Superfluid flow with external localized repulsive potential

VIII BHs Workshop@Lisbon

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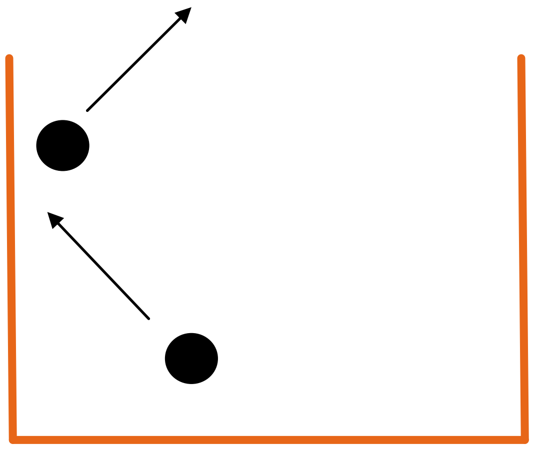
I *Introduction*

AdS spacetime

AdS/CFT(CMP)

Strongly coupled gauge theory



AdS Boundary

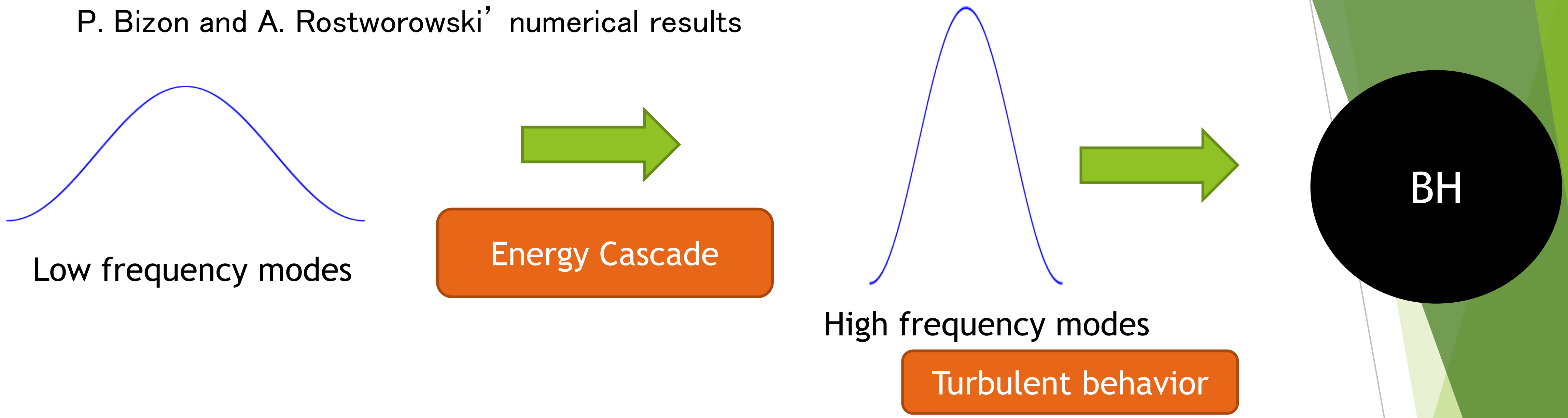
AdS is ***non-linearly*** unstable

Singularity generically appears from regular initial data!

AdS spacetime $\approx$ Confining BOX

P. Bizon and A. Rostworowski (2011)

P. Bizon and A. Rostworowski' numerical results



Q. Does any similar turbulent behavior occur under ***the black hole background***?

Maybe ***Yes!***

Cosmic censorship is violated!?

B. E. Niehoff, J. E. Santos, B. Way (2015)

S. R. Green, S. Hollands, A. Ishibashi, R. W. Wald arXiv:1512.02644

Superradiant instabilities of Kerr-AdS black holes trigger ***energy cascade***

Key question: In a wide class of AdS BHs, does turbulent behavior occur?

Condensed matter physics:

Vortex formation, Superfluid flow, …..

