\rho_d} + \frac{a^2Q_d^0}{\rho_d}\right\} \times \left\{-3\mathcal{H}C_e^2 + 3w\mathcal{H} - \frac{a^2Q_d^0}{\rho_d}\right\}\Delta_d$$
$$-[\mathcal{H}+3\mathcal{H}C_e^2 - 6w\mathcal{H} + \frac{2a^2Q_d^0}{\rho_d} - \frac{w}{1+w}\frac{\rho_d'}{\rho_d}]\Delta_d' - k(\frac{a^2\delta Q_{pd}^{(VI)}}{\rho_d}) - k^2C_e^2\Delta_d$$
$$+3(w'\mathcal{H}+w\mathcal{H}')\Delta_d - k^2(1+w)\Psi + \frac{a^2\delta Q_d^{0(VI)}}{\rho_d}[\mathcal{H}(1-3w) - \frac{w}{1+w}\frac{\rho_d'}{\rho_d} + \frac{a^2Q_d^0}{\rho_d}] + (\frac{a^2\delta Q_d^{0(VI)}}{\rho_d})'.$$

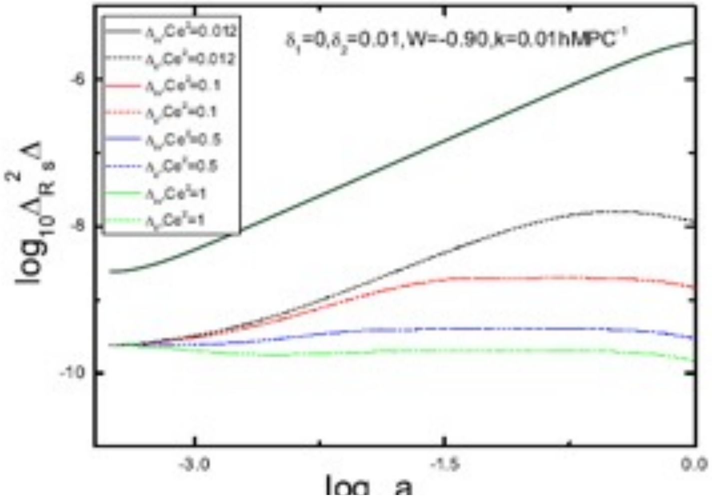
Introducing the interaction

$$Q_m^\nu = \left[\frac{3\mathcal{H}}{a^2}(\delta_1\rho_m+\delta_2\rho_d),0,0,0\right]^T$$
$$Q_d^\nu = \left[-\frac{3\mathcal{H}}{a^2}(\delta_1\rho_m+\delta_2\rho_d),0,0,0\right]^T,$$

In the subhorizon approximation k>>aH

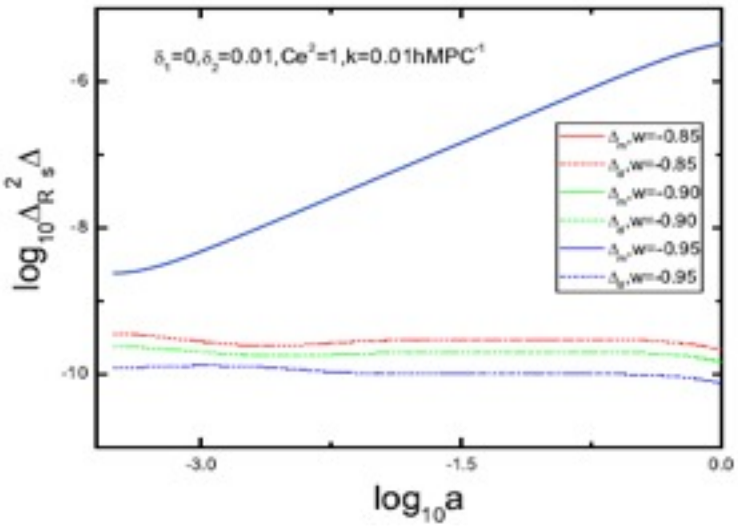
$$\delta Q_m^{0(VI)} \simeq \frac{3\mathcal{H}}{a^2}(\delta_1\delta\rho_m^I+\delta_2\delta\rho_d^I) \qquad \delta\rho_m^I \simeq \delta\rho_m$$
$$\delta Q_d^{0(VI)} \simeq \frac{3\mathcal{H}}{a^2}(\delta_1\delta\rho_m^I+\delta_2\delta\rho_d^I) \qquad \delta\rho_d^I \simeq \delta\rho_d.$$

