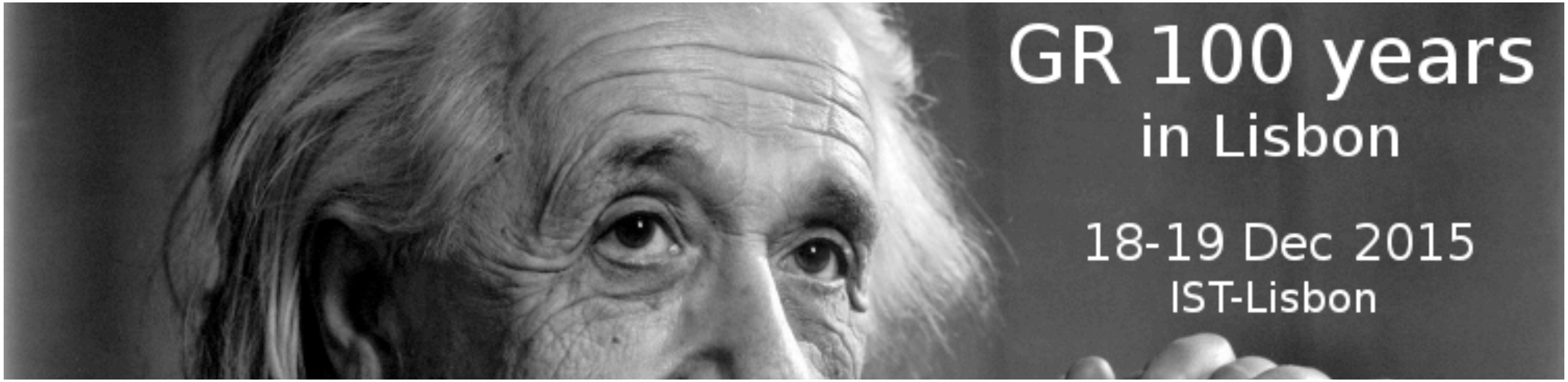


Extremal charged black holes: Equal absorbers and scatterers of EM and G radiation

Luís Carlos Bassalo Crispino
(crispino@ufpa.br)
Universidade Federal do Pará – UFPA
&
Atsushi Higuchi (University of York),
Ednilton Santos de Oliveira (Universidade Federal do Pará), and
Samuel Richard Dolan (University of Sheffield).

Summary

- **Introduction & Basic Concepts**

- **Absorption and Scattering by the Schwarzschild Black Hole**

- **Absorption and Scattering by the Reissner-Nordström Black Hole**

- **Final Remarks**

Introduction & Basic Concepts

Absorption and Scattering by Black Holes

(Differential) Scattering cross section

$$\frac{d\sigma_{sc}}{d\Omega} = \frac{\text{number of particles scattered per unit time in the solid angle } d\Omega}{\text{incident flux}}$$

Absorption cross section

$$\sigma_{abs} = \frac{\text{number of absorbed particles per unit time}}{\text{incident flux}}$$

Reissner-Nordström Black Hole

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

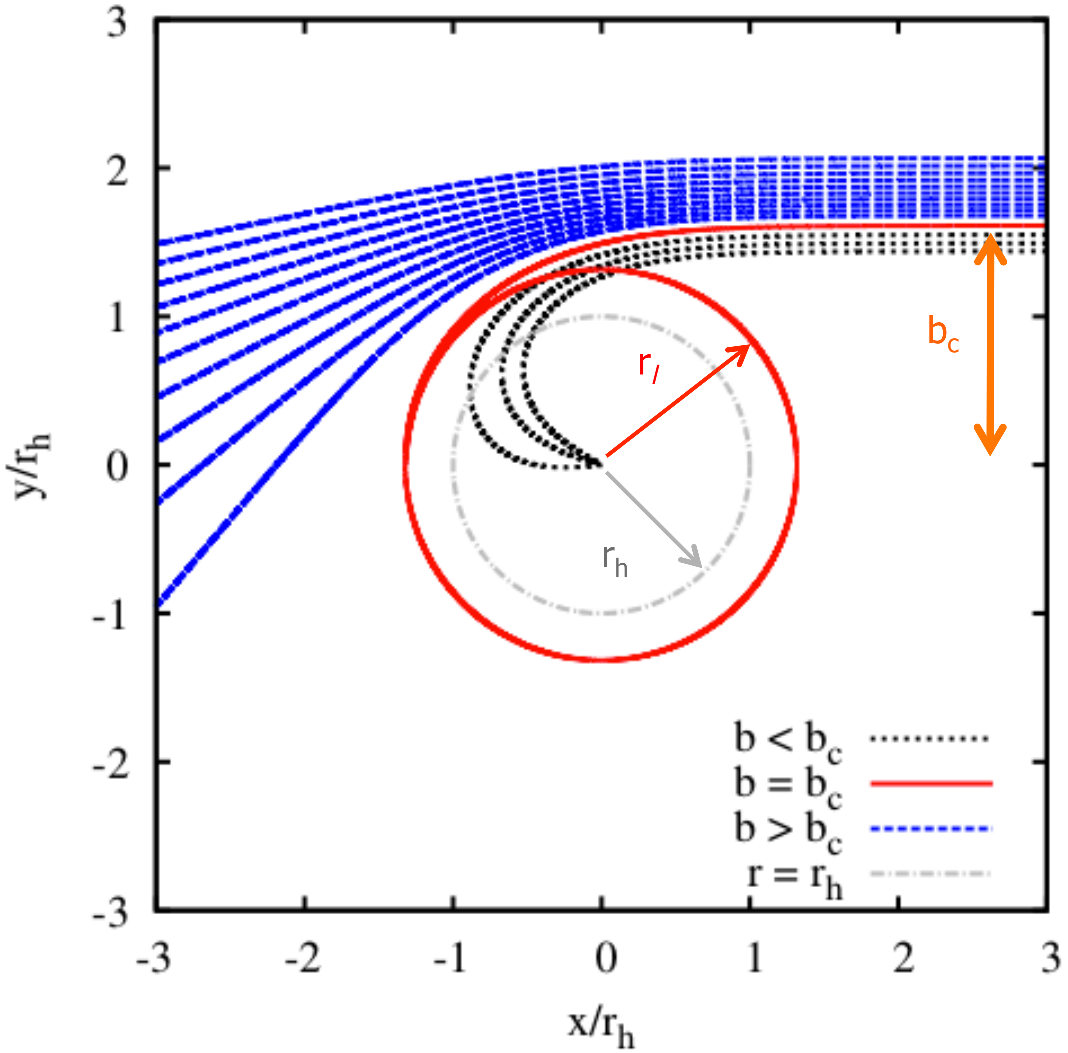
$$f = 1 - 2M/r + Q^2/r^2$$

- **Q = 0 ➔** Schwarzschild Black Hole;
- **0 < |Q| < M ➔** Typical Reissner-Nordström Black Hole;
- **|Q| = M ➔** Extreme Reissner-Nordström Black Hole.

Absorption by Schwarzschild black holes

Absorption Cross Section of Schwarzschild Black Holes [s=0]

- Geodesic (classical) absorption (high-frequency limit)



Absorption Cross Section of Schwarzschild Black Holes [s=0]

- Geodesic (classical) absorption (high-frequency limit)

$$\theta = \pi/2$$

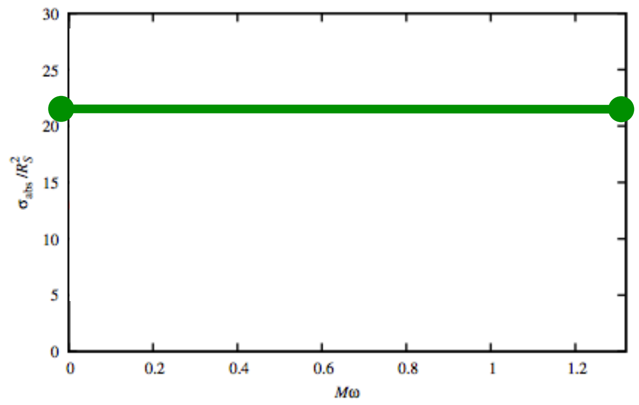
$$2L_{\text{geo}} = -f(r)\dot{t} + f(r)\dot{r} + r^2\dot{\varphi}^2 = 0,$$

$$\dot{r}^2 + L^2\frac{f(r)}{r^2} = E^2,$$

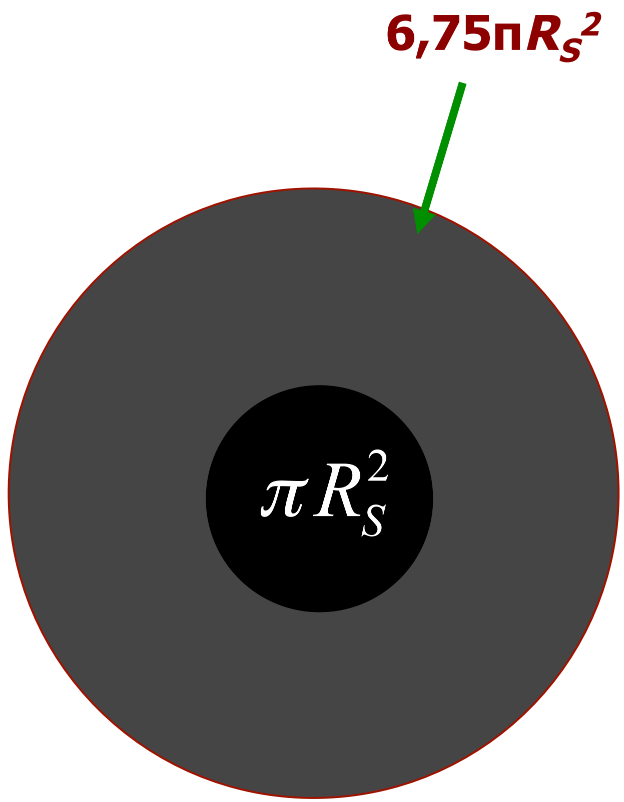
$$V_{\text{eff}} = L^2 f(r)/r^2.$$

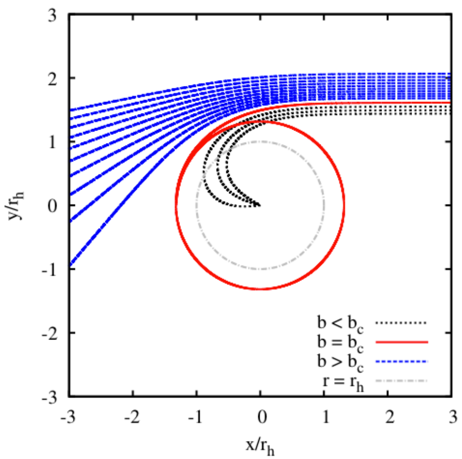
Absorption Cross Section of Schwarzschild Black Holes [s=0]

