A general formula for the redshift drift at low *z* *

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Summary

- The redshift – low-z approximation
- The redshift drift – low-z approximation
- Examples: LTB, ΛCDM
- Conclusions

The redshift

$$z = \frac{\lambda_o - \lambda_e}{\lambda_e}$$

$$1 + z = \frac{\nu_e}{\nu_o} = \frac{(k_\alpha u^\alpha)_e}{(k_\alpha u^\alpha)_o} ,$$

Integration of the null geodesics equation is needed to obtain k_α at the observer, which must be done numerically in cases with few or no symmetries at all.

$z << 1 \rightarrow z = d\lambda / \lambda \rightarrow$

$$z = \frac{(k_\alpha u^\alpha)_e - (k_\alpha u^\alpha)_o}{(k_\alpha u^\alpha)_o} \simeq \frac{\mathrm{d}(k_\alpha u^\alpha)}{(k_\alpha u^\alpha)_o} ,$$

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$$\mathrm{d}(k_\alpha u^\alpha) = (k_\alpha \dot{u}^\alpha) = (k_{\alpha;\beta} k^\beta u^\alpha)_o \mathrm{d}v + (k_\alpha u^\alpha{}_{;\beta} k^\beta)_o \mathrm{d}v \,.$$

$$\dot{A}_\nu \equiv (k_\mu \nabla^\mu) A_\nu$$

where *v* is the affine parameter on the ray.

Using

$$k_{\beta;\alpha} k^\alpha \quad = \quad 0 \,,$$

$$\nabla_\mu u_\nu = -u_\mu \dot{u}_\nu + \frac{1}{3}\Theta h_{\mu\nu} + \sigma_{\mu\nu} + \omega_{\mu\nu}$$

$$h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$$

$$\Theta = \tilde{\nabla}_\mu u^\mu$$

$$\sigma_{\mu\nu} = \tilde{\nabla}_{<\mu} u_{\nu>}$$

$$\omega_{\mu\nu} = \tilde{\nabla}_{[\mu} u_{\nu]}$$

$$\mathrm{d}(k_\alpha u^\alpha) \quad = \quad \left[(\sigma_{\alpha\beta} k^\alpha k^\beta)_o - \frac{1}{3}\Theta_o (k_\alpha u^\alpha)_o^2 + (k_\alpha \dot{u}^\alpha)_o (k_\alpha \dot{u}^\alpha)_o \right] \mathrm{d}v$$

In terms of the vector

$$n^\alpha = -\frac{1}{k_\gamma u^\gamma} k^\alpha + u^\alpha$$

$$n_\alpha n^\alpha = -1\,,$$

$$n^\alpha u_\alpha = 0$$

which is collinear with the projection of *k* $^\alpha$ on *t* = const. (and points towards the origin),

$$\begin{aligned} \mathrm{d}(k_\alpha u^\alpha) &= \left[(\sigma_{\alpha\beta} k^\alpha k^\beta)_o - \frac{1}{3}\Theta_o (k_\alpha u^\alpha)^2_o + (k_\alpha \dot{u}^\alpha)_o (k_\alpha \dot{u}^\alpha)_o\right] \mathrm{d}v \\ &= (k_\gamma u^\gamma)^2_o \left(\sigma_{\alpha\beta} n^\alpha n^\beta - \frac{1}{3}\Theta - (n_\alpha \dot{u}^\alpha)\right)_o \delta v\,. \end{aligned}$$

$$k^\rho n_\rho = k^\rho u_\rho,$$

$$\delta\ell \equiv [-(k^\gamma n_\gamma)]\mathrm{d}v$$

is the distance (in the rest space of the observer) between the source and the observer

$$z = \left(\frac{1}{3}\Theta - \sigma_{\alpha\beta} n^\alpha n^\beta + n_\alpha \dot{u}^\alpha\right)_o \delta\ell_o\,.$$

(Plebansky and Krasinski, 2006)

$$z = \left(\frac{1}{3}\Theta - \sigma_{\alpha\beta}n^{\alpha}n^{\beta} + n_{\alpha}\dot{u}^{\alpha}\right)_{o} \delta\ell_{o}\,.$$

* This expression is valid for small *z* , and for any observer and any gravitational theory.