Growth of structures



for smaller C_e^2, DE perturbation grows.

w further away from -1, DE perturbation grow more and will influence the DM perturbation

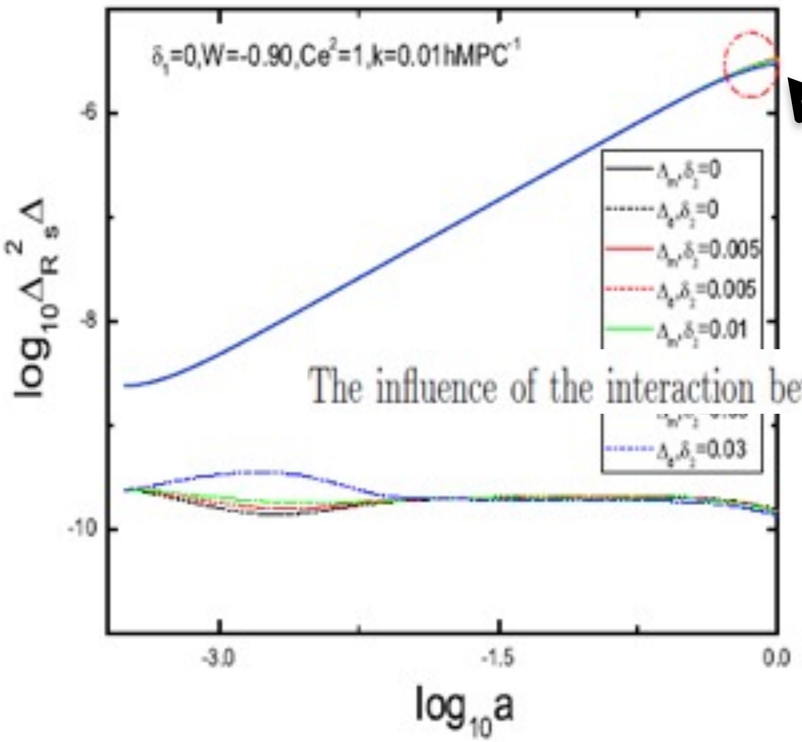


for fixed C_e^2, DE perturbation grows when w deviates from -1

When C_e^2 is not so tiny and w close to -1, in subhorizon approximation, DE perturbation is suppressed