- Geodesic (classical) absorption
  (high-frequency limit)



$$\sigma_{\text{geo}} = \pi b_c^2 = \pi \frac{r_l^2}{f(r_l)}.$$





Scalar Absorption Cross Section of Schwarzschild Black Holes

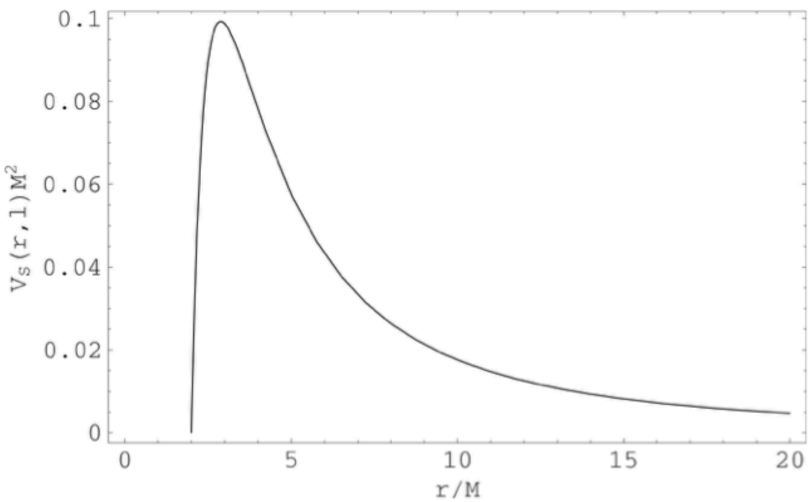
- Partial wave method

$$\frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}g^{ab}\partial_b\Phi\right) = 0.$$

$$\Phi_\omega = \sum_{lm}\frac{\phi(r)}{r}Y_l^m(\theta,\varphi)e^{-i\omega t},$$

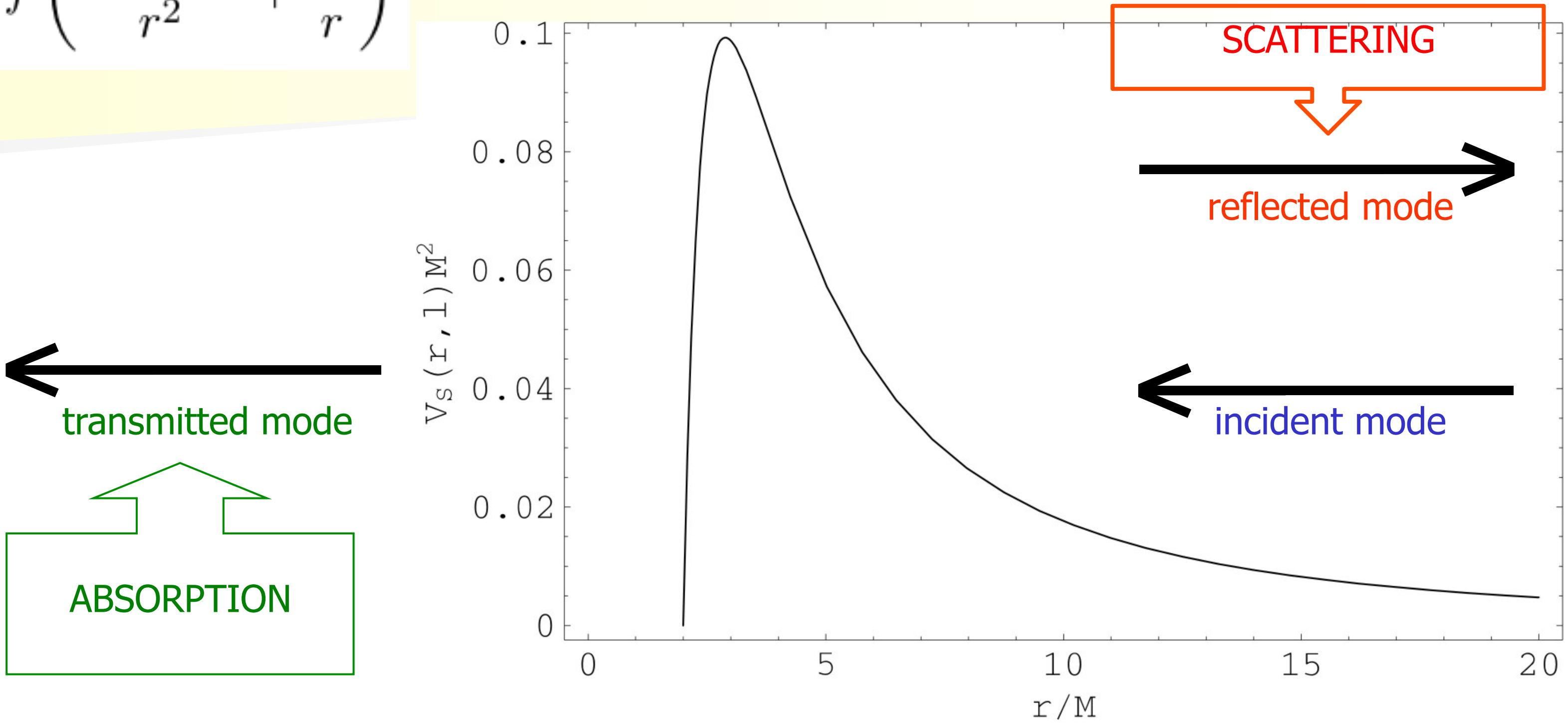
$$\left(-\frac{d}{dx^2} + V_\phi(r) - \omega^2\right)\phi(r) = 0,$$

$$V_\phi(r) = f\left(\frac{l(l+1)}{r^2} + \frac{f'}{r}\right)$$



Scalar Absorption Cross Section of Schwarzschild Black Holes

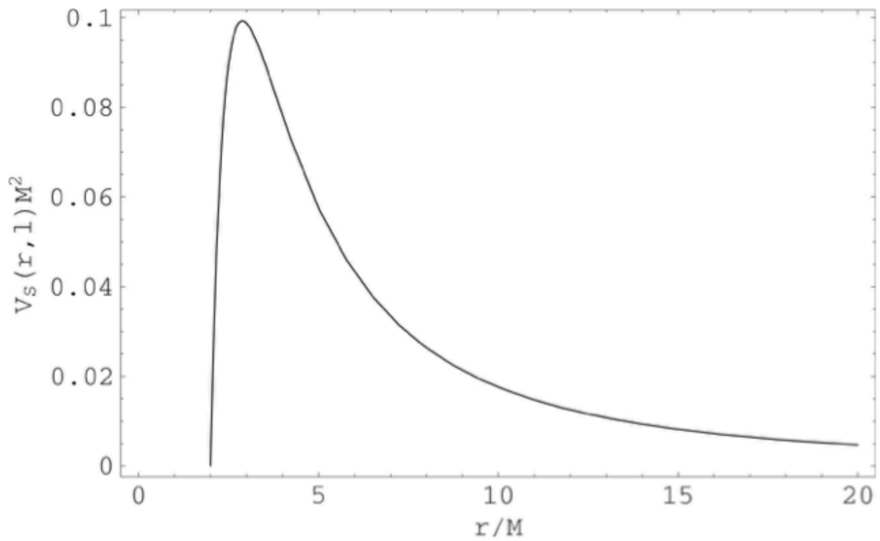
$$V = f\left(\frac{l(l+1)}{r^2} + \frac{f'}{r}\right)$$



$$\psi_{\omega l}(r) \approx \begin{cases} A_{\omega l}^{(tr)} e^{-i\omega x} & (x \to -\infty, r \approx r_h); \\ A_{\omega l}^{(in)} e^{-i\omega x} + A_{\omega l}^{(out)} e^{i\omega x} & (x \to +\infty, r \to \infty). \end{cases}$$

Absorption Cross Section of Schwarzschild Black Holes [s=0]

- Partial wave method



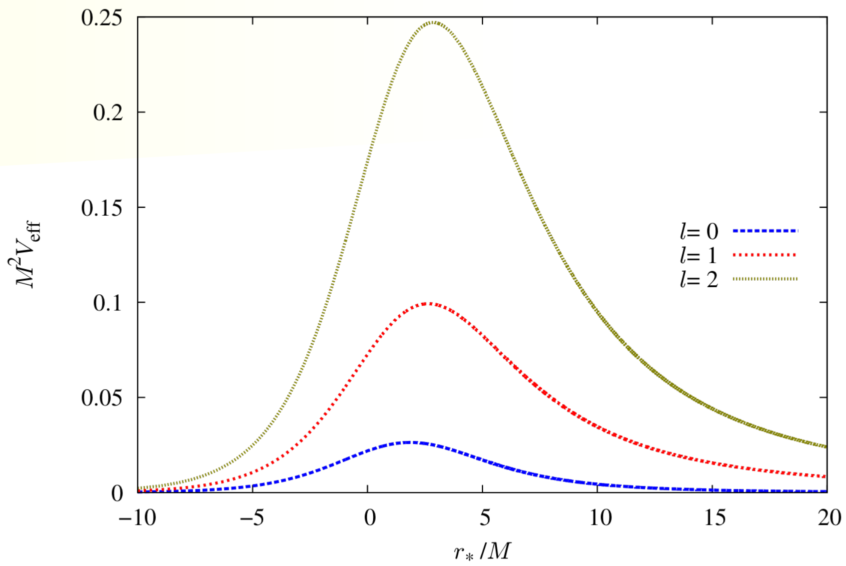
$$\phi^{in}(r) \sim \begin{cases} R_I + \mathcal{R}_{\omega l} R_I^* & x \to +\infty \ (r \to +\infty), \\ \mathcal{T}_{\omega l} R_{II} & x \to -\infty \ (r \to r_h), \end{cases}$$

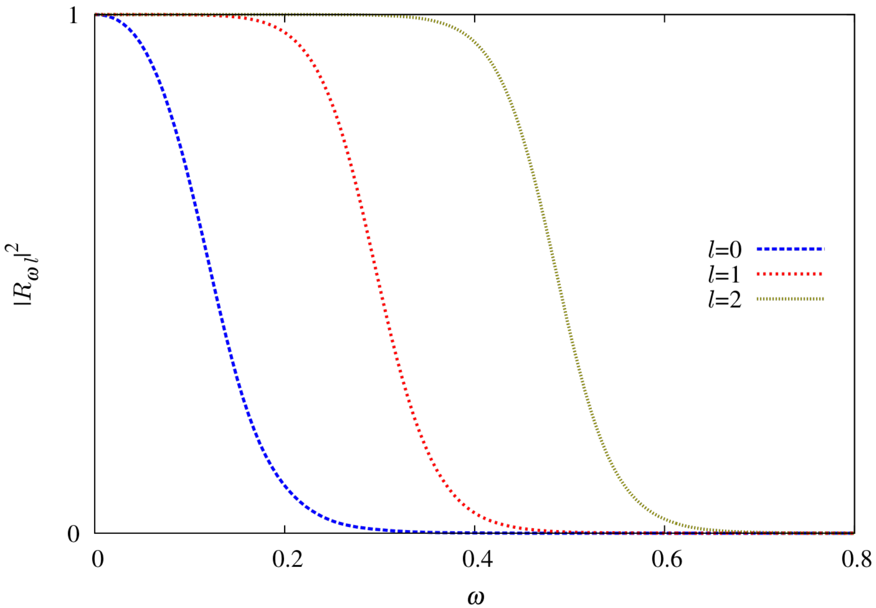
$$|\mathcal{R}_{\omega l}|^2 + |\mathcal{T}_{\omega l}|^2 = 1.$$

