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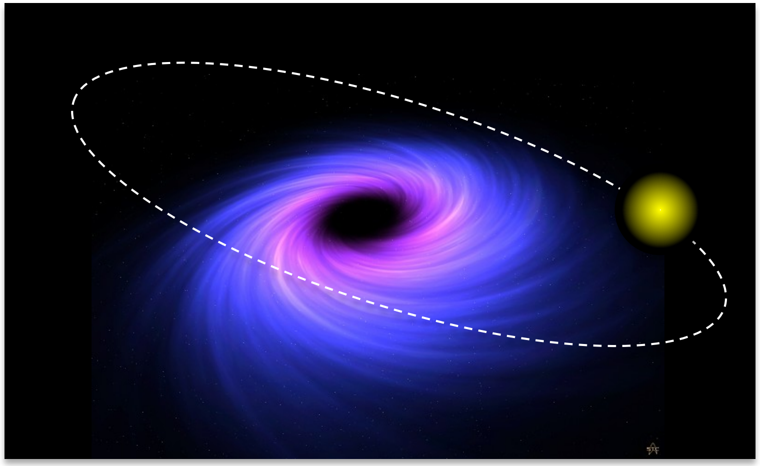
Superradiant scattering in astrophysical binary systems



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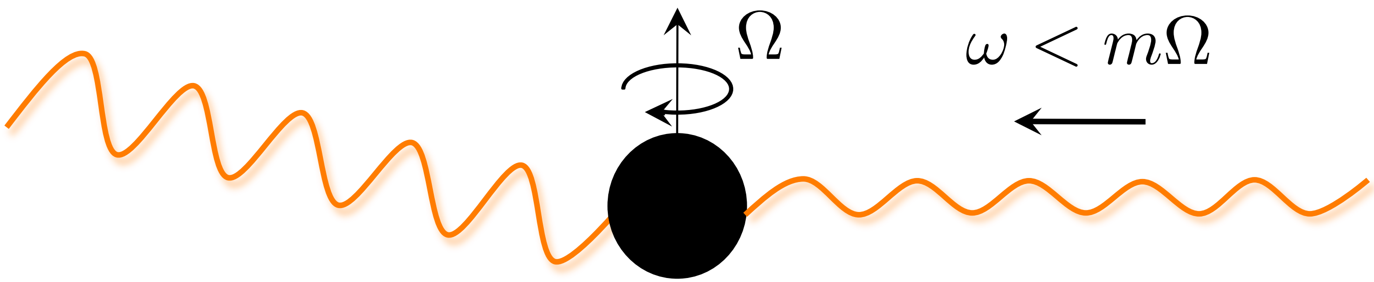
Phys. Lett. B749, 226 (2015) [arXiv:1501.07605 [gr-qc]] + work in progress

VIII Black Holes Workshop, IST Lisbon, 22 December 2015

Black hole superradiance

Scattering in the Kerr space-time can result in amplification of particular wave modes:



Amplification of up to 4.4% for EM waves and 138% for GW!

[Zeldovich (1971); Starobinsky & Churilov (1973); Teukolsky & Press (1973-74)]

Does this occur in astrophysical systems?

Black hole superradiance

Angular velocity of a Kerr black hole:

$$\Omega = \frac{ac}{r_+^2 + a^2} \simeq 10 \text{ kHz} \left(\frac{10 M_\odot}{M}\right) \left(\frac{\tilde{a}}{1 + \sqrt{1 - \tilde{a}^2}}\right)$$

where $\tilde{a} = ac^2/GM$ and $J = aMc$.

Need sources of low-frequency radiation
dominated by superradiant modes
close to a black hole

Black hole superradiance

The scattering problem can be studied using the Newman-Penrose scalars: [Newman & Penrose (1962)]

$$\psi_0 = -C_{\mu\nu\rho\sigma} l^\mu m^\nu l^\rho m^\sigma \ , \qquad \psi_4 = -C_{\mu\nu\rho\sigma} n^\mu \bar{m}^\nu n^\rho \bar{m}^\sigma$$

.

$$\begin{aligned} l^\mu &= \left[\frac{r^2+a^2}{\Delta}, 1, 0, \frac{a}{\Delta}\right] \ , \\ n^\mu &= \frac{1}{2\rho^2}\left[r^2+a^2, -\Delta, 0, a\right] \ , \\ m^\mu &= \frac{1}{\sqrt{2}\bar{\rho}}\left[ia\sin\theta, 0, 1, \frac{i}{\sin\theta}\right] \ , \end{aligned}$$