The Influence of the DE perturbation on the structure formation

Growth of structures



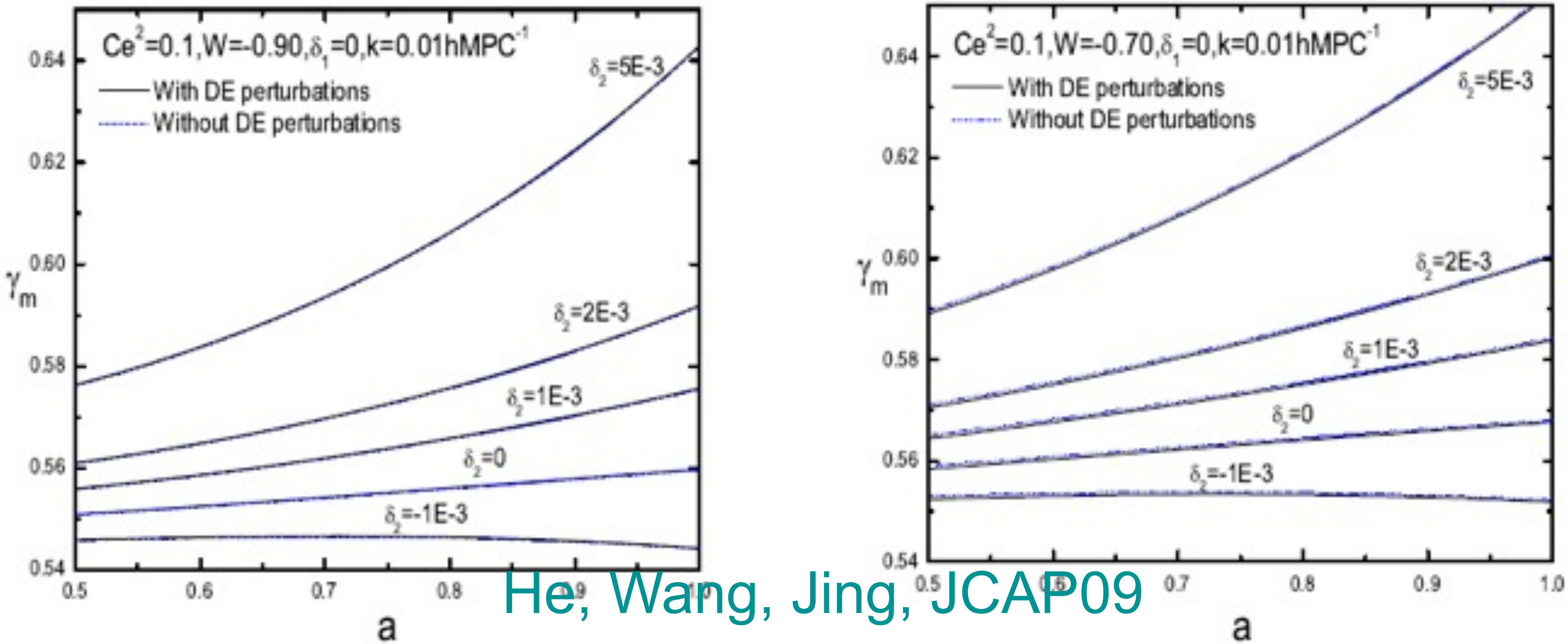
The influence of the interaction between dark sectors on the structure formation

The influence of the interaction between dark sectors can start to appear in the very recent epoch.

To see more clearly of the influence of different parameters on the growth history of the DM perturbation introduce the growth index γ with the definition [8]

$$\gamma_m = (\ln \Omega_m)^{-1} \ln \left(\frac{a}{\Delta_m} \frac{d\Delta_m}{da} \right).$$

Growth of structures



He, Wang, Jing, JCAP09

The interaction influence on the growth index overwhelm the DE perturbation effect.

This opens the possibility to reveal the interaction between DE&DM through measurement of growth factor in the future.

Layzer-Irvine equation for DM

Layzer-Irvine equation describes how a collapsing system reaches dynamical equilibrium in an expanding universe

For DM: the rate of change of the peculiar velocity is

$$\frac{\partial}{\partial t}(aV_m) = -\nabla_x(a\psi_m + a\psi_d) - 3H(\xi_1 + \xi_2/r)(aV_m)$$

Neglecting the influence of DE and the couplings, ---Newtonian mechanics

Multiplying both sides of this equation by $\sigma_{dm} a\vec{v}_{dm}$, integrating over the volume and using continuity equation,

$$\dot{T}_m + \dot{U}_{mm} + H(2T_m + U_{mm}) = -\int \psi_d \frac{\partial}{\partial t}(\sigma_m \hat{\varepsilon}) - 3H(\xi_1 + \xi_2/r)T_m$$
$$+3H\left\{\xi_1 U_{md} + \xi_2 U_{dm} + 2\xi_1 U_{mm} + 2\xi_2 U_{dd}\right\}.$$

describes how DM reaches dynamical equilibrium in the collapsing system in the expanding universe.

