

# Resolution of black hole singularities in Palatini gravity

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Based on arXiv:1504.07015[hep-th], 1507.07763[hep-th] (PRD), 1508.03272[hep-th] (PRD), 1509.02430[hep-th] and 1511.03755[gr-qc].

VIII Black Holes Workshop, Lisbon, December 22th, 2015

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Singularity theorems Curvature divergences Geodesic completeness

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- The theorems on singularities (Hawking, Penrose) tell us that space-time singularities are unavoidable in the context of GR.
- Fundamentally different from singularities on the fields living on a fixed space-time background! (e.g. Coulomb's divergence).
- If space-time breaks down when a singularity, how can we even speak of a singularity as something occurring at some "location"?.

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- Fundamentally different from singularities on the fields living on a fixed space-time background! (e.g. Coulomb's divergence).
- If space-time breaks down when a singularity, how can we even speak of a singularity as something occurring at some "location"?.
- It is hard to rigorously capture the intuitive notion of a singularity (the theorems on singularities offer little clue about this).

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- If we see space-time singularities as indicative of a physically troublesome region, a natural guess is that something is going on ill with the geometry.
  - The blow up of curvature scalars (curvature divergences) tell us that we have a space-time singularity.
  - Then if you want non-singular space-time time, just make all the curvature invariants finite.

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- Difficulty 1: theorems on singularities still apply. Pay the price of violation of energy conditions.
- Difficulty 2: nothing in the singularity theorems speak of curvature invariants.

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#### Geodesic completeness

A more powerful characterization of space-times containing singularities is provided by the notion of geodesic completeness (Geroch, Penrose, Hawking,...).

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#### Geodesic completeness

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That observers may experience intense tidal forces or large deformations is irrelevant as long as they exist.

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#### Thus:

- In a singular space-time there exist geodesic curves which cannot be extended to arbitrarily large values of the affine parameter (i.e., they start or terminate at some finite value).
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  - The fundamental criterium to be considered for space-time singularities is geodesic completeness.

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  - Space-time singularities are an artifact of the classical description provided by GR, which breaks down at high curvature/short-scales: modified gravity.
  - The bulk of "quantum gravity" effects can be captured by some effective theory of gravity in which singularities are avoided.

Background geometry Theories and approach Geodesics

## Background geometry

Consider the following electrovacuum geometry (derived first in 1207.6004 [gr-qc] (PRD))

$$ds^{2} = -A(x)dv^{2} + \frac{2}{\sigma_{+}}dvdx + r^{2}(x)d\Omega^{2}$$

where

$$\begin{split} A(x) &= \frac{1}{\sigma_{+}} \left[ 1 - \frac{r_{S}}{r} \frac{(1 + \delta_{1} G(r))}{\sigma_{-}^{1/2}} \right] ; \ \delta_{1} = \frac{1}{2r_{S}} \sqrt{\frac{r_{q}^{3}}{l_{c}}} ; \ \sigma_{\pm} = 1 \pm \frac{r_{c}^{4}}{r^{4}(x)} ; \\ G(z) &= -\frac{1}{\delta_{c}} + \frac{1}{2} \sqrt{z^{4} - 1} \left[ f_{3/4}(z) + f_{7/4}(z) \right] \end{split}$$

where  $r_c = \sqrt{l_c r_q}$ : "core" radius,  $l_c$  some length scale,  $r_q^2 = 2G_N q^2$ : charge radius,  $r_S = 2M_0$ : Schwarzschild radius,  $\delta_c \simeq -0.572$  a constant.

For  $z = r/r_c \gg 1 \rightarrow$  Reissner-Nordström space-time:

$$A(x) \approx 1 - \frac{r_{\rm S}}{r} + \frac{r_{\rm q}^2}{2r^2} + O\left(\frac{1}{r^4}\right)$$

For  $z \simeq 1$  ( $N_q$ : number of charges):

$$A(x) \simeq \frac{N_q}{4N_c} \frac{(\delta_1 - \delta_c)}{\delta_1 \delta_c} \sqrt{\frac{r_c}{r - r_c}} + \frac{N_c - N_q}{2N_c} + \dots$$

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From 
$$\left(\frac{dr}{dx}\right)^2 = \frac{\sigma_-}{\sigma_+^2}$$
 we find

$$r^2(x) = \frac{x^2 + \sqrt{x^4 + 4r_c^4}}{2}$$

which has a minimum at x = 0. This is reminiscent of a wormhole geometry.



WH structure persists regardless of the fact that the curvature scalars at the wormhole throat be all finite (δ<sub>1</sub> = δ<sub>c</sub>) or divergent (δ<sub>1</sub> ≠ δ<sub>c</sub>).

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#### Theories and approach

This space-time is an exact solution of quadratic gravity [1207.6004 [gr-qc] (PRD)]:

$$\begin{split} S_{Quad} &= \frac{1}{2\kappa^2}\int d^4x \sqrt{-g}\left[R+l_{\epsilon}^2(aR^2+R_{\mu\nu}R^{\mu\nu})\right] \\ &- \frac{1}{16\pi}\int d^4x \sqrt{-g}F_{\mu\nu}F^{\mu\nu} \end{split}$$

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and also of Born-Infeld gravity [1311.0815 [hep-th] (PRD)]:

$$\begin{split} S_{BI} &= \frac{1}{\kappa^2 \varepsilon} \int d^4 x \left[ \sqrt{-|g_{\mu\nu} - l_{\varepsilon}^2 R_{\mu\nu}(\Gamma)|} - \lambda \sqrt{-g} \right] \\ &- \frac{1}{16\pi} \int d^4 x \sqrt{-g} F_{\mu\nu} F^{\mu\nu} \end{split}$$

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- Standard electromagnetic Maxwell field as matter → energy conditions are satisfied.
- Formulated in the Palatini approach: metric and connection as independent fields:
  - Second-order field equations.
  - Vacuum equations are Minkowski or (A)dS solutions → no extra propagating dofs.
  - In GR (and Lovelock), metric and Palatini formulations coincide.