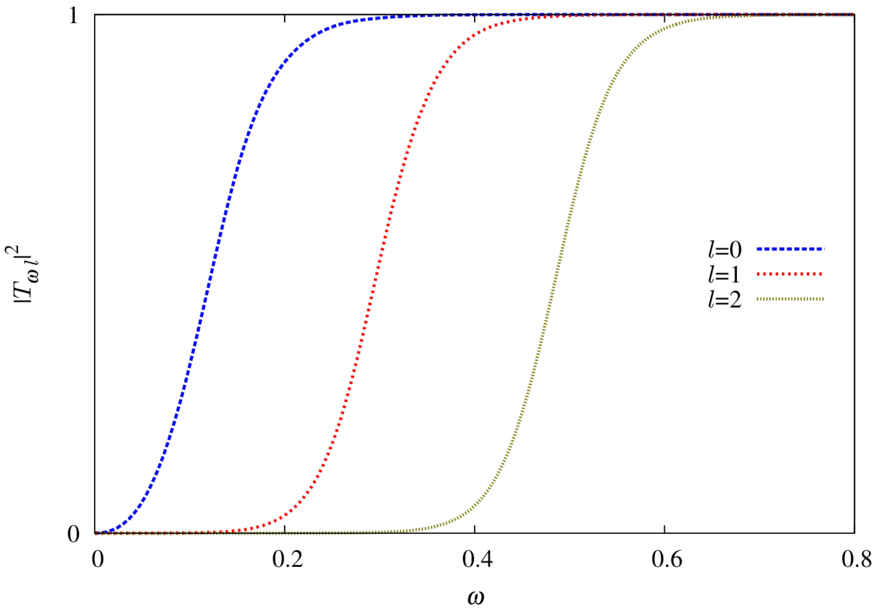
Scalar Absorption Cross Section of Schwarzschild Black Holes

$$\left(-\frac{d}{dx^2}+V_\phi(r)-\omega^2\right)\phi(r)=0,$$

$$|\mathcal{R}_{\omega l}|^2+|\mathcal{T}_{\omega l}|^2=1$$

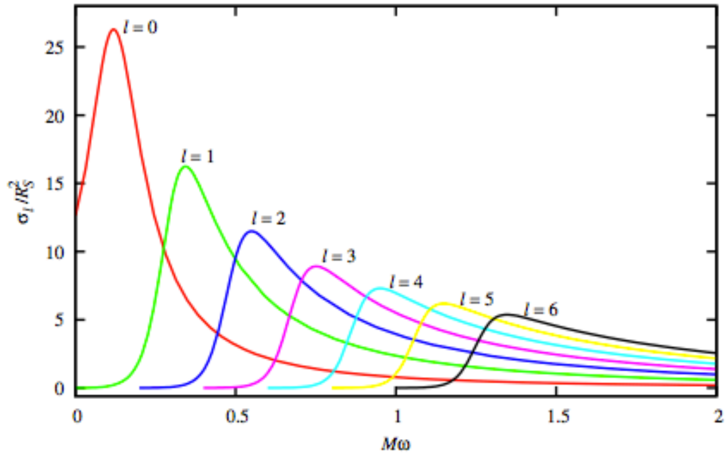




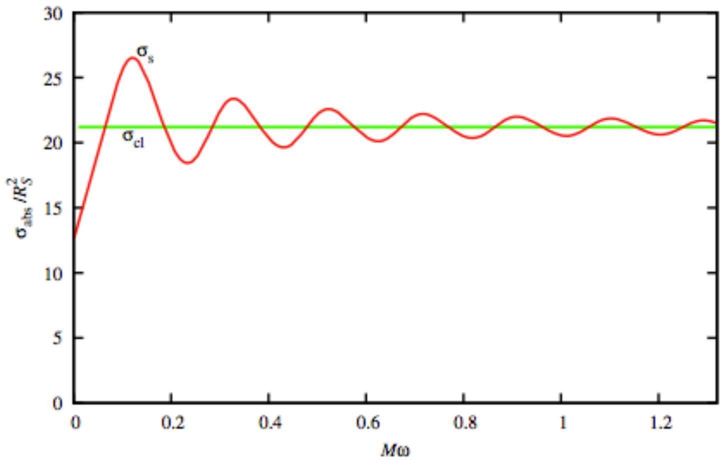


Absorption Cross Section of Schwarzschild Black Holes [s=0]

- Partial wave method

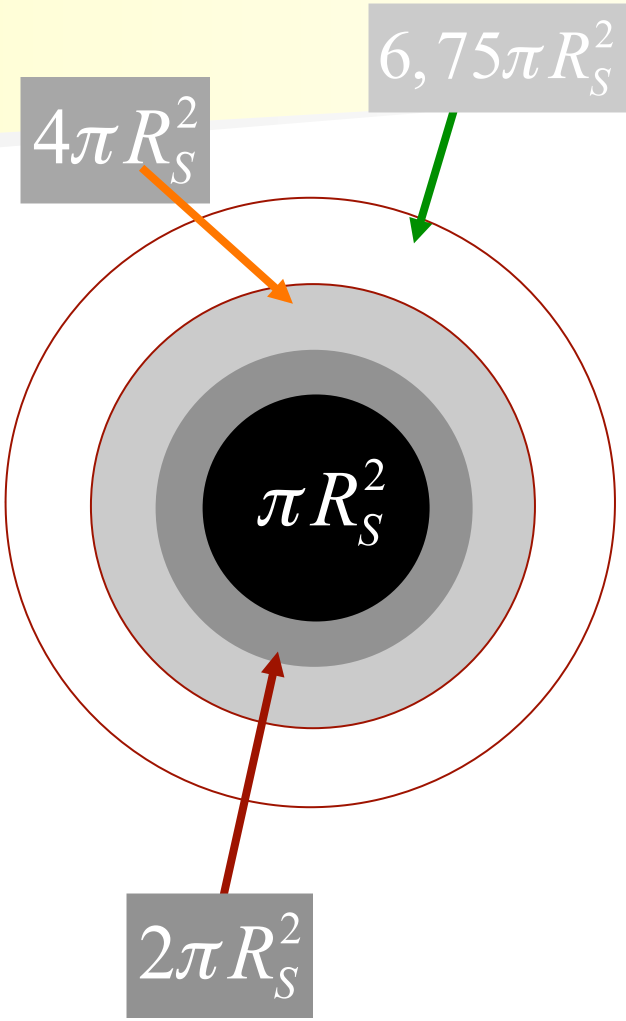


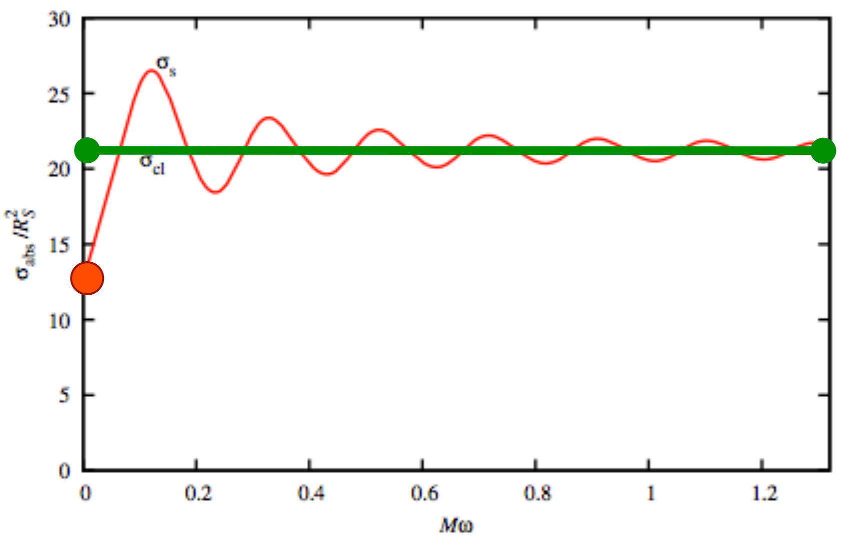
$$\sigma_{abs} = \sum_{l}^{\infty} \sigma_l,$$



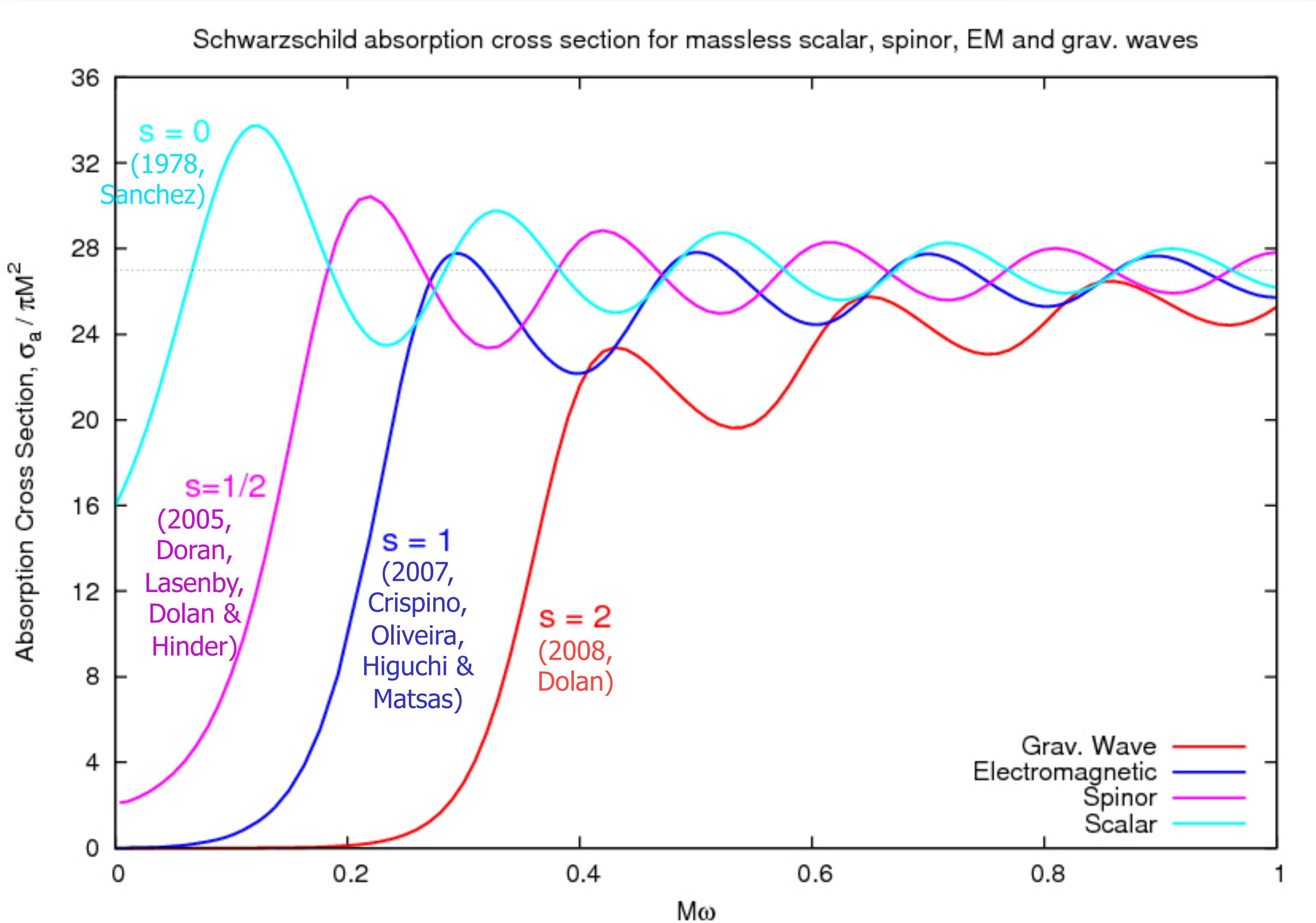
$$\sigma_l = \frac{\pi}{\omega^2}(2l+1)\left|\mathcal{T}_{\omega l}\right|^2 .$$

