

Explanation of the data files:

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1) There are 10 data files. Each file gives the data for one black hole solution. These solutions belong to the phase space plotted in Fig. 2 of the paper (henceforth referred to as THE PAPER),

<http://arxiv.org/pdf/1403.2757.pdf>

reproduced in Fig. 4 (UPDATE IN FINAL VERSION) of the review.

i.e. they have a nodeless scalar field with azimuthal harmonic index  $m=1$ . The control parameters of the solution are the scalar field frequency  $w$  and the horizon radius  $r_H$  (capital R in the review). Their values appear in the file name; thus e.g.

"w=0,97-2nd-0,25.txt" means  $w=0.97$ , 2nd branch and  $r_H=0.25$ .

We will explain the "branch" below.

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2) The ten solutions have control parameters

w=0,98

rh=0.2278: q=0 (Kerr)

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w=0.97

rh=0.14 (2nd branch): q=0.014

rh=0.25 (2nd branch): q=0.585

rh=0.2 (1st branch): q=0.988

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w=0.9

rh=0.2 (1st branch): q=0.988

rh=0.15 (2nd branch): q=0.762

rh=0.08 (2nd branch): q=0.649

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w=0.8

rh=0.1 (1st branch): q=0.99

rh=0.14 (2nd branch): q=0.919

rh=0.1 (2nd branch): q=0.877

You see also the value of the normalized Noether charge  $q$  (see page 3 of the paper). This parameter varies between 0 (for Kerr black holes) and 1 (for boson stars). Curves with fixed  $q$  in the phase space (Fig. 2 in the paper) describe a spiral. So lines with constant  $w$  intersect a curve with fixed  $q$  in various points. The branch # classifies these points.

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3) The data columns in each file are organized as:

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X_1  theta_1 F1 F2 F0 Phi W
X_2  theta_1 F1 F2 F0 Phi W
...
X_251 theta_1 F1 F2 F0 Phi W

X_1  theta_2 F1 F2 F0 Phi W
X_2  theta_2 F1 F2 F0 Phi W
...
X_251 theta_2 F1 F2 F0 Phi W

X_1  theta_30 F1 F2 F0 Phi W
X_2  theta_30 F1 F2 F0 Phi W
...
X_251 theta_20 F1 F2 F0 Phi W
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where

a)  $F_i, \phi, W$  are the functions used in eq. 4 and 5 of the paper that describe the line element and the scalar field.

b) the first column relates to the radial coordinate as follows. It gives a coordinate  $X$ , which takes values,

$$X_k = (k-1)/250 \quad (k=1, \dots, 251)$$

This coordinate  $X$  is a compactified radial coordinate,  $0 \leq X \leq 1$ . It relates to another radial coordinate,  $x$ , as

$$X = x/(1+x). \text{ Finally, } x \text{ relates to the radial coordinate } r, \text{ used in eqs. 4 and 5 as } x = \sqrt{r^2 - r_H^2}.$$

The radial coordinate  $r$  (denoted  $R$  in the review) is not the Boyer-Lindquist radial coordinate in the Kerr limit. The transformation between  $r$  and the BL radial coordinate (for the Kerr metric) is given in the review.

c) The second column has values of  $\theta$

$$\theta_k = (k-1) \cdot \pi/29/2 \quad (k=1, \dots, 30)$$

The solution is even under  $z \rightarrow -z$

d) So each solution is given on a  $251 \times 30$  grid, corresponding to 7530 points (for above and on the equatorial plane).