J.H.He, B.Wang, E.Abdalla, D.Pavon, JCAP(10)

Virial condition

If the DE is distributed homogeneously, $\sigma_d = 0,$

For DM: $\dot{T}_m + \dot{U}_{mm} + H(2T_m + U_{mm}) = -3H(\xi_1 + \xi_2/r)T_m + 6H\xi_1 U_{mm}.$

For a system in equilibrium $\dot{T}_m = \dot{U}_{mm} = 0,$

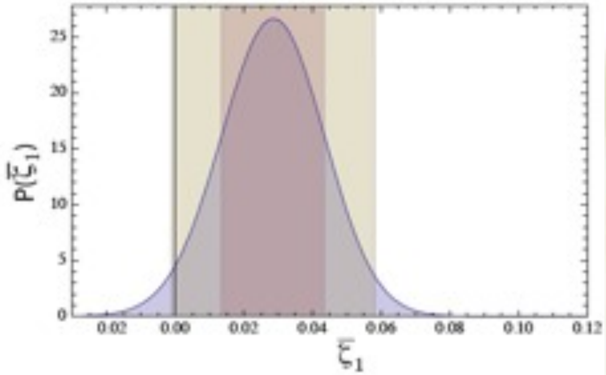
Virial Condition: $(2 + \bar{\xi}_1 + \bar{\xi}_2/r)T_m + (1 - 2\bar{\xi}_1)U_{mm} = 0$

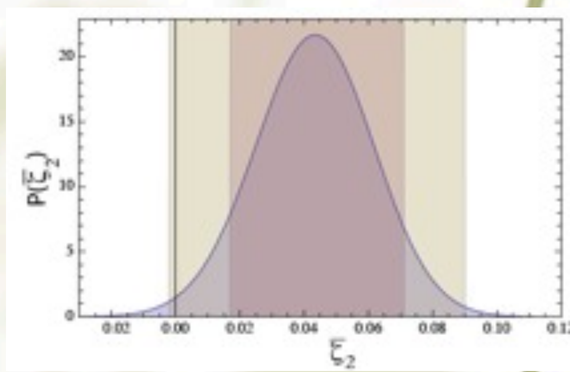
presence of the coupling between DE and DM changes the equilibrium configuration of the system

Galaxy clusters are the largest virialized structures in the universe

Comparing the mass estimated through naïve virial hypothesis with that from WL and X-ray

$$f_1 = M_X/M_{vir} ,$$
$$f_2 = M_{WL}/M_{vir} ,$$
$$f_3 = M_X/M_{WL} .$$





33 galaxy clusters optical, X-ray and weak lensing data

E. Abdalla, L.Abramo, L.Sodre, B.Wang, PLB(09)

Layzer-Irvine equation for DE

For DE: starting from $\frac{\partial}{\partial t}(aV_d) = -\nabla_x(a\Psi) - \frac{C_e^2}{1+w}\nabla_x \cdot (a\Delta_d) + 3H\left[(w-c_a^2) + \frac{1+w-c_a^2}{1+w}(\xi_1 r + \xi_2)\right](aV_d)$

Multiplying both sides of this equation by $aV_d\rho_d\hat{\varepsilon}$ integrating over the volume and using continuity equation, we have:

For DE:

$$(1+w)\dot{T}_d + \dot{U}_{dd} + H[2(1+w)T_d + U_{dd}] = -3H\left\{2(C_e^2+\xi_2)U_{dd} + 2\xi_1 U_{mm} + \xi_1 U_{md} + (C_e^2+\xi_2)U_{dm}\right\}$$
$$-\int \psi_m \frac{\partial}{\partial t}(\sigma_d\hat{\varepsilon}) - c_e^2\int V_d\nabla_x(\sigma_d)\hat{\varepsilon} + 3H\left[(1+w)(w-2C_a^2) + (1+w-2C_a^2)(\xi_1 r + \xi_2)\right]T_d,$$

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J.H.He, B.Wang, E.Abdalla, D.Pavon, JCAP(10)

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The time and dynamics required by DE and DM to reach equilibrium are different in the collapsing system.

J.H.He, B.Wang, E.Abdalla, D.Pavon, JCAP(10)

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DE does not fully cluster along with DM.

J.H.He, B.Wang, E.Abdalla, D.Pavon, JCAP(10)

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The time and dynamics required by DE and DM to reach equilibrium are different in the collapsing system.

DE does not fully cluster along with DM.