Scalar Absorption Cross Section of Schwarzschild Black Holes





Absorption Cross Section of Schwarzschild Black Holes



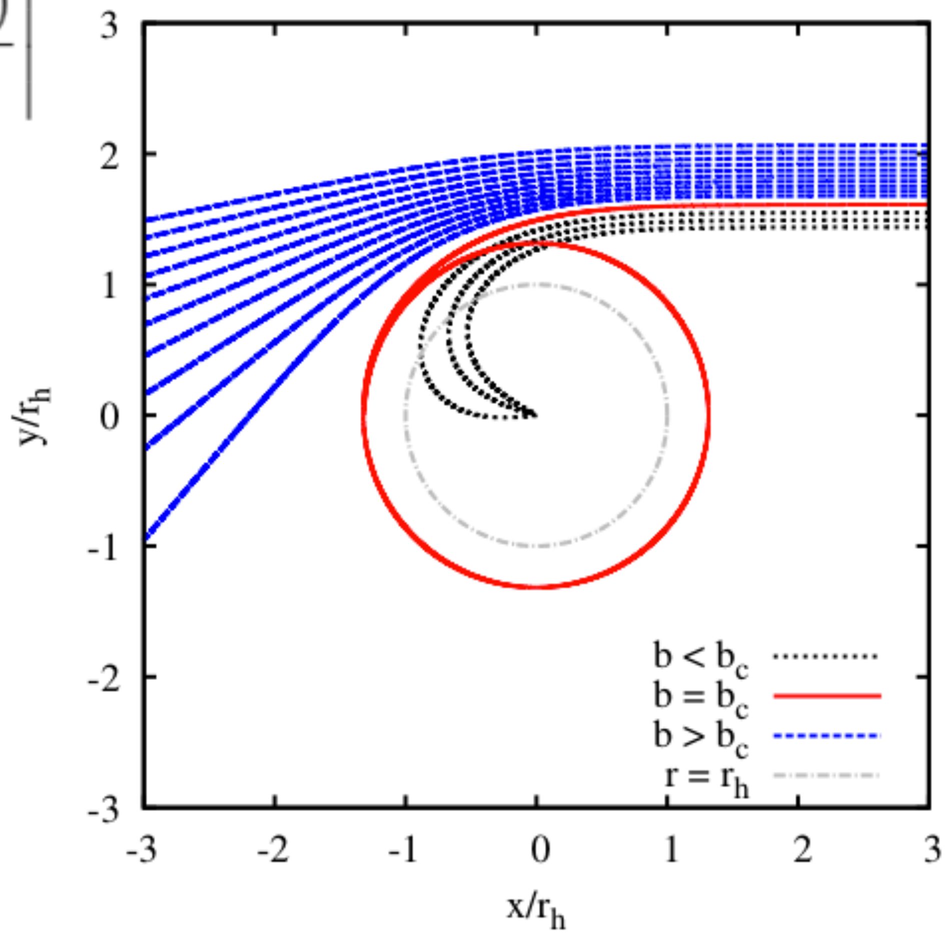
Schwarzschild black hole total absorption cross section for massless waves with spin 0, ½, 1 e 2.
[Courtesy: Samuel Richard Dolan, 2008.]

Scattering by Schwarzschild black holes

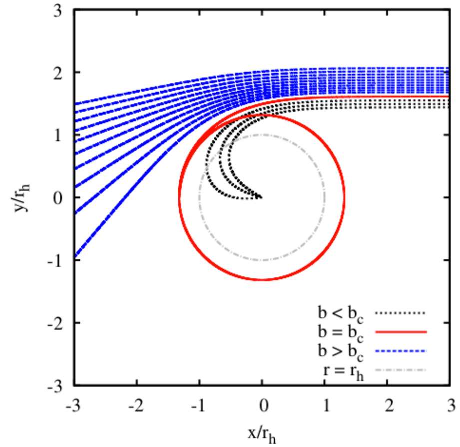
Scattering Cross Section of Schwarzschild Black Holes
Massless Scalar Field

- Geodesic (classical) scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{\sin\Theta}\sum b(\Theta)\left|\frac{db(\Theta)}{d\Theta}\right|$$



Scattering Cross Section of Schwarzschild Black Holes
Massless Scalar Field



- Geodesic (classical) scattering

$$\theta = \pi/2$$

$$\left(\frac{du}{d\varphi}\right)^2 = \frac{1}{b^2} - f(1/u)u^2,$$

$$E = -f\dot{t} \qquad\qquad L = r^2\dot{\varphi}$$

$$\frac{d^2u}{d\varphi^2} = -\frac{u^2}{2}\frac{df(1/u)}{du} - uf(1/u)$$

Scattering Cross Section of Schwarzschild Black Holes
Massless Scalar Field

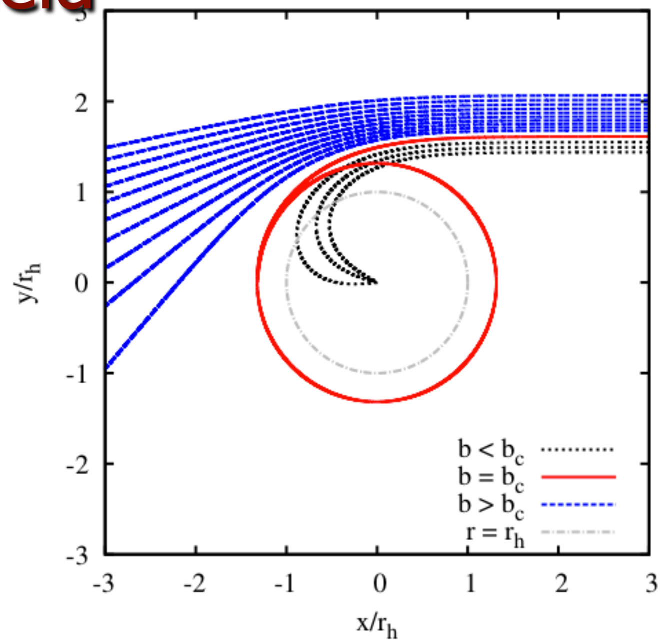
- Geodesic (classical) scattering

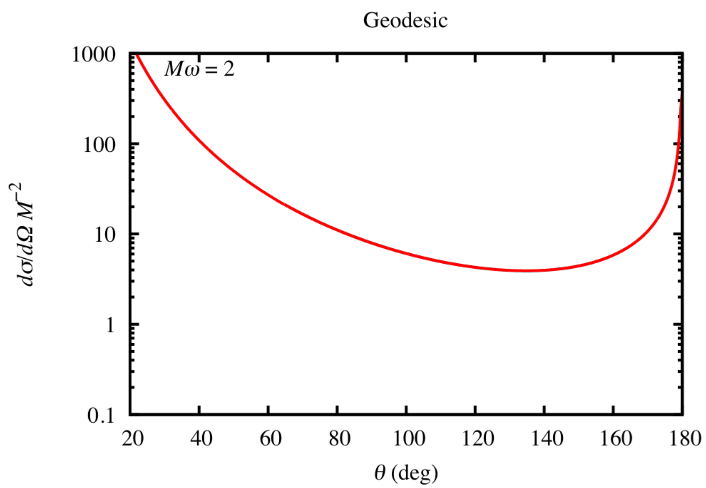
$$\left(\frac{du}{d\varphi}\right)^2 = \frac{1}{b^2} - f(1/u)u^2,$$

$$\alpha = \int_0^{u_0} \left[\frac{1}{b^2} - f(1/u)u^2\right]^{-1/2} du$$

$$\Theta(b) = 2\alpha(b) - \pi$$

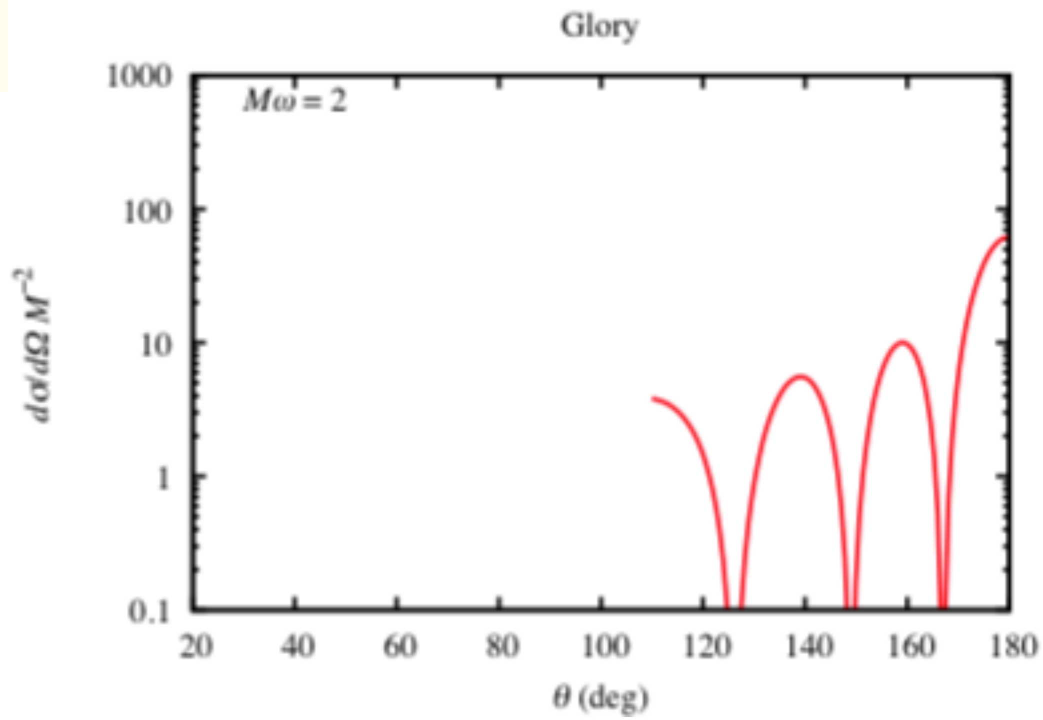
$$\frac{d\sigma}{d\Omega} = \frac{1}{\sin\Theta} \sum b(\Theta) \left|\frac{db(\Theta)}{d\Theta}\right|,$$





Scattering Cross Section of Schwarzschild Black Holes
Massless Scalar Field

- Glory approximation



$$\frac{d\sigma_{\rm sc}}{d\Omega} = 2\pi\omega b_g^2 \left|\frac{db}{d\theta}\right|_{\theta=\pi} J_{2s}^2(\omega b_g \sin\theta)$$

Scattering Cross Section of Schwarzschild Black Holes
Massless Scalar Field

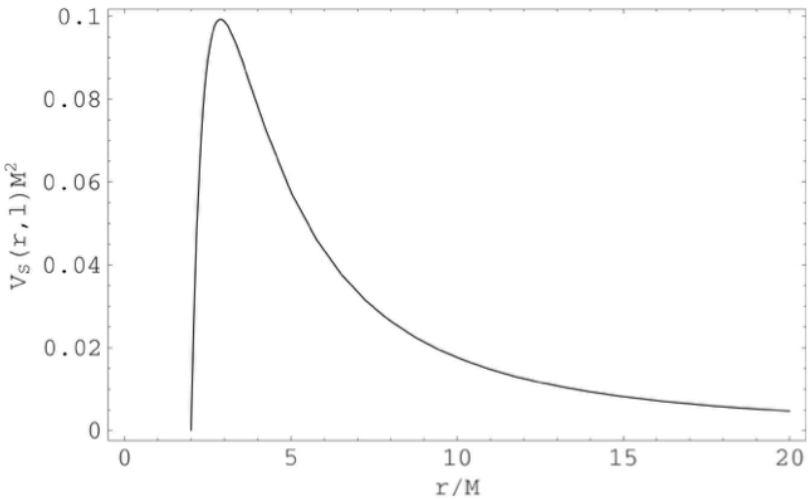
- Partial wave method

$$\frac{1}{\sqrt{-g}}\partial_a\left(\sqrt{-g}g^{ab}\partial_b\Phi\right)=0.$$

$$\Phi_\omega=\sum_{lm}\frac{\phi(r)}{r}Y_l^m(\theta,\varphi)e^{-i\omega t},$$

$$\left(-\frac{d}{dx^2}+V_\phi(r)-\omega^2\right)\phi(r)=0,$$

$$V_\phi(r)=f\left(\frac{l(l+1)}{r^2}+\frac{f'}{r}\right)$$



Scattering Cross Section of Schwarzschild Black Holes Massless Scalar Field

- Partial wave method

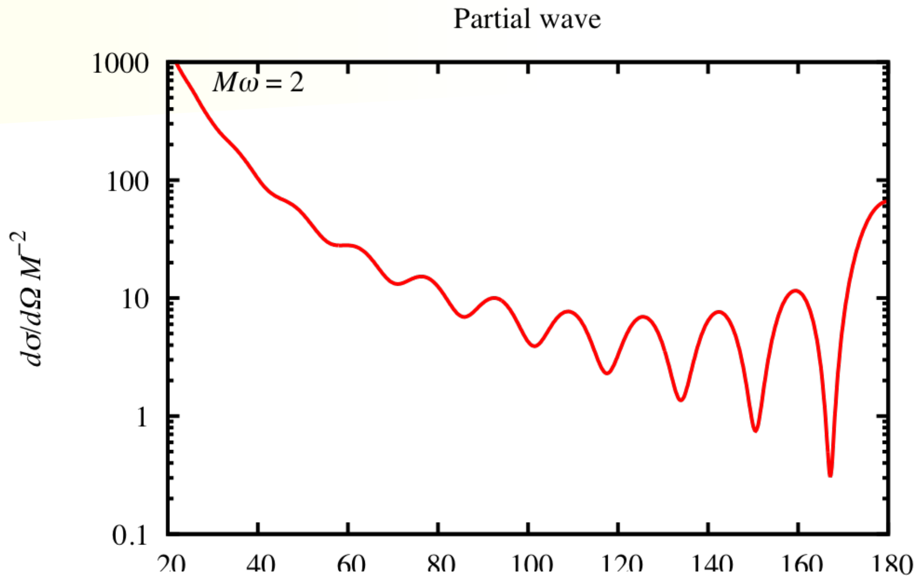
$$\phi^{in}(r) \sim \begin{cases} R_I + \mathcal{R}_{\omega l} R_I^* & x \to +\infty \ (r \to +\infty), \\ \mathcal{T}_{\omega l} R_{II} & x \to -\infty \ (r \to r_h), \end{cases}$$