[Kinnersley (1969)]

$$\Delta = r^2 + a^2 - 2Mr \qquad\qquad \bar{\rho} = r + ia\cos\theta$$

Black hole superradiance

The NP scalars admit a mode decomposition of the form:

.

$$\bar{\rho}^{4}\psi_4 = \sum_{j,m,\omega} e^{-i\omega t+im\phi} S_{jm}(\theta) R_{jm}(r)$$

which yields an angular equation (spheroidal harmonics) and the radial Teukolsky equation: [Teukolsky & Press (1973-74)]

.

$$\Delta\partial_r^2 R_{jm} + 2(s+1)(r-M)\partial_r R_{jm} + \left(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r - A_{jm}\right) R_{jm} = 0$$

$$K(r) = (r^2+a^2)\omega - ma$$

Scattering of plane gravitational waves

We then obtain:

.
$$\frac{dE^{jm}_{out}}{dt} = [1 + Z_{jm}(\omega)]\,\frac{dE^{jm}_{in}}{dt}$$

where:

$$Z_{jm}(\omega) \propto \begin{cases} -(\omega - m\Omega)\ , & \omega > 0 \\ -(-\omega + m\Omega)\ , & \omega < 0 \end{cases}$$

Thus, superradiant modes have:

$$(\omega > 0 \wedge m > 0) \qquad \vee \qquad (\omega < 0 \wedge m < 0)$$

Also:

$$-1 \leq Z_{jm}(\omega) \lesssim 1.38$$

[Teukolsky & Press (1973-74)]

Scattering of plane gravitational waves

A source sufficiently far from the BH will produce plane waves

.
Assume a generic plane wave at spatial infinity as boundary condition:

$$h_{ij} = \frac{1}{2} e^{-i\omega t + i\mathbf{k}\cdot\mathbf{r}} \left[h_+ \mathbf{e}^+_{ij} + h_\times \mathbf{e}^\times_{ij} \right] + \text{c.c.}$$

$$\mathbf{e}^+_{ij} = \begin{pmatrix} \sin^2\phi_0 - \cos^2\theta_0 \cos^2\phi_0 & -(1+\cos^2\theta_0)\cos\phi_0 \sin\phi_0 & \sin\theta_0 \cos\theta_0 \cos\phi_0 \\ -(1+\cos^2\theta_0)\cos\phi_0 \sin\phi_0 & \cos^2\phi_0 - \cos^2\theta_0 \sin^2\phi_0 & \sin\theta_0 \cos\theta_0 \sin\phi_0 \\ \sin\theta_0 \cos\theta_0 \cos\phi_0 & \sin\theta_0 \cos\theta_0 \sin\phi_0 & -\sin^2\theta_0 \end{pmatrix}$$

.

$$\mathbf{e}^\times_{ij} = \begin{pmatrix} \cos\theta_0 \sin(2\phi_0) & -\cos\theta_0 \cos(2\phi_0) & -\sin\theta_0 \sin\phi_0 \\ -\cos\theta_0 \cos(2\phi_0) & -\cos\theta_0 \sin(2\phi_0) & \sin\theta_0 \cos\phi_0 \\ -\sin\theta_0 \sin\phi_0 & \sin\theta_0 \cos\phi_0 & 0 \end{pmatrix}$$

Scattering of plane gravitational waves

Boundary condition determined by NP scalars at spatial infinity (flat space):

$$\psi_4 \simeq (h_+ - ih_\times)\omega^2 e^{-i\omega t}\sum_{jm}\left[a_j^{(2)}\frac{e^{-ikr}}{(kr)^5} + b_j^{(2)}\frac{e^{ikr}}{kr}\right]{}_{-2}Y^*_{jm}(\hat{k})\,{}_{-2}Y_{jm}(\hat{r})$$
$$+ \;(\omega \leftrightarrow -\omega, h_{+,\times} \leftrightarrow h^*_{+,\times})$$

The incident energy flux for each angular mode is then:

$$\frac{dE^{jm}_{in}}{dt} = \frac{\pi^2}{8}\begin{cases}|h_+ - ih_\times|^2|{}_{-2}Y_{jm}(\theta_0,\phi_0)|^2\;, & \omega > 0\\ |h_+ + ih_\times|^2|{}_{-2}Y_{jm}(\theta_0,\phi_0)|^2\;, & \omega < 0\end{cases}$$