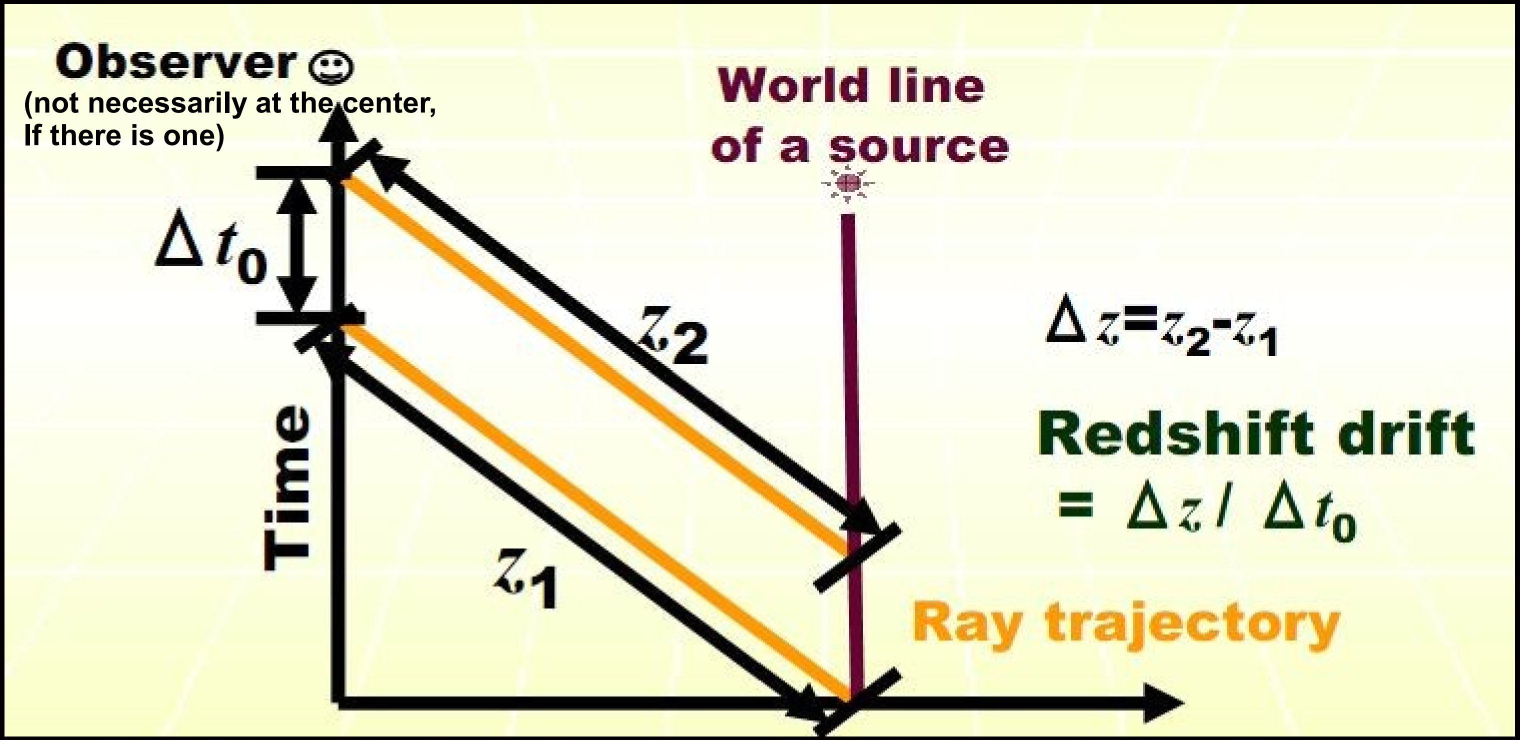
* All the quantities are evaluated at the position of the observer (no integration of the geodesics is needed).