The energy conservation breaks down inside the collapsing system.

J.H.He, B.Wang, E.Abdalla, D.Pavon, JCAP(10)

Spherical collapse model

Homogenous DE : $\rho_\lambda^{cluster} = \rho_\lambda + \sigma_\lambda \qquad \sigma_d = 0$

In the background:

$$\dot\rho_m + 3H\rho_m = 3H(\xi_1\rho_m + \xi_2\rho_d),$$

The energy balance equation

$$\dot\rho_d + 3H(1+w)\rho_d = -3H(\xi_1\rho_m + \xi_2\rho_d)$$

Friedmann equation $3M_p^2H^2 = \rho_m + \rho_d$

In the spherical region:

Raychaudhuri's equation – dynamical motion of the spherical region

$$\dot\theta = -\frac{1}{3}\theta^2 - 4\pi G\sum_\lambda(\rho_\lambda + 3p_\lambda) \qquad \theta = 3\frac{\dot R}{R}$$

Local expansion

The energy balance equation in the spherical region:

$$\dot\rho_m^{cluster} + 3h\rho_m^{cluster} = 3H(\xi_1\rho_m^{cluster} + \xi_2\rho_d) \qquad h = \dot R/R.$$

DM perturbation equation

$$\frac{d^2 ln\delta_m}{dlna^2} + [\frac{1}{2} - \frac{3}{2}w(1-\Omega_m)]\frac{dln\delta_m}{dlna} + \left(\frac{dln\delta_m}{dlna}\right)^2 =$$

$$-(3\xi_1 + 6\frac{\xi_2}{r})\frac{dln\delta_m}{dlna} - \frac{3}{r}[\xi_2 + 3\xi_1\xi_2 + 3\xi_2^2/r - \xi_2\frac{dlnr}{dlna} + \xi_2(\frac{dlnH}{dlna}+1)] + \frac{3}{2}\Omega_m$$

Spherical collapse model

Inhomogenous DE : **DE does not trace DM**

DE and DM have different four velocities $u^a_{(d)} \neq u^a_{(m)}$ $\quad u^a_{(d)} = \gamma(u^a_{(m)} + v^a_d)$

The non-comoving perfect fluids

$$T^{ab}_{(m)} = \rho_m u^a_{(m)} u^b_{(m)}$$
$$T^{ab}_{(d)} = \rho_d u^a_{(d)} u^b_{(d)} + p_d h^{ab}_{(d)}$$

Rest on DM frame, we obtain the energy momentum tensor for DE,

$$T^{ab}_{(d)} = \rho_d u^a u^b + p_d h^{ab} + 2u^{(a} q^{b)}_{(d)} \qquad q^a = (\rho_d + p_d) v^a_d$$

energy flux of DE observed in DM frame

The timelike part of the conservation law, $u_b \nabla_a T^{ab}_{(\lambda)} = u_b Q^b_{(\lambda)}$

- DM $\quad \dot{\rho}^{cluster}_m + 3h\rho^{cluster}_m = 3H(\xi_1 \rho^{cluster}_m + \xi_2 \rho^{cluster}_d)$
- DE $\quad \dot{\rho}^{cluster}_d + 3h(1+w)\rho^{cluster}_d = -\vartheta(1+w)\rho^{cluster}_d - 3H(\xi_1 \rho^{cluster}_m + \xi_2 \rho^{cluster}_d)$

$\vartheta = \nabla_x v_d$

Additional expansion due to peculiar velocity of DE relative to DM

Spherical collapse model

The spacelike part of the conservation law $h^c_b \nabla_a T^{ab}_{(\lambda)} = h^c_b Q^b_{(\lambda)}$

Only DE has non-zero component $\dot{q}^a_{(d)} + 4hq^a_{(d)} = 0$

$q^a = (\rho_d + p_d) v^a_d$ is the DE flux observed in DM rest frame

∇_x

Only keep linear, we obtain: $\dot{\vartheta} + h(1-3w)\vartheta = 3H(\xi_1 \Gamma + \xi_2)\vartheta$

$$\dot{\rho}_m^{cluster} + 3h\rho_m^{cluster} = 3H(\xi_1 \rho_m^{cluster} + \xi_2 \rho_d^{cluster})$$

$$\dot{\rho}_d^{cluster} + 3h(1+w)\rho_d^{cluster} = -\vartheta(1+w)\rho_d^{cluster} - 3H(\xi_1 \rho_m^{cluster} + \xi_2 \rho_d^{cluster})$$