Scattering of plane gravitational waves

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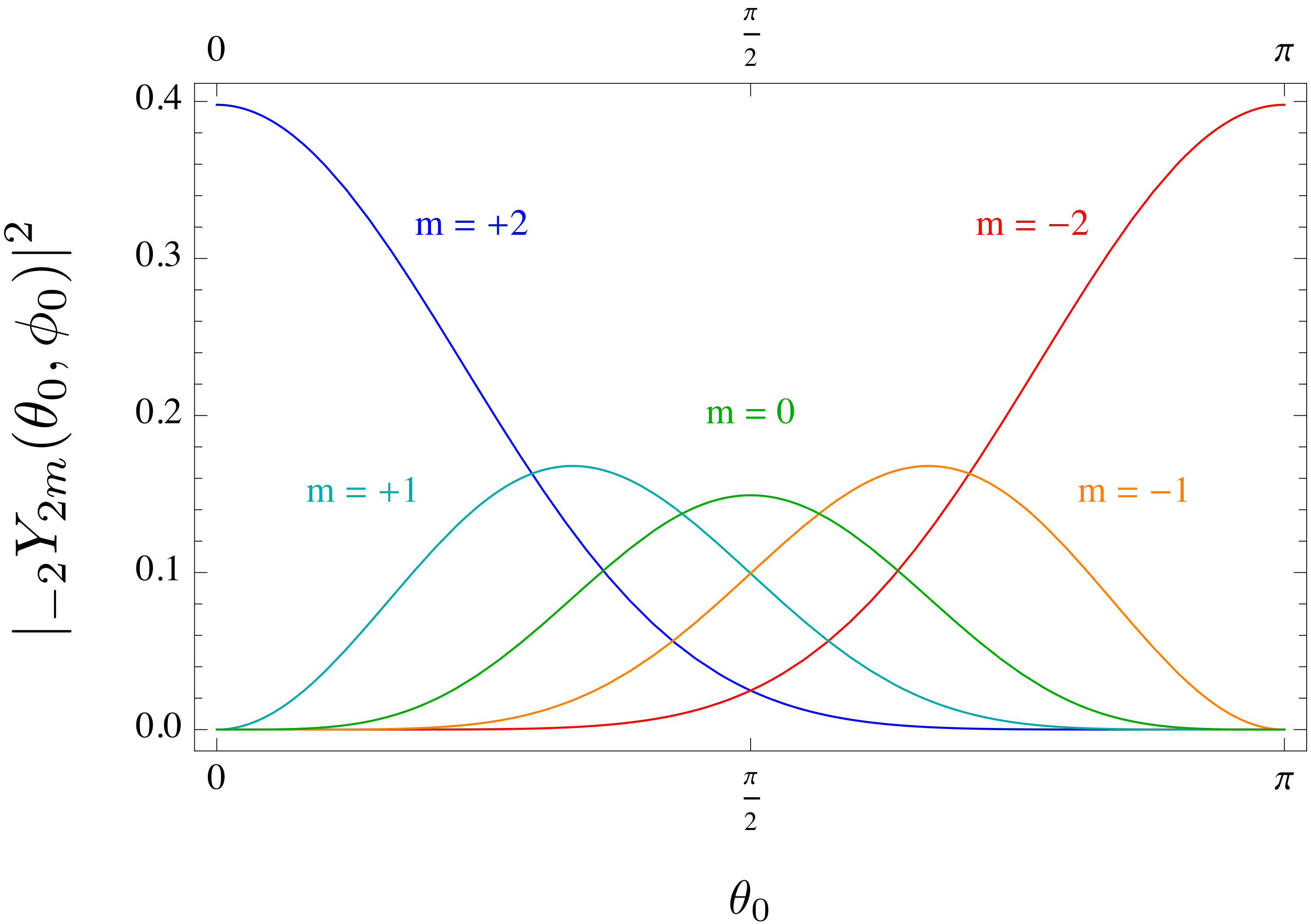


Scattering of plane gravitational waves

Conclusion:

For amplification of a plane wave we need:

1) polarization: $|h_+ - ih_\times| \neq |h_+ + ih_\times|$

2) incident direction parametrically close to the BH spin axis

3) low frequency: $|\omega| < |m|\Omega$

Can we obtain this in realistic systems?