AdS/CFT(CMP)

BH physics

turbulent behavior

Today's talk:

As a first step, we explore how steady superfluid flow is broken via AdS/CFT

The model of Bose-Einstein Condensation

1-dim. ***Non-linear Schrodinger model***

$$i\partial_t \varphi - i v \partial_x \varphi = -\partial_{xx}\varphi - \varphi + |\varphi|^2 \varphi + U(x)\varphi$$

The limit of application: ***Weakly interacting dilute Bose gas system***

Extension to ***strongly coupled*** gauge theory

AdS/CFT(CMP)

$$\mathcal{L} = -|\nabla\psi - iA\psi|^2 - m^2|\psi|^2 - V(x,u)|\psi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

u: radial coordinate of AdS

Holographic superconductor model

Hartnoll, Herzog, Horowitz (2008)

Q. How steady superfluid flow is broken?

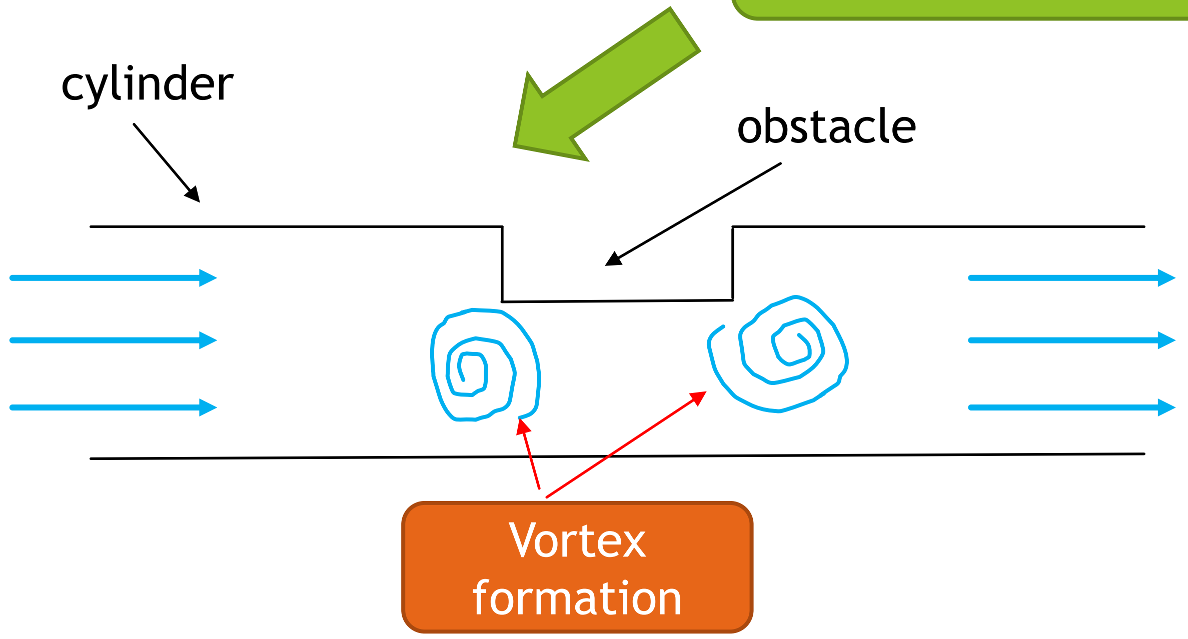
Landau's criterion

$V > V_L \qquad V_L \equiv \mathrm{Min}\,\varepsilon(p)/p$

ε(p): elementary excitation energy

Experimental results: Ex). ^{4}He

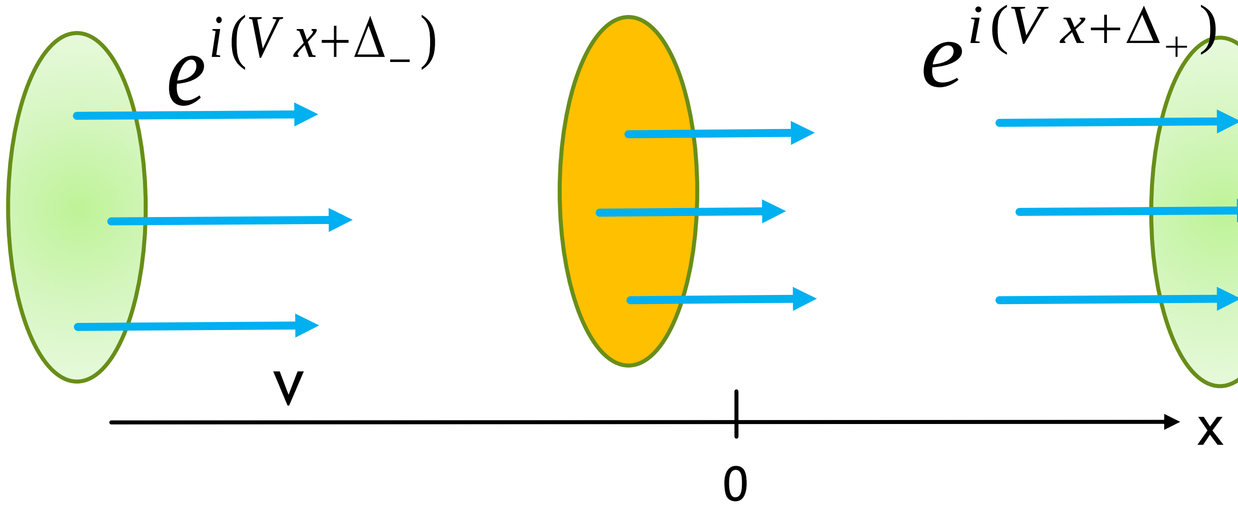
Critical Velocity $V_c < V_L$

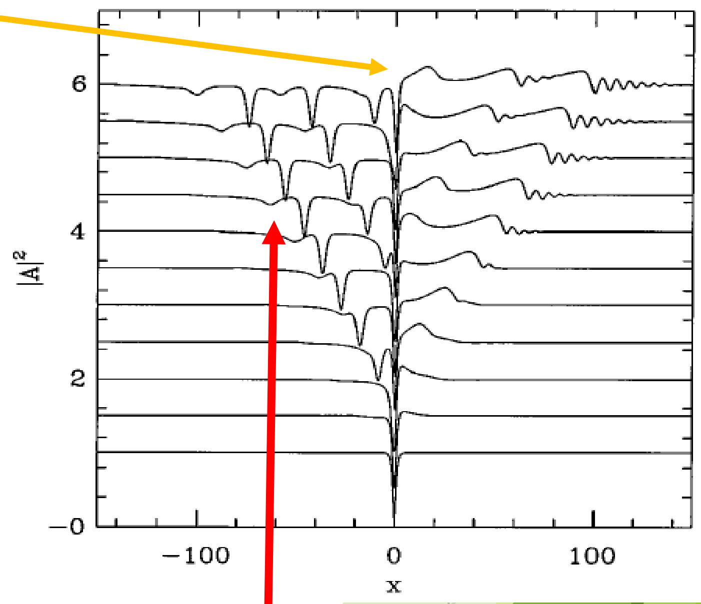


Hakim's analysis(1997)

$$i\partial_t\varphi - iv\partial_x\varphi = -\partial_{xx}\varphi - \varphi + |\varphi|^2\varphi + U(x)\varphi$$

$U(x) = g\,\delta(x),\quad g>0\quad$ *repulsive potential*





Hakim(1997) PRE55

- When g< g_c two steady flow solutions appear
- When g= g_c the two solutions coalesce and disappear.
- g> g_c ***the gray soliton*** solutions are created by the obstacle.