+

Raychaudhuri's equation

We can describe the spherical collapse model when DE does not trace DM

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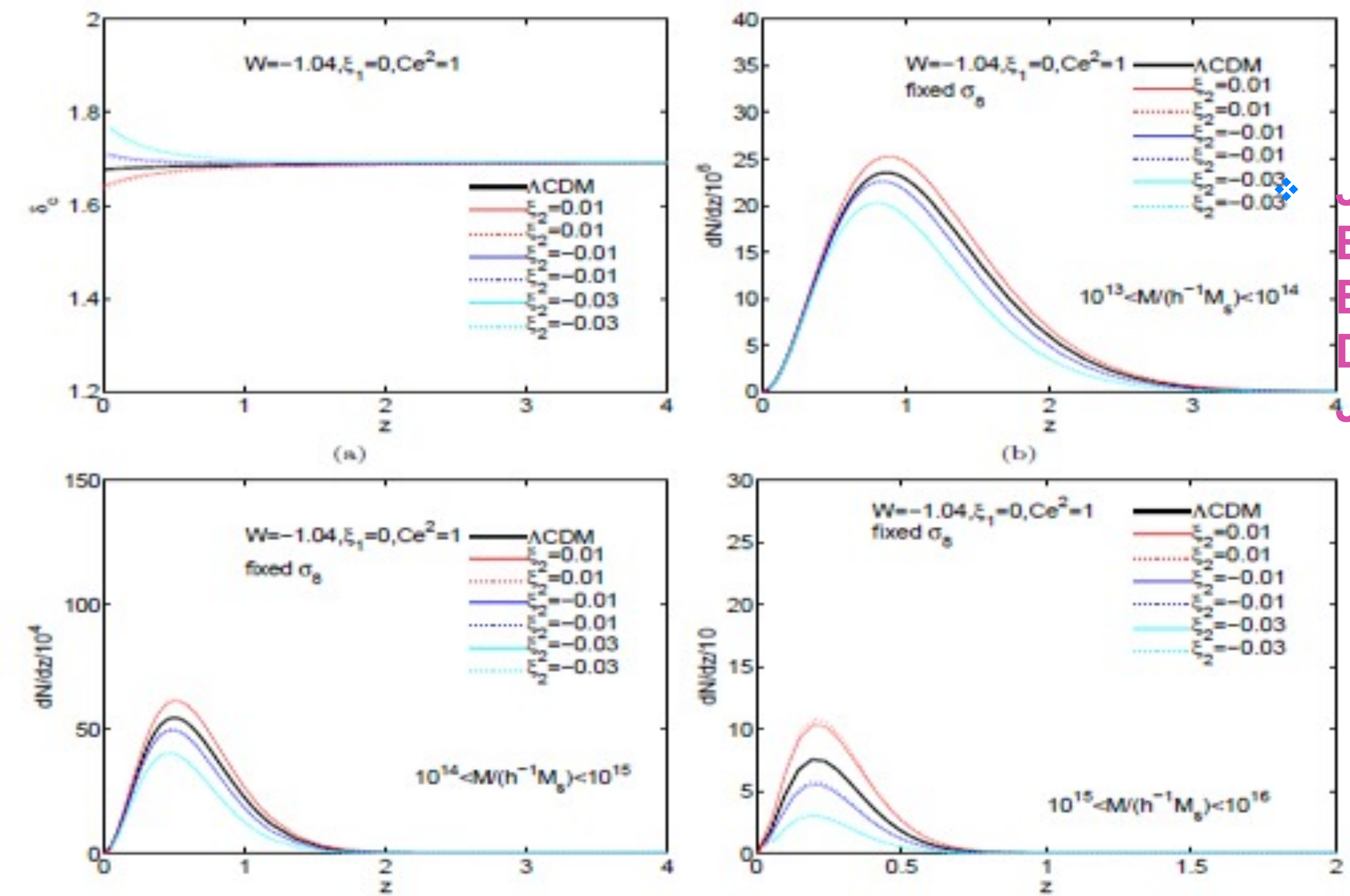
Press-Schechter Formalism