* *z* does not depend of the vorticity (for *z<<1*).

* The shear and the acceleration introduce anisotropic effects in *z*.

* In isotropic models, the redshift observed at the center is due solely to the expansion (for small *z*).

The redshift drift (Sandage, 1962)



(C. Yoo)

* Potential direct and "clean" probe of the expansion since it does not rely on details of the source (e.g. "standard candles").

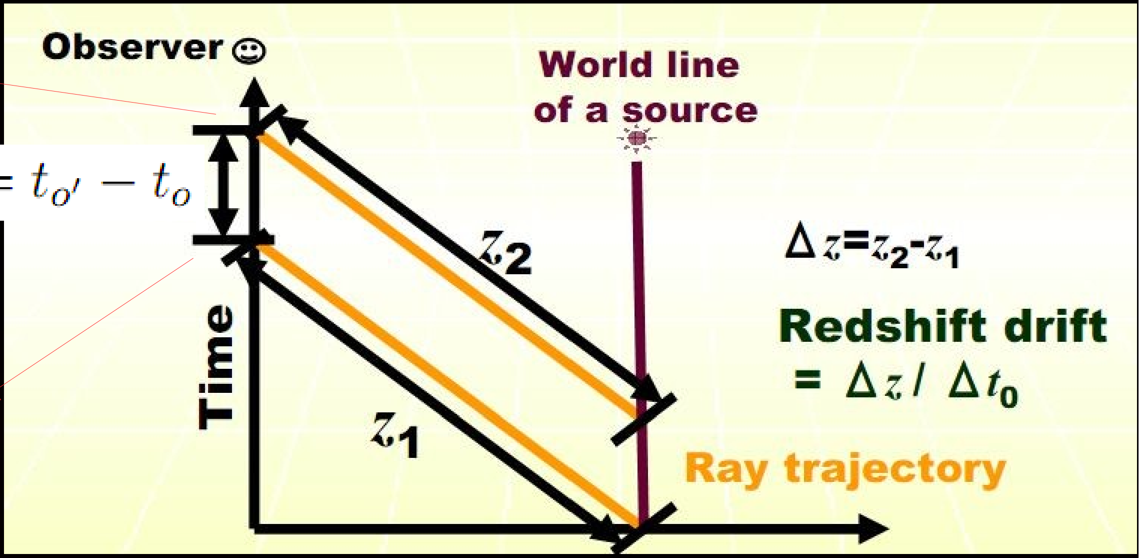
To calculate the redshift drift, integration of the null geodesics is needed → we can try a low-z approximation.

The redshift drift for *z<< 1* can be calculated as follows:

$$z_{o'} = \left(\frac{1}{3}\Theta - \sigma_{\alpha\beta}n^\alpha n^\beta + n_\alpha \dot{u}^\alpha\right)_{o'} \delta l_{o'}$$

$$\delta t_o = t_{o'} - t_o$$

$$z_o = \left(\frac{1}{3}\Theta - \sigma_{\alpha\beta}n^\alpha n^\beta + n_\alpha \dot{u}^\alpha\right)_{o} \delta l_{o}$$



We will consider from now on that the acceleration is zero, a condition satisfied if the pressure is homogeneous (for a perfect fluid).

Developing $z_{o'}$ in series around t_o, we get

$$\frac{z_{o'} - z_o}{\delta t_o} = \left[\frac{\mathrm{d}\sigma_{\alpha\beta}}{\mathrm{d}t} n^\alpha n^\beta - \sigma_{\alpha\beta}\frac{\mathrm{d}}{\mathrm{d}t}(n^\alpha n^\beta) + \frac{1}{3}\frac{\mathrm{d}\Theta}{\mathrm{d}t}\right]_o \delta l_o - \left[\left(\sigma_{\alpha\beta} n^\alpha n^\beta\right) + \frac{1}{3}\Theta\right]_o \frac{\mathrm{d}\delta l}{\mathrm{d}t}\bigg|_o$$

(Up to this point, the development is independent of the gravitational theory).