Quadrupole formula (TT gauge)

$$h_{ij} = \frac{2G}{c^4}\frac{1}{|\mathbf{r}-\mathbf{r}_p|}\left[P_i{}^k P_j{}^l - \frac{1}{2}P_{ij}P^{kl}\right]\ddot{Q}_{jk}\left(t - \frac{|\mathbf{r}-\mathbf{r}_p|}{c}\right)$$

where

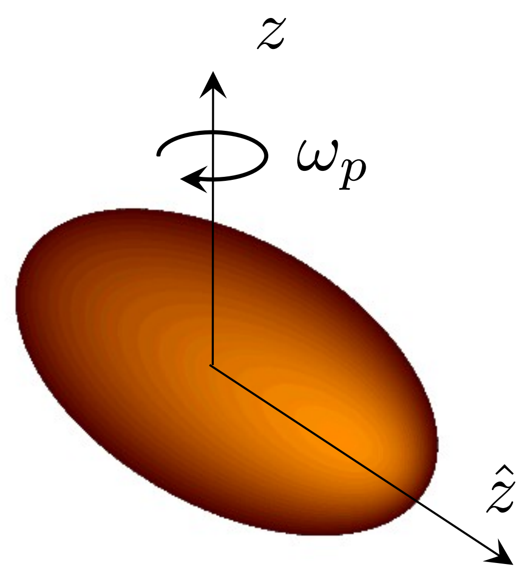
$$P_{ij} = \delta_{ij} - n_i n_j \qquad\qquad Q_{ij} = -I_{ij} + \frac{1}{3}I_k{}^k\delta_{ij}$$

Note that for an oscillating quadrupole at large distance:

$$\ddot{Q}_{ij} \propto e^{-i\omega(t-|\mathbf{r}-\mathbf{r_p}|/c)} \rightarrow e^{-i\omega t+i\mathbf{k}\cdot\mathbf{r}}e^{ik|\mathbf{r}_p|}$$

Example 1: rotating ellipsoid

[Bonazzola & Gourgoulhon (1995)]



In ellipsoid frame:

$$Q_{\hat{i}\hat{j}} = \begin{pmatrix} -Q_{\hat{z}\hat{z}}/2 & 0 & 0 \\ 0 & -Q_{\hat{z}\hat{z}}/2 & 0 \\ 0 & 0 & Q_{\hat{z}\hat{z}} \end{pmatrix}$$

In BH frame:

$$\ddot{Q}_{ij} = \frac{3}{2} Q_{\hat{z}\hat{z}} \omega_p^2 \sin\beta \left[-\cos\beta e^{-i\omega_p t} T_{ij}^{(1)} + 2\sin\beta e^{-2i\omega_p t} T_{ij}^{(2)} \right] + c.c.$$

$$T_{ij}^{(1)} = \begin{pmatrix} 0 & 0 & \mp i \\ 0 & 0 & 1 \\ \mp i & 1 & 0 \end{pmatrix}, \qquad T_{ij}^{(2)} = \begin{pmatrix} 1 & \pm i & 0 \\ \pm i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Example 1: rotating ellipsoid

For example, for the double-frequency mode we obtain:

$$h_+ - ih_\times \propto {}_{-2}Y^*_{2,\mp 2}(\theta_p, \phi_p)$$
$$h_+ + ih_\times \propto {}_{-2}Y_{2,\pm 2}(\theta_p, \phi_p)$$

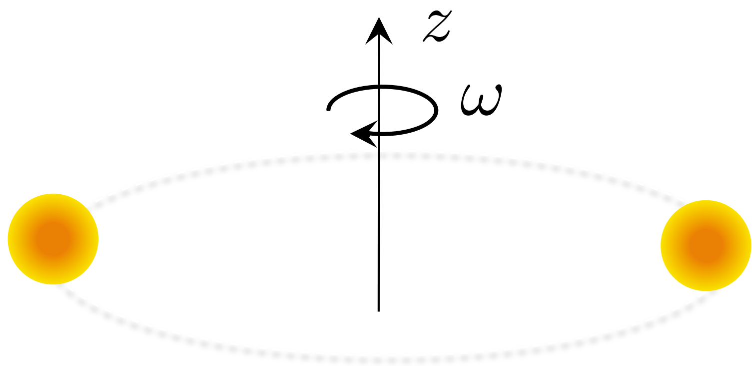
so that waves are indeed polarized and amplification can occur when the ellipsoid is parametrically close to the BH spin axis.

Application: **pulsars orbiting stellar black holes!**

$$\omega_p \sim \Omega \sim \mathrm{few\ khZ}$$

Advanced LIGO/VIRGO

Example 2: compact binary



For a Keplerian circular orbit with two equal mass stars:

$$\ddot{Q}_{ij} = -2\omega_p^2 M R^2 e^{-2i\omega t} T_{ij}^{(2)} + c.c$$

The emitted GW are polarized like in the previous example!

