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On the angular momenta of two classical black holes

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Abstract

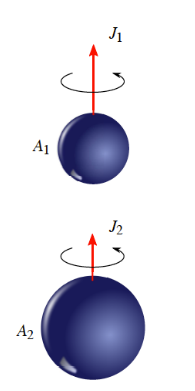
- Using exact two Kerr-NUT black holes exact axisymmetric solutions we make some remarks on possible interaction of two classical black holes with respect to their angular momenta.
  - "Double Kerr-NUT spacetimes: spinning strings and spinning rods"
    Patricio S. Letelier and Samuel R. Oliveira, **Phys. Lett. A 238**, 101 (1998)
  - "On the interaction between two Kerr black holes"
    Carlos A.R. Herdeiro and Carmen Rebelo, **JHEP 10**, (2008)

Ilustration

2 static BHs

- with core masses m_i
- at the axis positions z_i
- with angular parameters $a_i = J_i / m_i$
- with horizon areas A_i

Aligned



Neugebauer & Hennig [2014]

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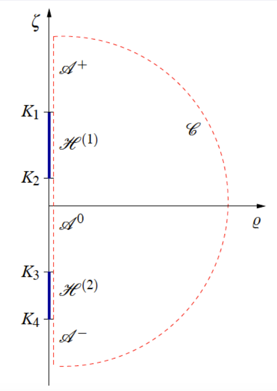
- Rotation is repulsive
  - No repulsion if $a_1 = a_2 \Rightarrow$ same directions
  - Maximum repulsion for $a_1 = -a_2 \Rightarrow$ opposite directions
- Repulsion is not enough to keep the BHs apart.
  - There exists a conical singularity between them: Letelier & Oliveira [1998]
  - Non-existence proof: Neugebauer & Hennig [2009]

Ilustration in Bach-Weyl setup

2 static BHs

- represented by two rods
- Killing horizons $\mathscr{H}^{(i)}$
- segments on the axis $\mathscr{A}^{o\pm}$
- spatial infinity $\mathscr{C}$

Aligned



Neugebauer & Hennig [2014]

Belinsky-Zakharov Method

- A 4-soliton solution:

$$ds^2 = g_{ab}(r,z)\,dx^a dx^b - f(r,z)\left(dr^2 + dz^2\right)$$

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$$g_{ab} = \frac{1}{r^4}\prod_{k=1}^{4}\mu_k\left(g^o_{ab} - \sum_{k,l=1}^{4}\frac{\Gamma^{-1}_{kl}N^{(k)}_a N^{(l)}_b}{\mu_k\mu_l}\right),\ etc.$$

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- $r = 0$ is the axis? By construction $\det g_{ab} = -r^2$.
- asymptotically flat?

Weyl-Lewis-Papapetrou coordinates

- Weyl-Lewis-Papapetrou form:

$$ds^2 = e^{2\psi}(dt + wd\phi)^2 - r^2 e^{-2\psi} d\phi^2 - e^{2(\nu-\psi)}\left(dr^2 + dz^2\right)$$

  with $\psi(r,z),\ w(r,z),\ \nu(r,z)$.
- Conic singularities if for small circles centred on the $z-$axis on planes of constant z we have ratio circumference/radius $\neq 2\pi$.
- WLP coordinates can be global coordinates in the axisymmetric, stationary and asymptotically flat vacuum region outside of rotating black holes.

  Chrusciel & Costa [2008]

Minkowsky case

- Set all $m_i = b_i = 0$, $a_i \neq 0$ in the 4 soliton solution. One gets

$$ds^2 = (dt + w_o d\phi)^2 - r^2 d\phi^2 - e^{2\nu_o}\left(dr^2 + dz^2\right)$$

- with

$$w_o = -2(a_1 + a_2),\ e^{\nu_o} = \frac{64}{c_4}\left((z_2 - z_1)^2 + (a_2 - a_1)^2\right).$$

- which represents a spinning string with angular momentum $J = -(a_1 + a_2)/2$ and tension or pressure $\lambda = (1 - e^{\nu_o})/4$.

The functions on $r = 0$

- Set $r = 0$ outside the BHs in the 4 soliton solution.
- One gets long expressions in terms of the parameters of the solution.
- Fix the parameters so it leaves just a conical singularity between the BHs
- Use $F_z = \int T_z^z \sqrt{g_2} dr d\phi$ with Einstein Equations with delta distribution:

$$F_z = \frac{1}{8\pi} \int e^{-\nu} \left[\nu_r + r\left(\psi_z^2 - \psi_r^2\right) + \frac{e^{4\psi}}{4r}\left(w_r^2 - w_z^2\right) \right] d\phi dr.$$

- Integrating in a small disk centered in the symmetry axis one gets

$$F_z = \lambda = \left(1 - e^{-\nu_o}\right)/4.$$

Conclusions

- Exact solutions can offer several interesting solutions based on careful interpretation
- Applications to physical setups or in numerical computation
- 2 BHs angular momenta interaction with orbital motion?
- Temperature and Entropy?

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