$$R_I = e^{-i\omega x} \sum_{i=1}^{N} \frac{A_\infty^j}{r^j},$$

$$R_{II} = e^{-i\omega x} \sum_{i=1}^{N} (r - r_h)^j A_{r_h}^j$$

Scattering Cross Section of Schwarzschild Black Holes Massless Scalar Field

- Partial wave method

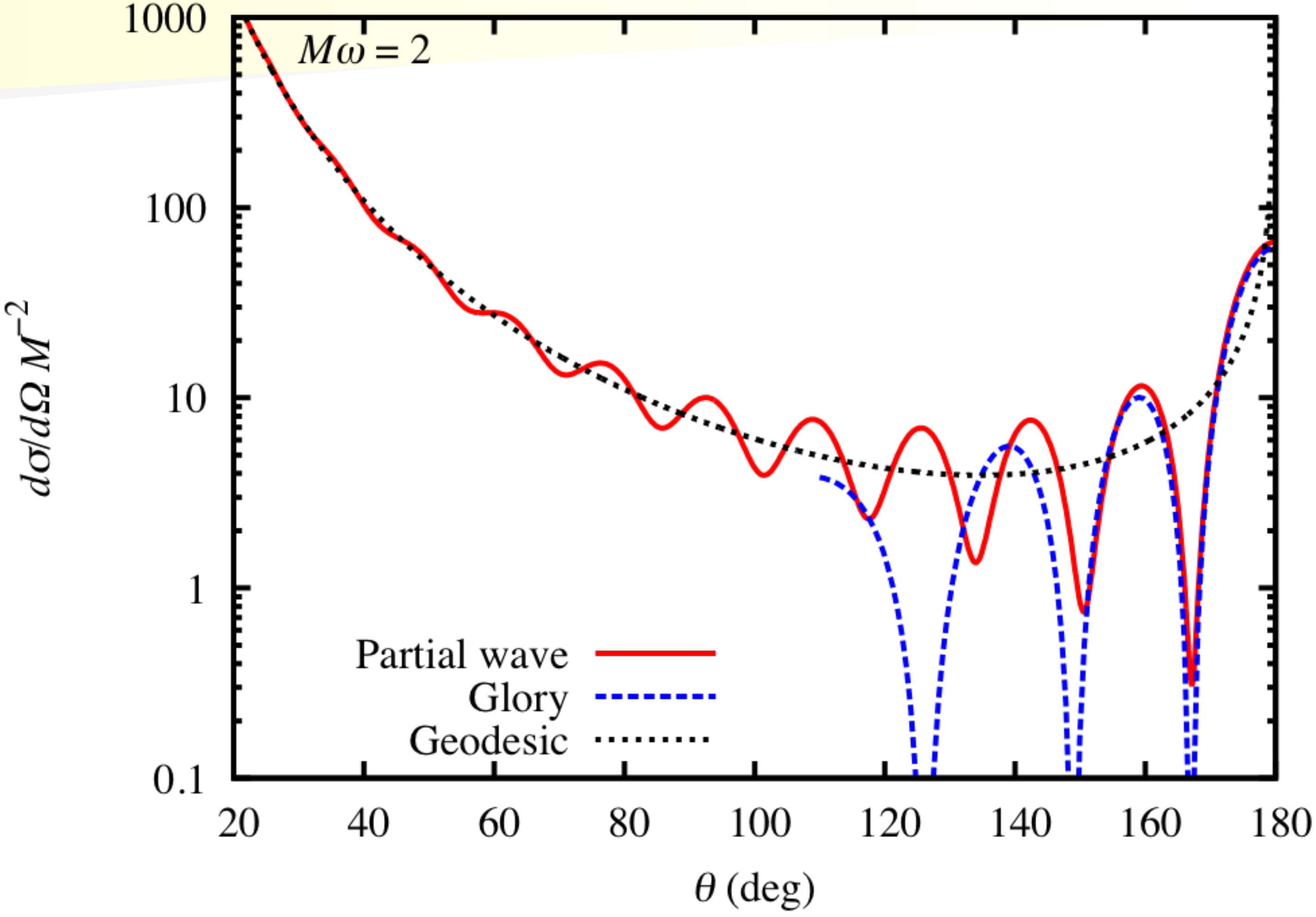


$$\frac{d\sigma}{d\Omega} = |g(\theta)|^2,$$

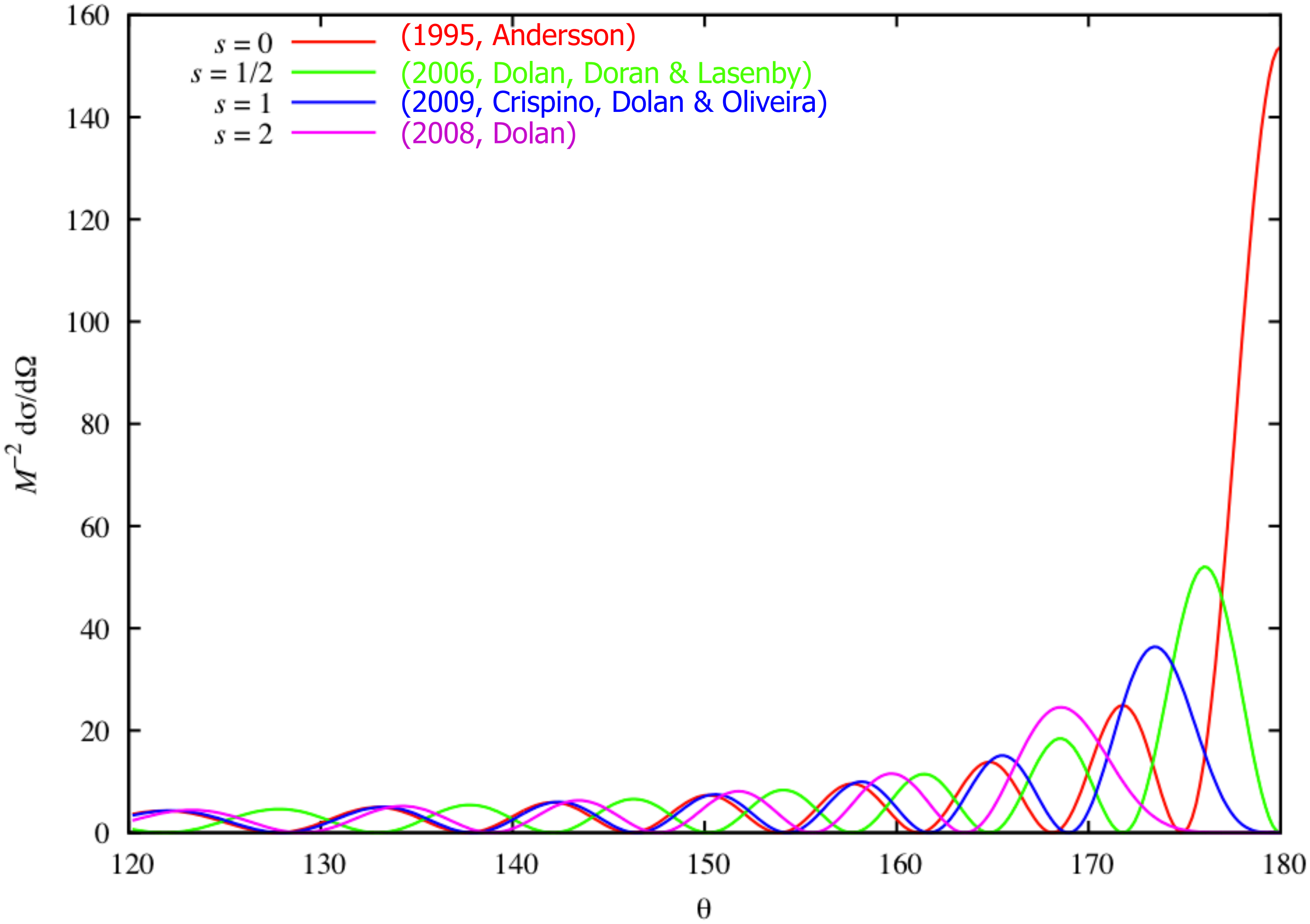
$$g(\theta) = \frac{1}{2i\omega} \sum_{l=0}^{\infty} (2l+1) \left[e^{2i\delta_l(\omega)} - 1 \right] P_l(\cos\theta),$$

$$e^{2i\delta_l}(\omega) \equiv (-1)^{l+1} \mathcal{R}_{\omega l}.$$

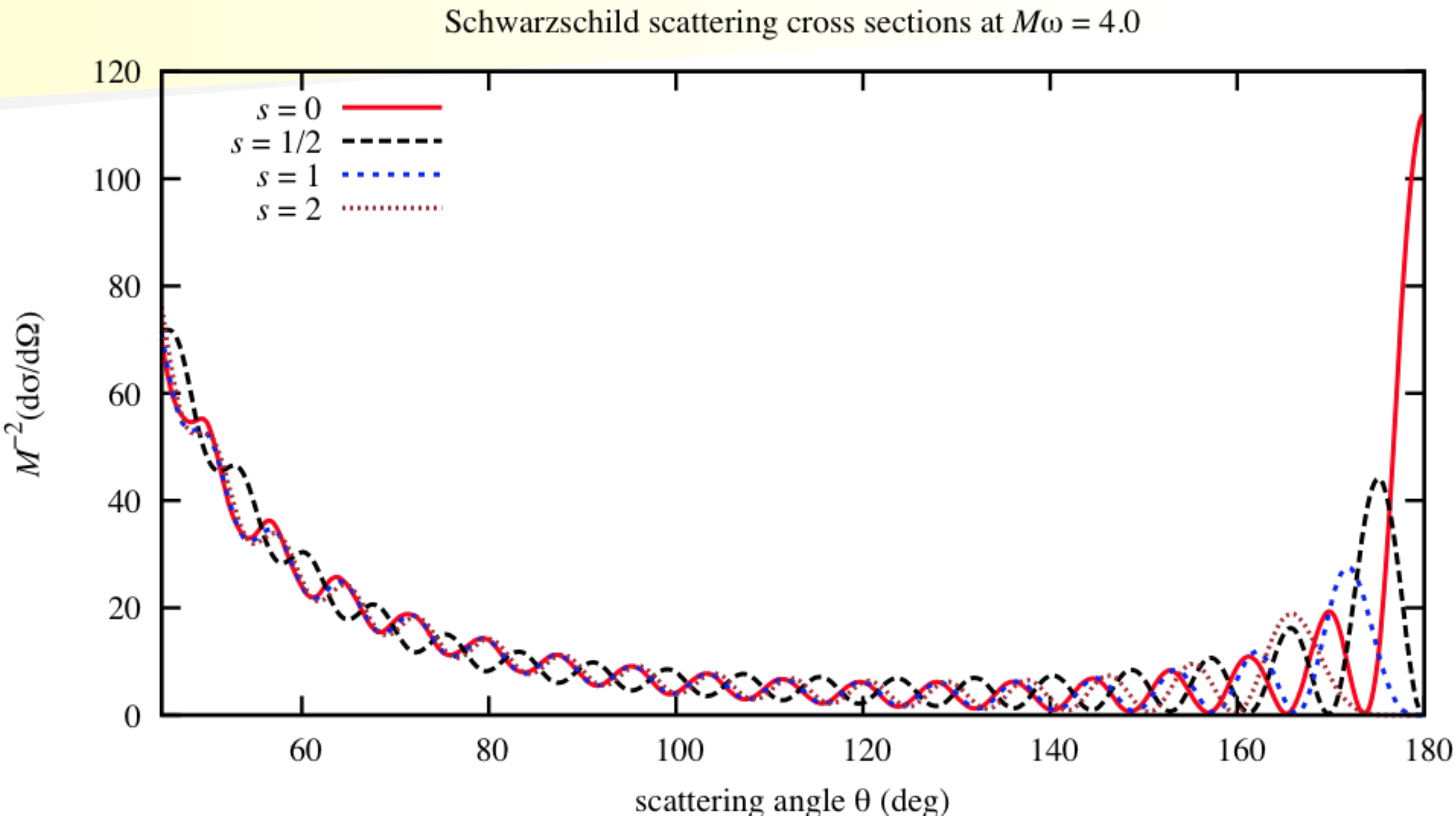
Scattering Cross Section of Schwarzschild Black Holes
Massless Scalar Field