Example 2: compact binary

Application: compact binary orbiting a supermassive BH

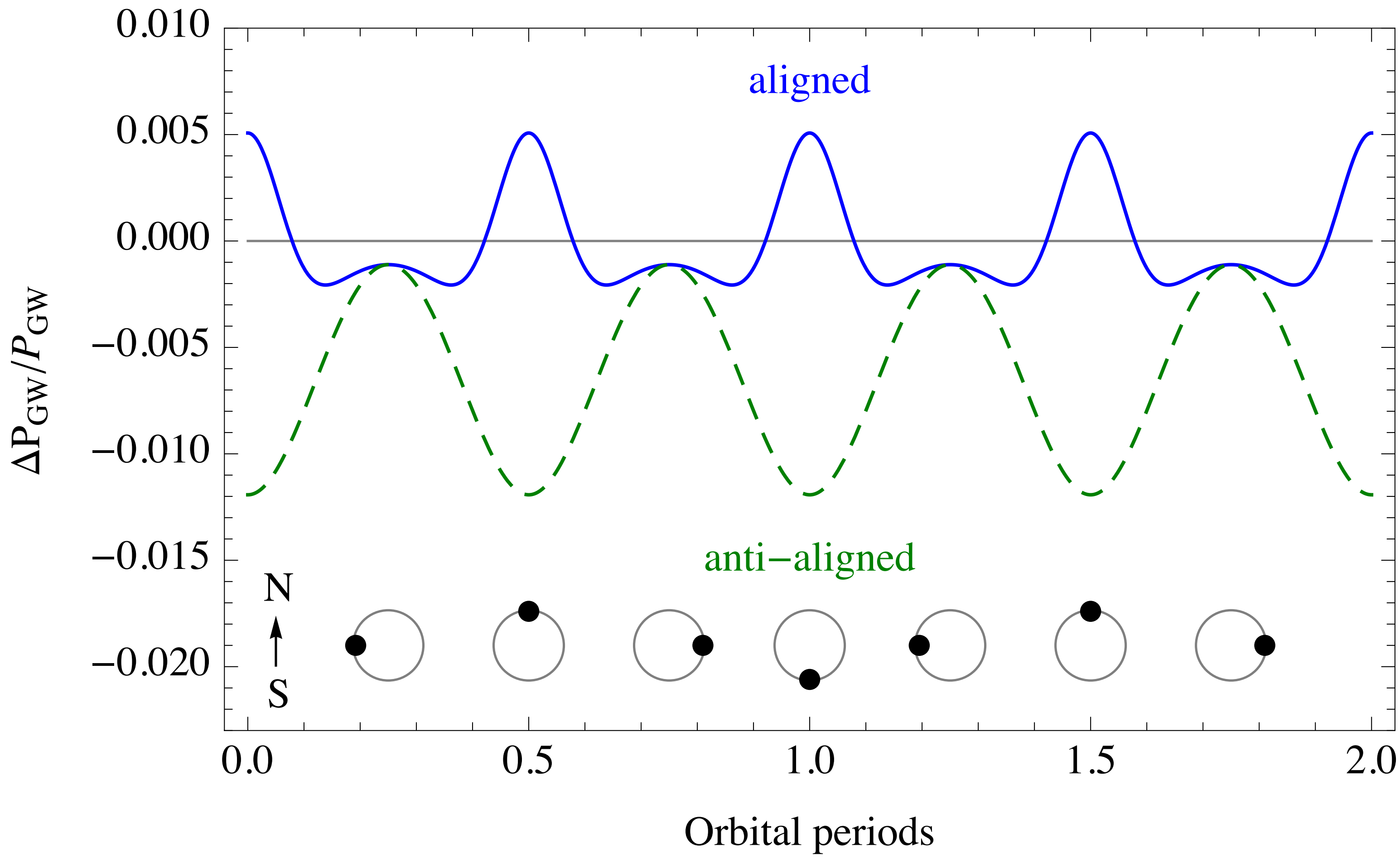
$$\Omega \leq 0.1 \left(\frac{10^9 M_\odot}{M_{BH}} \right) \text{mHz}$$

LISA

$$P_{binary} \gtrsim 17 \left(\frac{M_{BH}}{10^9 M_\odot} \right) \text{ hrs}$$

Note: Hulse-Taylor binary pulsar has period of 7.75 hours.

GW luminosity modulation



$\omega = 2\omega_p = 2\Omega \ , \quad \tilde{a} = 0.999$ $\qquad L_p = 10\lambda$

Summary

- Superradiant amplification requirements (plane waves):

1. Low frequency
2. Polarization
3. Incidence parametrically close to BH spin axis

- There are realistic systems that meet these criteria:

1. Pulsar + stellar BH
2. Compact binary + SMBH system

- Potential observational signatures like luminosity modulation