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## Geodesics

• A geodesic curve  $\gamma^{\mu} = x^{\mu}(\lambda)$  with tangent vector  $u^{\mu} = \frac{dx^{\mu}}{d\lambda}$  and affine parameter  $\lambda$  satisfies (e.g. Chandrasekhar's book):

$$\frac{d^2 x^{\mu}}{d\lambda^2} + \Gamma^{\mu}_{\alpha\beta} \frac{dx^{\alpha}}{d\lambda} \frac{dx^{\beta}}{d\lambda} = 0$$

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Comments:

- The metric defines a natural connection (Christoffel) and defines a set of geodesics.
- The independent connection can be used to define a different set of geodesics.
- Assuming the EEP and since matter is not coupled directly to the independent connection we assume geodesics to be those of the metric.

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By spherical symmetry there are two conserved quantities:  $L = r^2 d\varphi/d\lambda$  (angular momentum per unit mass) and  $E = Adt/d\lambda$  (total energy per unit mass).

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- By spherical symmetry there are two conserved quantities:  $L = r^2 d\varphi/d\lambda$  (angular momentum per unit mass) and  $E = Adt/d\lambda$  (total energy per unit mass).
- Rewrite the geodesic equation in terms of the geodesic tangent vector

$$\frac{1}{\sigma_+^2} \left(\frac{dx}{d\lambda}\right)^2 = E^2 - A\left(\kappa + \frac{L^2}{r^2(x)}\right)$$

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where k = 0(1) for null (time-like) geodesics.

Space-time singularities in GR Background geometry A geodesically complete space-time Conclusions Geodesics

Radial null geodesics (k = 0, L = 0) integrate this equation as

$$\pm E \cdot \lambda(x) = \begin{cases} 2F_1[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{r_1^{c}}{r^4}]r & \text{if } x \ge 0\\ 2x_0 - 2F_1[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{r_2^{c}}{r^4}]r & \text{if } x \le 0 \end{cases}$$



- The affine parameter λ(x) extends over the real axis and the space-time, no matter the behaviour of curvature scalars.
- In GR: r(λ) = ±Eλ, the affine parameter is only defined on the positive/negative side of the real axis because r(λ) is positive.

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For null and time-like geodesics with L ≠ 0, see the geodesic equation as a classical particle in a one effective dimensional potential

$$V(x) = A\left(\kappa + \frac{L^2}{r^2(x)}\right)$$

which near the WH behaves as  $V(x) \approx -a/|x|$ , with  $a = \left(\kappa + \frac{L^2}{r_c^2}\right) \frac{N_q(\delta_c - \delta_1)r_c}{2N_c\delta_c\delta_1}$ .

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- If δ<sub>1</sub> > δ<sub>c</sub> (Reissner-Nordström-like configurations) an infinite potential barrier arises which prevents any geodesic from reaching x = 0 (same as in GR).
- If δ<sub>1</sub> < δ<sub>c</sub> (Schwarzschild-like configurations) the potential becomes infinitely attractive as x → 0 and yields

$$\frac{d\lambda}{dx} \approx \pm \frac{1}{2} \left| \frac{x}{a} \right|^{\frac{1}{2}} \rightarrow \lambda(x) \approx \pm \frac{x}{3} \left| \frac{x}{a} \right|^{\frac{1}{2}}$$

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complete!  $\rightarrow$  compare to GR result  $\lambda(r) \approx \pm \frac{2}{3}r \left(\frac{r}{r_S}\right)^{\frac{1}{2}}$  incomplete! (again because r > 0).

If δ₁ = δ<sub>c</sub> (no curvature divergences) depending on E > V<sub>max</sub> geodesics either bounce or oscillate through the WH.

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Thus... null and time-like complete!  $\rightarrow$  the presence of the wormhole is essential for this.

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Consider f(R) = R - λR<sup>2</sup> with anisotropic fluid T<sub>μ</sub><sup>V</sup> = diag[-ρ, -ρ, αρ, αρ] (equivalent to some models of non-linear electrodynamics), [1509.02430 [hep-th], 1511.03755 [gr-qc]]:

$$ds^{2} = \frac{1}{f_{R}} \left( -A(x)dt^{2} + \frac{1}{A(x)}dx^{2} \right) + r^{2}(x)d\Omega^{2}$$

where  $f_R = 1 - 1/z^{2+2\alpha}$ ,  $r = r_c z$ ,  $r_c^{2+2\alpha} \equiv (4\lambda)\kappa^2(1-\alpha)C$ .

Wormhole by solving

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Wormhole by solving

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Radial null geodesics complete [same for time-like and with angular momentum]

$$\pm E\tilde{\lambda}(z) = -\frac{z}{\sqrt{1-z^{-2(\alpha+1)}}} + 2z_2F_1\left(\frac{1}{2}, -\frac{1}{2(\alpha+1)}; 1-\frac{1}{2(\alpha+1)}; z^{-2(\alpha+1)}\right)$$

Infinite affine time  $\rightarrow$  these wormholes lie beyond the reach of any observer or signal!.

# Conclusions

- Geodesics in these space-times are null and time-like complete for all spectrum of mass and charge no matter the behaviour of the curvature scalars.
- Not "designed": they arise in reasonable extensions of GR where the matter satisfies the energy conditions.

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#### THANK YOU FOR YOUR ATTENTION!

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