The time derivatives can be rewritten as follows ($u^\mu = \delta^\mu{}_o$):

$$\dot\sigma^{<\mu\nu>} - \cancel{\tilde\nabla^{<\mu}\dot u^{\nu>}} = -\frac{2}{3}\Theta\sigma^{\mu\nu} + \cancel{\dot u^{<\mu}\dot u^{\nu>}} - \sigma^{<\mu}{}_{\gamma}\sigma^{\nu>\gamma} - \omega^{<\mu}\omega^{\nu>} - \left(E^{\mu\nu} - \frac{1}{2}\pi^{\mu\nu}\right).$$

$$\dot T^{<ab>} = [h^{(a}{}_c h^{b)}{}_d - \frac{1}{3}h^{ab}h_{cd}]\dot T^{cd}$$

$$\dot{\Theta} - \cancel{\tilde{\nabla}_a \dot{u}^a} = -\frac{1}{3}\Theta^2 + \cancel{(\dot{u}_a \dot{u}^a)^2} - 2\sigma^2 + 2\omega^2 - \frac{1}{2}(\mu + 3p) + \Lambda\,.$$

$$(u \cdot \nabla) n^\alpha = -\frac{\dot{k}^\alpha}{k \cdot u} + \frac{u \cdot \dot{k}}{(k \cdot u)^2} k^\alpha\,.$$

The evolution of the relative distance between the source and the observer ($\delta\ell$) is given by

$$\frac{(\delta\ell)^{\cdot}}{\delta\ell} = \sigma_{\mu\nu} N^\mu N^\nu + \frac{1}{3}\Theta$$

$$X_\perp^\mu = N^\mu \delta\ell$$

Substituting all this in

$$\frac{z_{o'} - z_o}{\delta t_o} = \left[\frac{\mathrm{d}\sigma_{\alpha\beta}}{\mathrm{d}t} n^\alpha n^\beta - \sigma_{\alpha\beta}\frac{\mathrm{d}}{\mathrm{d}t}(n^\alpha n^\beta) + \frac{1}{3}\frac{\mathrm{d}\Theta}{\mathrm{d}t}\right]_o \delta l_o - \left[\left(\sigma_{\alpha\beta} n^\alpha n^\beta\right) + \frac{1}{3}\Theta\right]_o \frac{\mathrm{d}\delta l}{\mathrm{d}t}\bigg|_o$$

We obtain the expression for the redshift drift:

$$\frac{\delta z}{\delta t_o} = \left\{ -n_\mu n_\nu \left[-\frac{2}{3}\Theta\sigma^{\mu\nu} - \sigma^{<\mu}_{\ \ \alpha}\sigma^{\nu>\alpha} - \omega^{<\mu}\omega^{\nu>} - E^{\mu\nu} \right] + \frac{2\sigma_{\alpha\beta}}{(k\cdot u)^2}\left[(k\cdot u)\dot{k}^\beta - (u\cdot\dot{k})k^\beta\right] n^\alpha \right.$$
$$\left. + \frac{1}{3}\left[-\frac{1}{3}\Theta^2 - 2\sigma^2 + 2\omega^2 - \frac{1}{2}(\rho + 3p) + \Lambda \right] \right\}_o \delta\ell_o - \left[\sigma_{\alpha\beta} n^\alpha n^\beta - \frac{1}{3}\Theta\right]_o \frac{\mathrm{d}\delta\ell}{\mathrm{d}t}\bigg|_o$$

* **It is valid for $z << 1$, and for GR only (due to the evol. eqns. for the expansion and the shear).**

* **It is valid for any observer such that $u^\mu = \delta^\mu_0$.**

* **$\delta z/\delta t_o$ depends of all the kinematical quantities, and also of $E^{\mu\nu}$.**

* **The shear, the vorticity, and $E^{\mu\nu}$ introduce anisotropy.**

Example 1: the Lemâitre-Tolman-Bondi model.

