# Wave Extraction in Higher Dimensional Numerical Relativity

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# Overview

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# Motivation

- Why Higher Dimensional Black Hole (BH) collisions?
  - TeV gravity theories with large compactified extra dimensions were constructed to explain the hierarchy problem.
  - These predict a Planck scale as low as the order of a TeV. (Arkani-Hamed, Dimopoulos, Dvali 1998)
  - Current particle collisions could be probing trans-Planckian regimes - potential BH production at LHC.
- Why do we need Wave Extraction?
  - To calculate the properties of the final BH after merger, we need to know how much energy, and linear and angular momentum have been radiated away by gravitational waves (GW).

# Overview

# $D = 4$

- In 4D there exist several methods for extracting GW from numerical simulations e.g. Newman-Penrose $\Psi_4$, Regge-Wheeler-Zerilli-Moncrief master functions, Landau-Lifshitz pseudotensor.
- We will focus on the Newman-Penrose method and describe how it generalises to $D > 4$.

# Newman-Penrose formalism

- In the Newman-Penrose formalism the 10 components of the Weyl tensor are encoded into 5 complex scalars: $\Psi_0 \ldots \Psi_4$.
- Define a null tetrad onto which the Weyl tensor is projected

$$\ell^\mu = \frac{1}{\sqrt{2}}\left(e^\mu_{(0)} + e^\mu_{(1)}\right) \;\; k^\mu = \frac{1}{\sqrt{2}}\left(e^\mu_{(0)} - e^\mu_{(1)}\right)$$
$$m^\mu = \frac{1}{\sqrt{2}}\left(e^\mu_{(2)} + ie^\mu_{(3)}\right) \;\; \bar{m}^\mu = \frac{1}{\sqrt{2}}\left(e^\mu_{(2)} - ie^\mu_{(3)}\right)$$

- The "peeling property" of the Weyl tensor states that

$$\Psi_n \sim \frac{1}{r^{5-n}}$$

- $\Psi_4$ encodes information about outgoing radiation, defined as :

$$\Psi_4 = C_{\mu\nu\rho\sigma} k^\mu \bar{m}^\nu k^\rho \bar{m}^\sigma$$

- By comparing this to the linearised Einstein equations in TT gauge we get:

$$\Psi_4 = \ddot{h}_+ - i\ddot{h}_\times$$

## Newman-Penrose cont.

- Energy radiated through gravitational waves is given by:

$$-\frac{dE}{dt} = \lim_{r\to\infty} \frac{r^2}{16\pi} \int_{S^2} \left| \int_{-\infty}^{t} \Psi_4 \, dt' \right|^2 d\omega$$

- We can calculate the Weyl tensor by reconstructing the Riemann tensor from our 3+1 variables over a sphere far from the black hole collision where the Ricci tensor and Ricci scalar vanish.

# Higher Dimensional Newman-Penrose

- The peeling property of the Weyl tensor is not so simple in $D > 4$ . Following Godazgar & Reall (Phys. Rev. D 85, 084021) we contract the Weyl tensor over a null frame ($(I), (J)$ run over $D-2$ orthogonal spatial dimensions):

$$\ell^A = -\frac{\partial}{\partial r}, \;\; k^A = \pm\left(\frac{\partial}{\partial u} - \frac{1}{2}\frac{\partial}{\partial r}\right),$$

$$m^A_{(I)} \text{ spacelike and orthonormal } \text{ s.t. } k \cdot m_{(I)} = 0$$

- We want the object that is the leading order term in $1/r$, analogous to $\Psi_4$,

$$\Omega'_{(I)(J)} = C_{ABCD} k^A m^B_{(I)} k^C m^D_{(J)}$$

.

## Higher Dimensional Newman-Penrose cont

- Using Bondi coordinates to define asymptotic flatness, it is shown that

$$\Omega'_{(I)(J)} = -\frac{1}{2}\frac{e^{\hat{a}}_{(I)} e^{\hat{b}}_{(J)} \ddot{h}^{(1)}_{\hat{a}\hat{b}}}{r^{D/2-1}} + \mathcal{O}(r^{-D/2})$$

where $h^{(1)}_{\hat{a}\hat{b}}$ is the Bondi news function.

- By the definition of Bondi mass

$$\dot{M}(u) = -\frac{1}{32\pi}\int_{S^{D-2}} \dot{h}^{(1)}_{\hat{a}\hat{b}} \dot{h}^{(1)\hat{a}\hat{b}} d\omega$$

we have:

$$\dot{M}(u) = -\lim_{r\to\infty}\frac{r^{D-2}}{8\pi}\int_{S^{D-2}}\left(\int_{-\infty}^{u}\Omega'_{(I)(J)}(\hat{u},r,x)d\hat{u}\right)^2 d\omega$$

# Overview

# Modified Cartoon formalism

- We perform a $(D-1)+1$ splitting of our spacetime, and evolve the BSSN equations for $(\chi, \tilde{\gamma}_{AB}, K, \tilde{A}_{AB}, \tilde{\Gamma}^A)$
- Increasing $D$ makes simulations much tougher.
- Don't want to evolve full spacetime - use symmetry to make problem easier.
- Use Modified Cartoon formalism - allows simulation of higher $D$ than other methods, e.g. reduction by isometry. (Yoshino, Shibata 2010; Pretorius 2005)
- Evolve spacetimes with $SO(D-3)$ symmetry as a $3D$ hypersurface with additional functions defined upon it.