$$\frac{dn(M,z)}{dM} = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}_m}{3M^2}\frac{\delta_c}{\sigma}e^{-\delta_c^2/2\sigma^2}\left[-\frac{R}{\sigma}\frac{d\sigma}{dR}\right]$$



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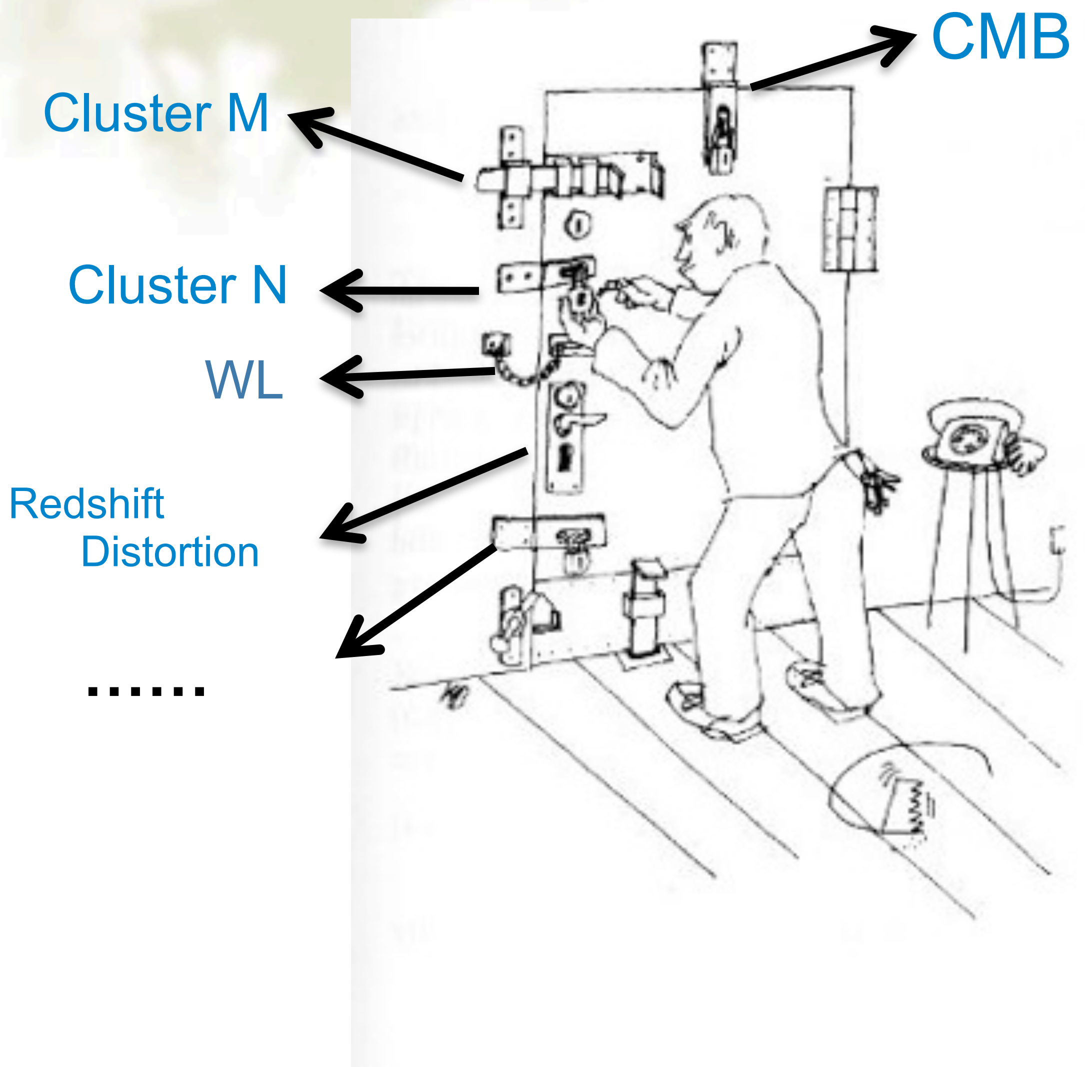
A lot of effort is required to disclose the signature on the interaction between DE and DM