Spherically-symmetric solution of GR with dust as a source:

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{[R_{,r}(t,r)]^2}{1+2E(r)}\mathrm{d}r^2 - [R(t,r)]^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2)\,,$$

$$R_{,t}^2 \;=\; 2E(r) + \frac{2M(r)}{R}\,,$$

$$\kappa\rho \;=\; \frac{2M_{,r}}{R^2 R_{,r}}.$$

This solution has been used to describe the accelerated expansion of the universe without the use of dark energy.

The acceleration and the vorticity are zero everywhere, and the shear is zero at the center.

$$\frac{\delta z}{\delta t_o} = \left\{ -n_\mu n_\nu \left[-\frac{2}{3}\Theta\sigma^{\mu\nu} - \sigma^{<\mu}_{\ \ \alpha}\sigma^{\nu>\alpha} - \omega^{<\mu}\omega^{\nu>} - E^{\mu\nu} \right] + \frac{2\sigma_{\alpha\beta}}{(k\cdot u)^2}\left[(k\cdot u)\dot{k}^\beta - (u\cdot\dot{k})k^\beta \right] n^\alpha \right.$$
$$\left. + \frac{1}{3}\left[-\frac{1}{3}\Theta^2 - 2\sigma^2 + 2\omega^2 - \frac{1}{2}(\rho + 3p) + \Lambda \right] \right\}_o \delta\ell_o - \left[\sigma_{\alpha\beta} n^\alpha n^\beta - \frac{1}{3}\Theta \right]_o \frac{\mathrm{d}\delta\ell}{\mathrm{d}t}\bigg|_o$$

$$\frac{(\delta\ell)^{\cdot}}{\delta\ell} = \sigma_{\mu\nu}N^\mu N^\nu + \frac{1}{3}\Theta$$

reduces to:

$$\frac{\delta z}{\delta t_o} = -\frac{1}{6}\rho_{m,o}\delta\ell_0$$

Hence, the RD in the LTB model is negative for an observer at the center, and for *z << 1*.

Using properties of the null geodesics and of the geometry, the same result was obtained by Yoo et al (2010).

Example 2: the ΛCDM model.

$$\frac{\delta z}{\delta t_o} = \left\{ -n_\mu n_\nu \left[-\frac{2}{3}\Theta\sigma^{\mu\nu} - \sigma^{<\mu}_{\ \ \alpha}\sigma^{\nu>\alpha} - \omega^{<\mu}\omega^{\nu>} - E^{\mu\nu} \right] + \frac{2\sigma_{\alpha\beta}}{(k\cdot u)^2}\left[(k\cdot u)\dot{k}^\beta - (u\cdot\dot{k})k^\beta \right] n^\alpha \right.$$
$$\left. + \frac{1}{3}\left[-\frac{1}{3}\Theta^2 - 2\sigma^2 + 2\omega^2 - \frac{1}{2}(\rho + 3p) + \Lambda \right] \right\}_o \delta\ell_o - \left[\sigma_{\alpha\beta}n^\alpha n^\beta - \frac{1}{3}\Theta \right]_o \frac{\mathrm{d}\delta\ell}{\mathrm{d}t}\bigg|_o$$

reduces to:

$$\frac{\delta z}{\delta t_o} = \frac{H_o^2}{2}(2\Omega_\Lambda - \Omega_{m,o})\delta\ell_o$$

Hence, the RD in the ΛCDM model is positive for *z << 1*.

This shows how our calculation of the RD can be useful to distinguish among cosmological models.

Conclusions

* We have obtained an expression for the redshift drift, valid for $z << 1$, for any observer comoving with the fluid, and for GR.

* The example analyzed showed that our result for the RD (which may be measured in the next decade) is a useful tool to differentiate cosmological models

* Application to anisotropic (Bianchi) models – in progress $\rightarrow$ directional effects.

* RD in less symmetrical models, such as Székeres.

* Limits on the position of off-center observers?