# Modified Cartoon cont.

- Define coordinates $(x, y, z, w_a)$, such that symmetry is in $z - w_a$ and $w_a - w_b$ planes.
- Transform to polar coordinates $(\rho, \varphi)$ in a $z - w_a$ plane.
- Apply symmetry conditions: $\partial_\varphi g_{AB} = 0$, $g_{A\varphi} = 0$ $A \neq \varphi$.
- Every BSSN variable we work with is constructed from $g$, so we can apply these conditions to everything we will work with.
- Transform back to Cartesians.
- Set $w = 0$.
- We need to introduce one new function for each (0,2) tensor e.g. $\tilde{\gamma}_{ww}, \tilde{A}_{ww}$.
- We can express $\partial_w$ in terms of derivatives and quantities in the $xyz$ plane.

# Riemann tensor decomposition

- We use this method, and the Gauss and Codazzi equations, to construct the full Riemann tensor from the BSSN modified cartoon variables we evolve.
- We introduce terms involving division by $z$, which we must regularise.

## Constructing the null frame

- We construct normalised basis vectors for the $D-2$ sphere, and evaluate these in our computational domain, giving 2 standard vectors

$$
\begin{aligned}
m_1 &= (0, -y^2 - z^2, xy, xz, 0, \cdots, 0), \\
m_2 &= (0, 0, -z, y, 0, \cdots, 0)
\end{aligned}
$$

and the remaining extra-dimensional vectors

$$
\begin{aligned}
m_3 &= (0, 0, 0, 0, 1, 0, \ldots, 0) \\
m_4 &= (0, 0, 0, 0, 0, 1, 0, \ldots, 0) \\
&\vdots \\
m_{D-2} &= (0, \ldots, 0, 1)
\end{aligned}
$$

- We orthonormalise with the radial vector using the Gram-Schmidt process.

# Constructing $\Omega'_{IJ}$

- Construct ingoing null radial vector

$$k^A = \frac{1}{2}\left(\frac{1}{\alpha}, -\frac{\beta^i}{\alpha} - r^i, 0\ldots, 0\right)$$

- Contracting with the Riemann tensor we get 4 components for $\Omega'_{11}, \Omega'_{12}, \Omega'_{22}, \Omega'_{ww}$ (NB only 3 are independent as $\Omega'_{IJ}$ is tracefree)

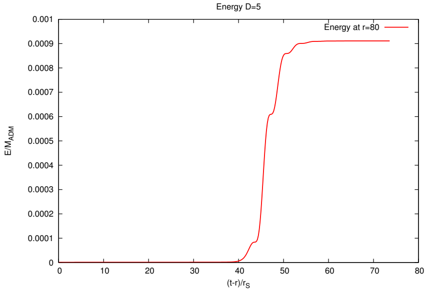
# Overview

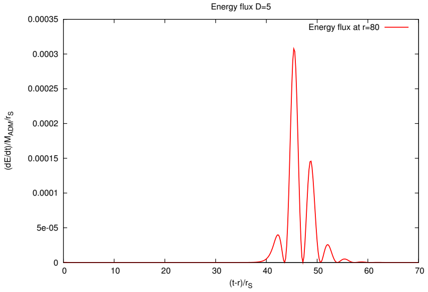
# Results

$$\dot{M}(u) = -\lim_{r\to\infty} \frac{r^{D-2}}{8\pi} \int ({}^{I}\Omega'^{2}{}_{11} + 2\,{}^{I}\Omega'^{2}{}_{12} + {}^{I}\Omega'^{2}{}_{22} + (D-4)\,{}^{I}\Omega'^{2}{}_{ww})\, d\omega$$

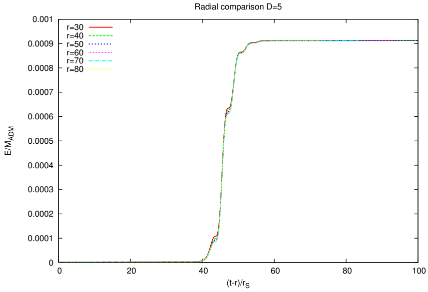
- Using Kodama-Ishibashi perturbative wave extraction, radiated energy for head on collision from rest in 5D has been calculated:
  $E_{\mathrm{rad}}/M_{\mathrm{ADM}} = 8.9 \pm 0.6 \times 10^{-4}$
  (Witek et al. Phys. Rev. D 82, 104014)
- We calculate: $E_{\mathrm{rad}}/M_{\mathrm{ADM}} = 9.06 \times 10^{-4}$

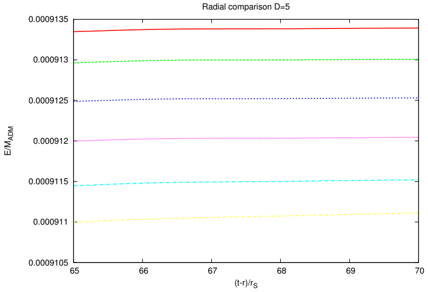
# Energy radiated in $D = 5$



# Extraction radius comparison



# Conclusions

- We have implemented a new method for calculating energy radiated in GWs in higher dimensional numerical relativity.
- Will allow us to probe higher dimensions than previously possible.
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Thank You.