Set Up of Holographic model

Probe approximation → Sch-AdS5:

$$ds_5^2 = -\frac{1-u^2}{u}dt^2 + \frac{du^2}{4u^2(1-u^2)} + \frac{1}{u}(dx^2 + dy^2 + dz^2).$$

$m^2 = -4$ (Analytic solution can be expected: Herzog(2011))

→ $$\mathcal{L} = -|\nabla\psi - iA\psi|^2 + 4|\psi|^2 - V(x,u)|\psi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$V(x,u) = g\,u\delta(x)$$

M_μ : Gauge invariant quantity

$$\psi = Re^{i\varphi} \qquad M_\mu := A_\mu - \nabla_\mu\varphi$$

Field Eqs.

$$D^\mu D_\mu\psi + 4\psi - V(x,u)\psi = 0,$$

$$\nabla_\nu F^{\nu\mu} = i[\psi^* D^\mu\psi - \psi(D^\mu\psi)^*],$$

$$D_\mu := \nabla_\mu - iA_\mu$$

→

$$\nabla^2 R - M^\mu M_\mu R - m^2 R - V(x,u)R = 0,$$

$$\nabla^\mu (M_\mu R^2) = 0$$

Momentum Conservation

$$\nabla_\nu F^{\nu\mu} = 2M^\mu R^2$$

We expand R and M_μ in a series of small parameter ε:

$$R = \epsilon R_1(u,x) + \epsilon^3 R_3(u,x) + \cdots ,$$

$$M_\mu = M_\mu^{(0)}(u,x) + \epsilon^2 M_\mu^{(2)}(u,x) + \cdots$$

0th order solution of M_μ

$$M_t^{(0)} = \mu_0(1-u), \qquad M_x^{(0)} = \xi(x), \qquad M_u^{(0)} = 0$$

μ_0 : chemical potential

Velocity of superfluid

AdS/CFT

1th order solution of R_1 → $R_1 = \rho(u)\zeta(x)$

Separation of variables

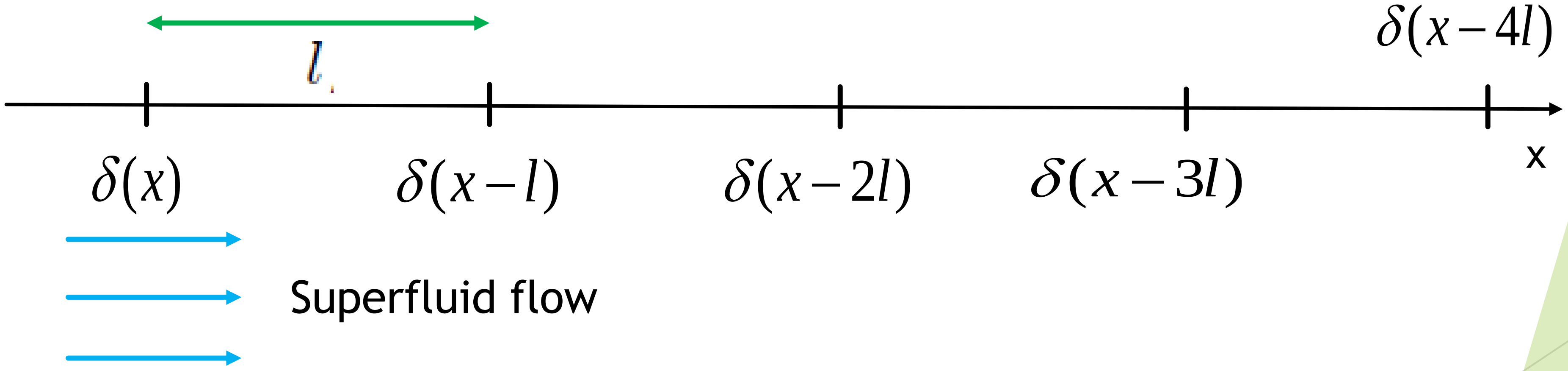
$$\zeta^2\xi = c_2.$$

Separation constant

$$\partial_x^2\zeta - \xi^2\zeta - g\delta(x)\zeta = -\kappa^2\zeta$$

$$4u^2(1-u^2)\rho'' - 4u(1+u^2)\rho' + \left[4 + \frac{\mu_0^2\, u(1-u)}{1+u} - \kappa^2 u\right]\rho = 0.$$

We assume a ***periodic potential*** V:



Bloch's theorem → ζ must be a periodic function of x

Solution: $0 \le x \le l$

$$\zeta^2 = \frac{1-\alpha^2}{2}\cos 2\kappa(x - l/2) + \frac{1+\alpha^2}{2}$$

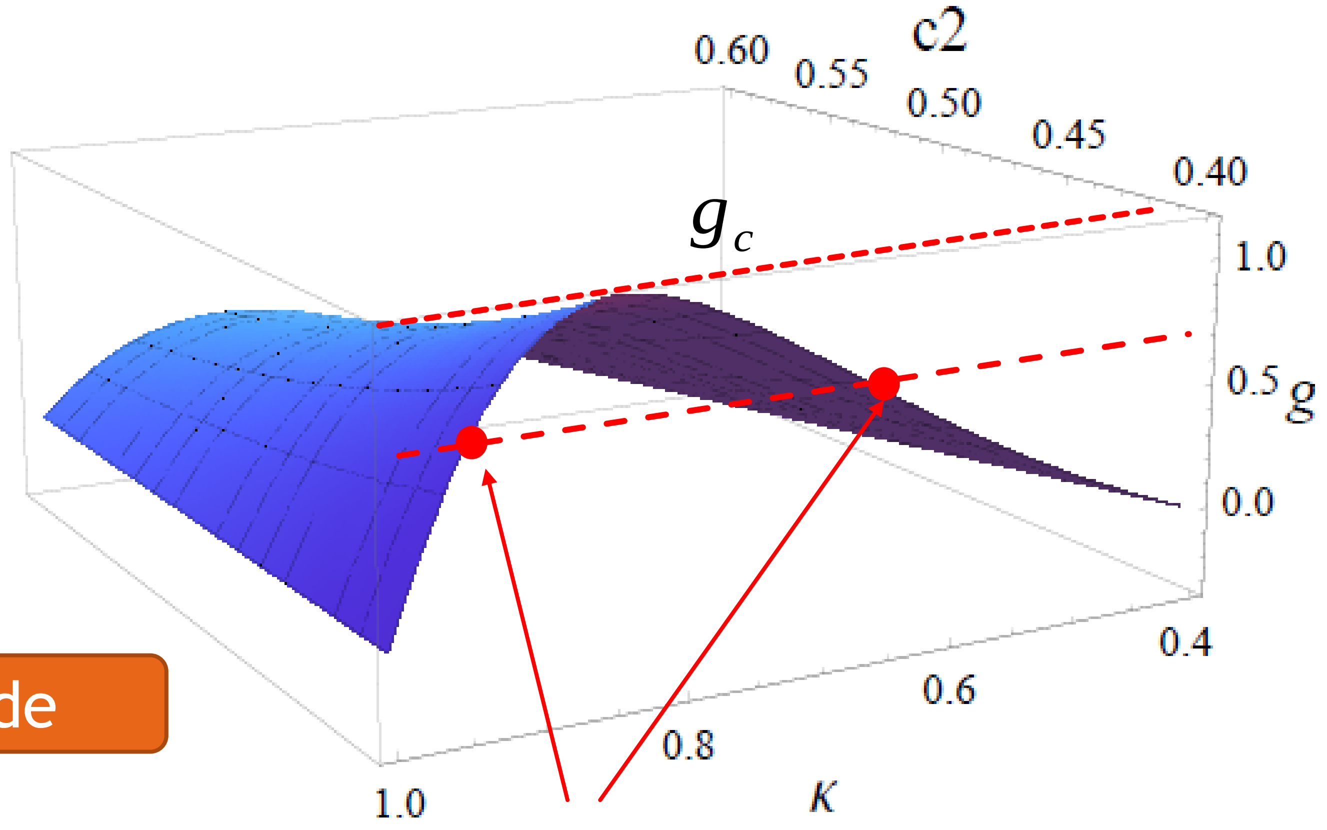
$$c_2 = \alpha\kappa$$

Asymptotic Condition for ρ

$\rho \to O(u)$ Normalizable mode

$$\rho = \frac{u}{1+u} - \kappa^2 \frac{u\ln(1+u)}{4(1+u)} + O(\kappa^4),$$

$$\mu_0 = 2 + \frac{1}{2}\kappa^2 + O(\kappa^4).$$



Two solution coexist

Holographic extension to Hakim's result

2^{th} order solution of $O(\epsilon^2)$

Eqs. for $M_x^{(2)}$、 $M_t^{(2)}$

$$(1-u^2)M_x''^{(2)} - 2uM_x'^{(2)} = \frac{c_2\rho^2}{2u^2} \qquad 4u\,\partial_u^2 M_t^{(2)} + \frac{1}{1-u^2}\partial_x^2 M_t^{(2)} = \frac{2\mu_0}{u(1+u)}\rho^2\zeta^2$$

$$M_x^{(2)} = \frac{c_2}{32(1+u)} \times \Big[8(1-\kappa^2) + \kappa^2\{(2\ln 2)(1+u)\ln(1-u) - \ln(1+u)(4+2(1+u)\ln(1-u) - (1+u)\ln(1+u))\} - 2\kappa^2(1+u)\mathrm{Li}_2\left(\frac{1+u}{2}\right)\Big] - \frac{c_2}{192}\{48 - \kappa^2(48+\pi^2 - 6(\ln 2)^2\} + O(\kappa^4).$$

polygamma function

AdS/CFT → Current of superfluid flow

$$\langle J_x \rangle \sim M_x'^{(2)}(0) = -\frac{c_2}{8}(2 - \kappa^2(1-\ln 2)).$$

$$M_t^{(2)}(u,x) = \eta_{t0}(u) + \eta_{t1}(u)\cos 2\kappa(x - l/2) \quad \bullet\ \bullet\ \bullet$$

What happens near g= g_c ?

Y. Kato, S. Watabe (2010) PRL105 ← The analysis of non-linear Schrodinger Eq.

Dynamical density fluctuations (DDFs) is important near the critical velocity

↓

Spectral function of the local density ***I***