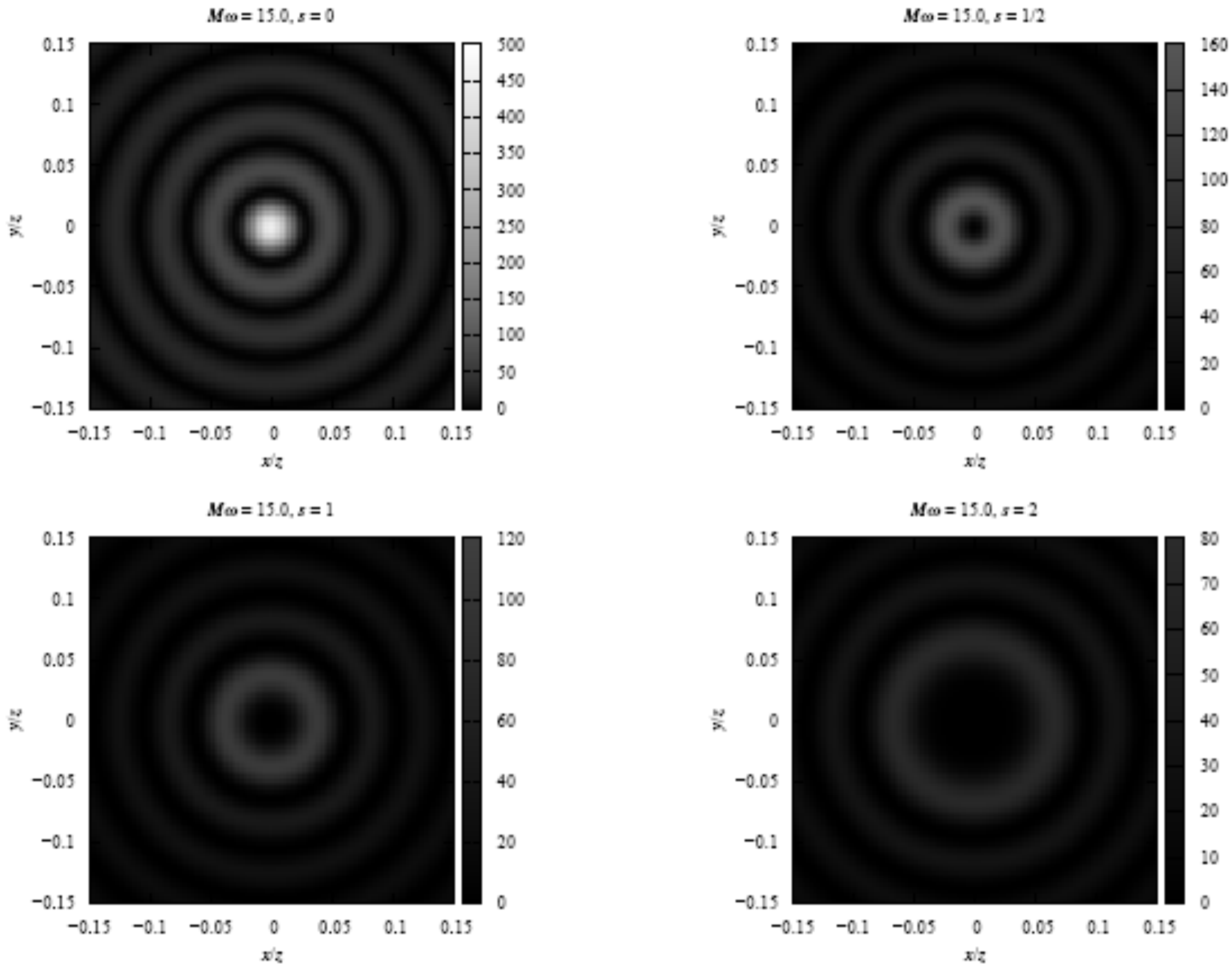
Scattering Cross Section of Schwarzschild Black Holes



Scattering Cross Section of Schwarzschild Black Holes

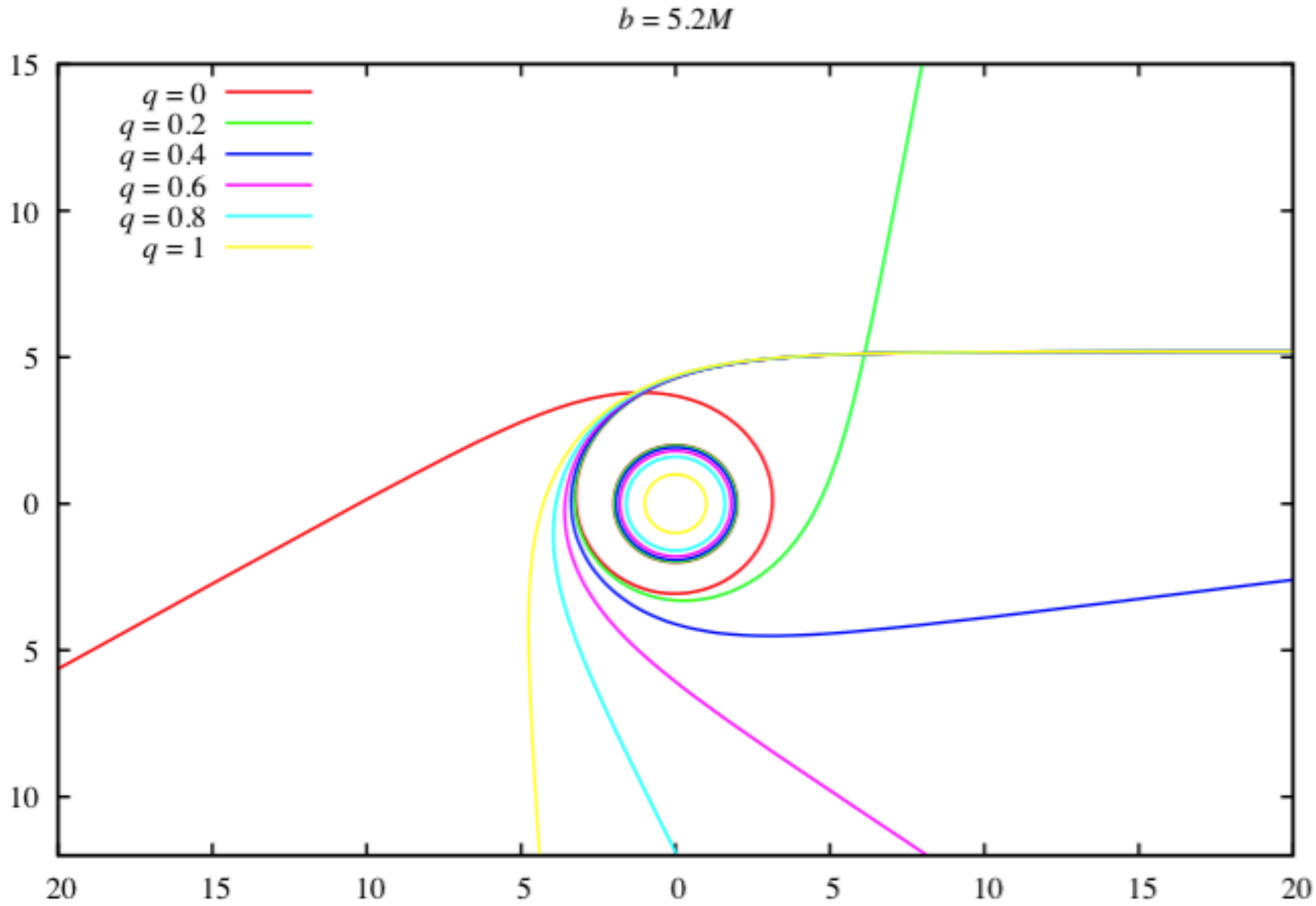


Scattering of Waves by Schwarzschild Black Holes
The Glory Effect



Electromagnetic absorption by Reissner-Nordström black holes

Absorption Cross Section of Charged Black Holes



Absorption Cross Section of Charged Black Holes

Wave Equations

Klein-Gordon

$$\Box\Phi = \nabla_\mu \nabla^\mu \Phi = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi \right) = 0.$$

Einstein-Maxwell

$$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Absorption Cross Section of Charged Black Holes

Partial Wave Method

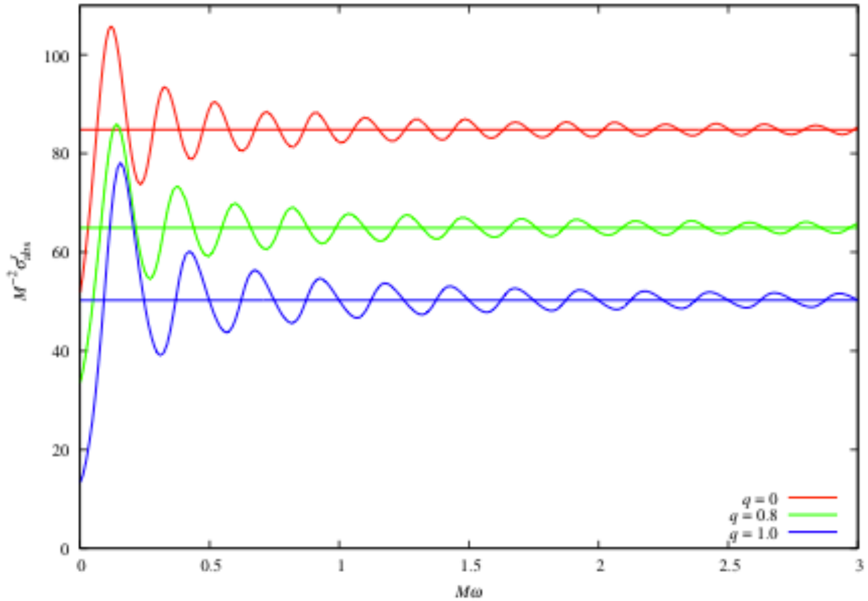
$$\sigma_{\text{abs}} \equiv -\frac{\text{integrated net (wave) flux at infinity}}{\text{incident (wave) current density}}$$