Understanding the interaction between DE and DM from

S. Micheletti, E. Abdalla, B. Wang, PRD(09)

Two fields describing each of the dark components: a fermionic field for DM, a bosonic field for the Dark

$$\mathcal{L} = \sqrt{-g}\{-V(\varphi)\sqrt{1-\alpha\partial^\mu\varphi\partial_\mu\varphi} + \frac{i}{2}[\bar{\Psi}\gamma^\mu\nabla_\mu\Psi - \bar{\Psi}\overleftarrow{\nabla}_\mu\gamma^\mu\Psi] - (M-\beta\varphi)\bar{\Psi}\Psi\},$$

where α is a constant with dimension MeV^{-4}, β a coupling between dark energy and dark matter fields, $V(\varphi)$ the tachyonic potential and g the determinant of the metric.

Friedmann-Robertson-Walker cosmology $g_{\mu\nu} = diag(1, -a(t)^2, -a(t)^2, -a(t)^2)$

equation of motion for the scalar field to be

$$\ddot{\varphi} = -(1-\alpha\dot{\varphi}^2)\left[\frac{1}{\alpha}\frac{dlnV(\varphi)}{d\varphi} + 3H\dot{\varphi} - \frac{\beta\bar{\Psi}\Psi}{\alpha V(\varphi)}\sqrt{1-\alpha\dot{\varphi}^2}\right],$$

with $H = \frac{\dot{a}}{a}$. We also have

$$\frac{d(a^3\Psi^\dagger\Psi)}{dt} = 0,$$
$$\frac{d(a^3\bar{\Psi}\Psi)}{dt} = 0.$$

From the latter, $\bar{\Psi}\Psi = \frac{\bar{\Psi}_0\Psi_0 a_0^3}{a^3}$. Moreover,

$$\rho_\varphi = \frac{V(\varphi)}{\sqrt{1-\alpha\dot{\varphi}^2}},$$
$$P_\varphi = -V(\varphi)\sqrt{1-\alpha\dot{\varphi}^2},$$
$$\rho_\Psi = M^*\bar{\Psi}\Psi$$
$$P_\Psi = 0,$$

$$\dot{\rho_\varphi} + 3H\rho_\varphi(\omega_\varphi + 1) = \beta\dot{\varphi}\frac{\bar{\Psi}_0\Psi_0 a_0^3}{a^3}$$
$$\dot{\rho_\Psi} + 3H\rho_\Psi = -\beta\dot{\varphi}\frac{\bar{\Psi}_0\Psi_0 a_0^3}{a^3}\,.$$

$$\omega_\varphi \equiv P_\varphi/\rho_\varphi = -(1 - \alpha\dot{\varphi}^2).$$

similar to the one usually used as a phenomenological model,

RHS does not contain the Hubble parameter H explicitly, but it does contain the time derivative of the scalar field, which should behave as the inverse of the cosmological time, replacing thus the Hubble

The Friedmann equation for a flat universe reads

$$H^2 = \frac{1}{3M_{pl}^2}\left[M^*\frac{\bar{\Psi}_0\Psi_0 a_0^3}{a^3} + \frac{V(\varphi)}{\sqrt{1-\alpha\dot{\varphi}^2}}\right],$$

$$V(\varphi) = \frac{m^{4+n}}{\varphi^n}\,, n > 0\,.$$

Summary

- Motivation to introduce the interaction between DE & DM
- Is the interaction allowed by observations?

  CMB+SNIa+BAO+Age

  Galaxy cluster scale tests
- Alleviate the coincidence problem
- Understanding the interaction from field theory

Thanks!!!