$$I(x,\omega) = \sum_l |\langle l | \psi^+(x)\psi(x) | g\rangle|^2 \delta(\omega - E_l + E_g)$$

g: ground state
l: excited state

$$I(x,\omega) = \omega^{d-2} F(x, \omega |g-g_c|^{-\frac{1}{2}})$$

d=1 → $I(x,\omega) \to \infty$ for $\omega \to |g-g_c|^{\frac{1}{2}}$

Holographic calculation of $I(x,\omega)$

➡ Construction of retarded Green function G_R of perturbed ψ in the bulk

$$D^2\delta\psi - V(x,u)\,\delta\psi = 0 \;\Longrightarrow\; D^2 G_R - V(x,u)\,G_R = \delta(t-t', x-x', u-u')$$

Separation of variables $\delta\psi = \chi_\kappa(x)\times R(u;\kappa)$

$$(D_x{}^2 - V(x))\chi_\kappa = -\kappa^2\,\chi_\kappa \qquad \sum_\kappa \chi_\kappa{}^{+}(x)\,\chi_\kappa(x') = \delta(x-x')$$

$$\delta\psi \cong \alpha_+(\omega,\kappa)\,u^{\lambda_+} + \alpha_-(\omega,\kappa)\,u^{\lambda_-} \qquad u\to 0$$

Normalizable mode (u^{λ_+})

$$I\;(\omega,x) \approx \mathrm{Im}\Big[\sum_\kappa \frac{\alpha_+(\omega,\kappa)}{\alpha_-(\omega,\kappa)}\chi_\kappa{}^{+}(x)\,\chi_\kappa(x)\Big]$$

A. Ishibashi, K. M., T. Okamura work in progress

Summary

- We investigated holographic superfluid flow as a first step to analyze ***turbulence*** of rotating AdS BH

- We obtained ***analytic solutions with external repulsive potential*** and reproduced Hakim' s results in holographic setting

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Future plan

- Derivation of spectral function I(ω, x) in the holographic setting
- Construction of ***gray soliton solution*** in rotating hairly AdS solutions

Thank you!