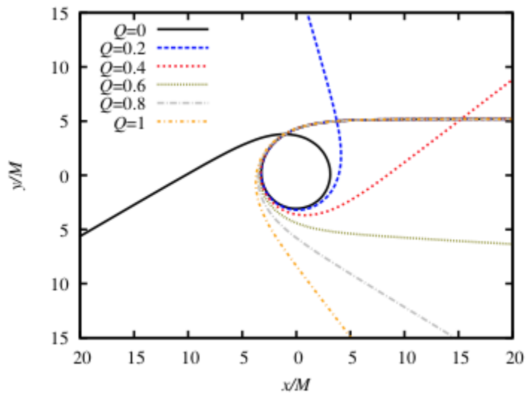
$$\sigma_{\text{abs}} = \sum_{l=s}^{\infty} \sigma_{\text{abs}}^{(l)} = \sum_{l=s}^{\infty} \sum_{\lambda} \frac{\pi}{N\omega^2}(2l+1)|T_{\omega l}^{\lambda}|^2$$

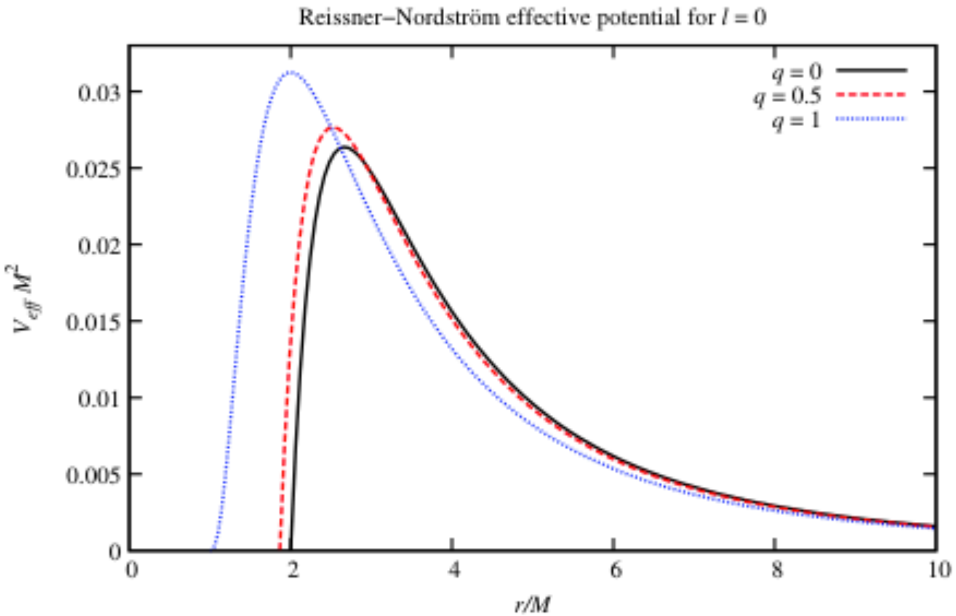
Absorption Cross Section of Charged Black Holes [s=0 case]

Absorption Cross Section

Scalar Case



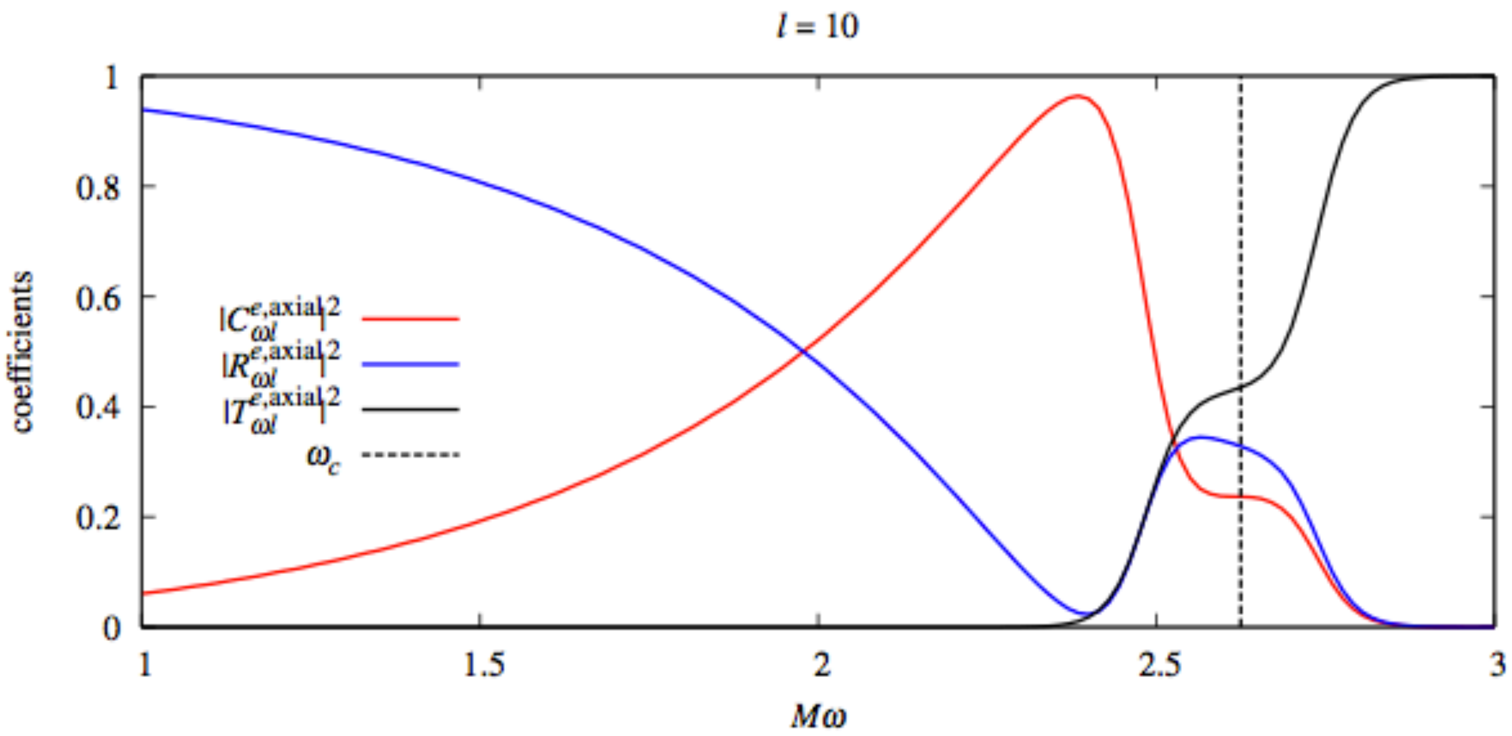




Absorption Cross Section of Charged Black Holes [s=1,2 cases]

Reflection, Transmission and Conversion Coefficients

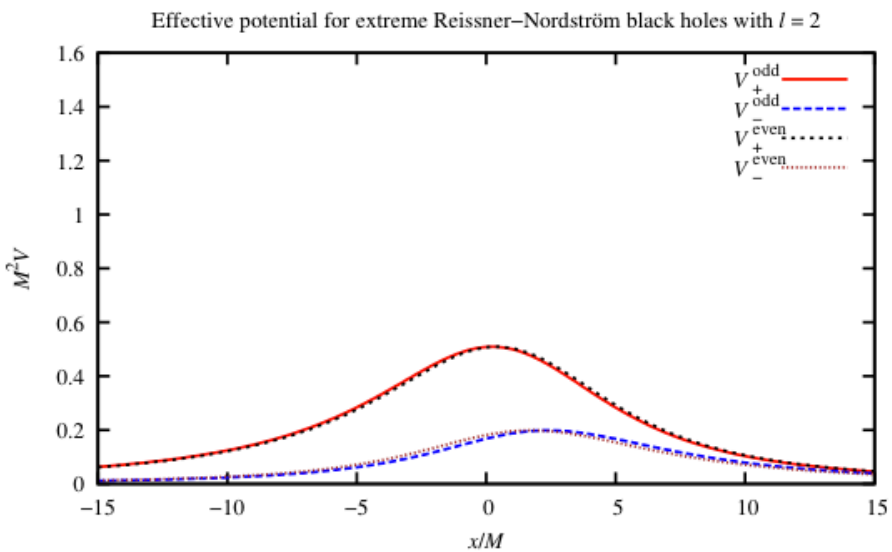
$$|R^{e,\lambda}_{\omega l}|^2 + |T^{e,\lambda}_{\omega l}|^2 + |C^{e,\lambda}_{\omega l}|^2 = 1.$$

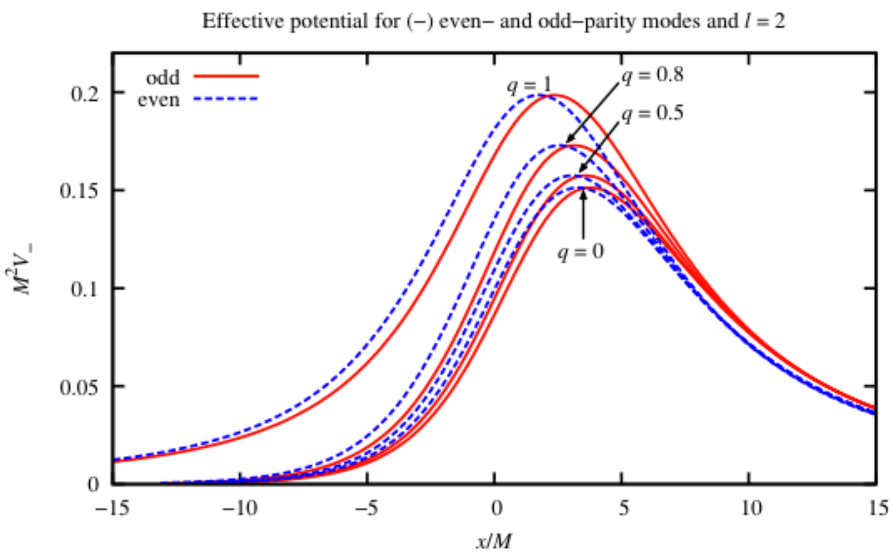


Absorption Cross Section of Charged Black Holes [s=1,2 cases]

Decoupled Equations

$$\frac{d^2}{dr_*^2}\varphi_\pm^\lambda + \left(\omega^2 - V_\pm^\lambda\right)\varphi_\pm^\lambda = 0,$$





Absorption Cross Section of Charged Black Holes [s=1,2 cases]

Asymptotic Conditions

$$\varphi_{\pm}^{\lambda} \propto \begin{cases} e^{-i\omega r_*} + A_{\pm}^{\lambda} e^{i\omega r_*}, & (r_* \to +\infty); \\ B_{\pm}^{\lambda} e^{-i\omega r_*}, & (r_* \to -\infty). \end{cases}$$

Purely EM incident wave:

$$\begin{aligned} F^{\lambda} &\approx \overbrace{F_{\text{in}}^{\lambda} e^{-i\omega r_*}} + F_{\text{out}}^{\lambda} e^{i\omega r_*}; \\ G^{\lambda} &\approx G_{\text{out}} e^{i\omega r_*}. \end{aligned}$$

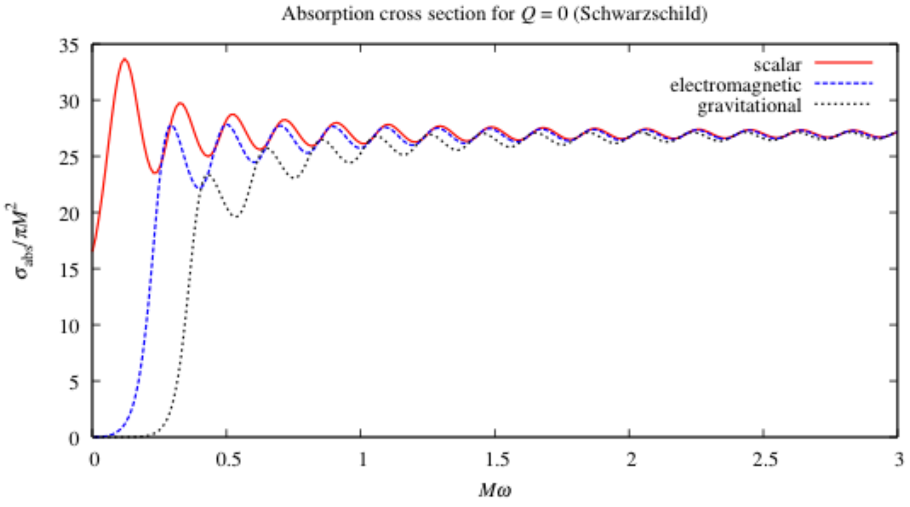
Purely G incident wave:

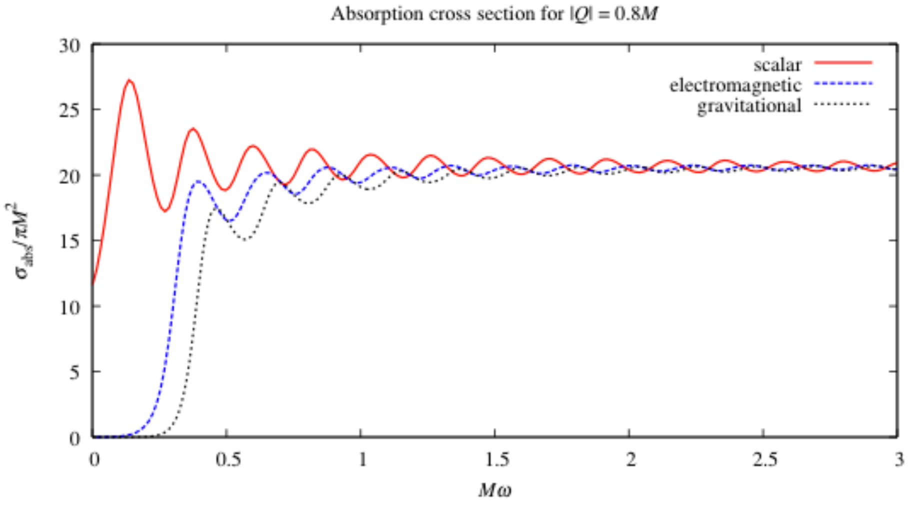
$$\begin{aligned} F^{\lambda} &\approx F_{\text{out}}^{\lambda} e^{i\omega r_*}; \\ G^{\lambda} &\approx \underbrace{G_{\text{in}} e^{-i\omega r_*}} + G_{\text{out}} e^{i\omega r_*}. \end{aligned}$$

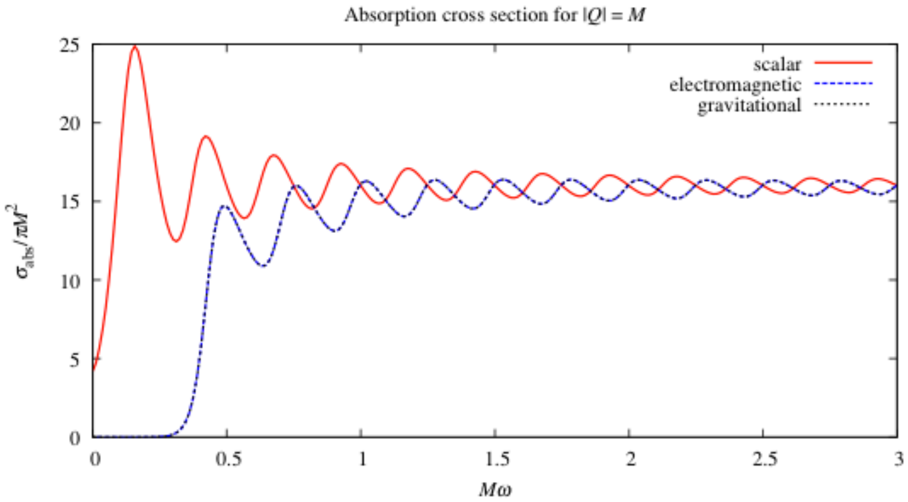
Absorption Cross Section of Charged Black Holes [s=1,2 cases]

Absorption Cross Section

Different Charges



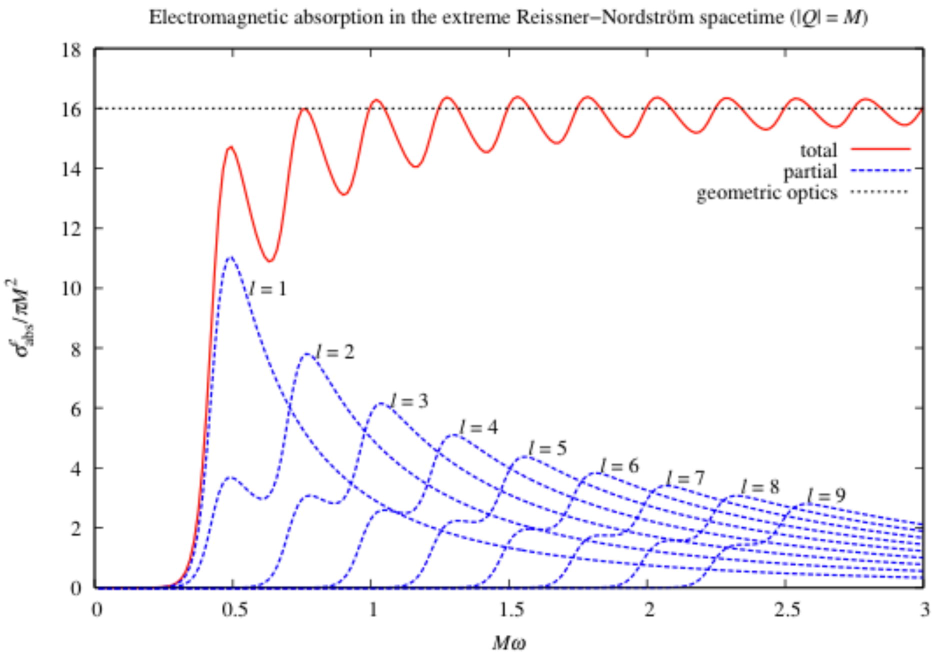


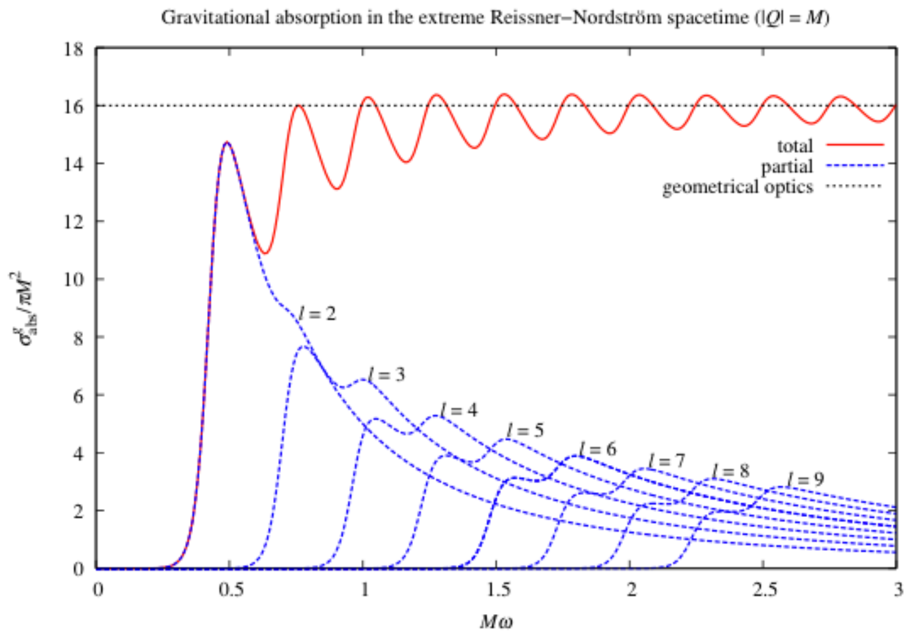


Absorption Cross Section of Charged Black Holes [s=1,2 cases]

Absorption Cross Section

Extreme Case

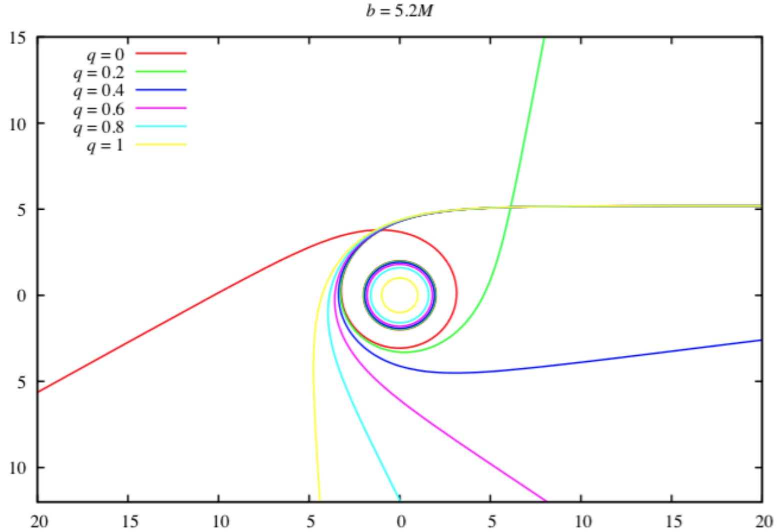


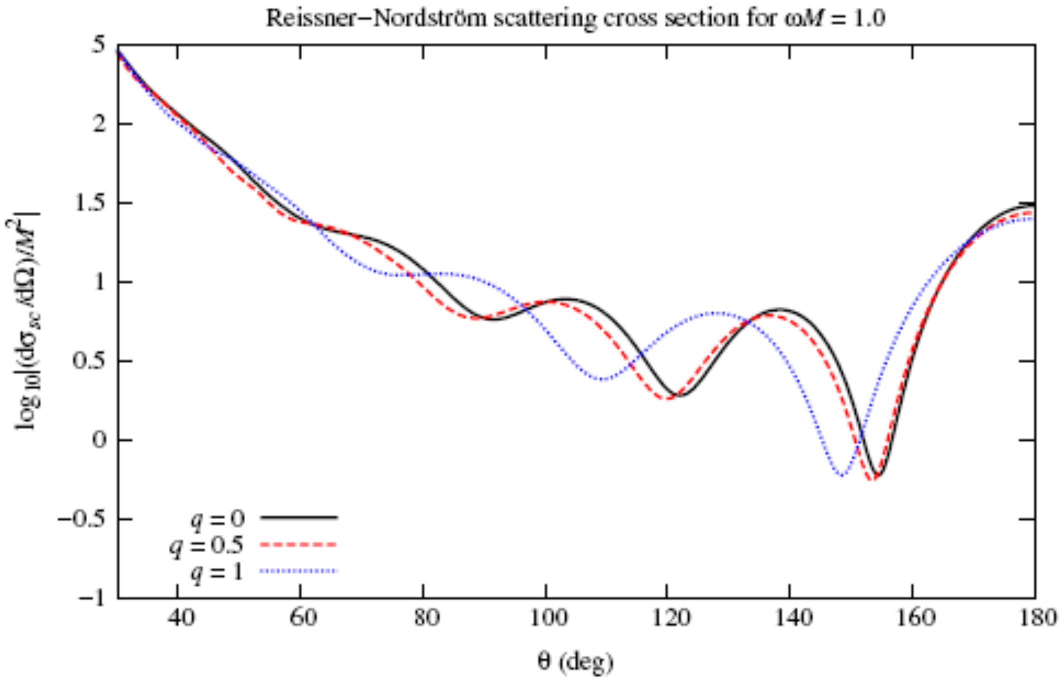


Scattering by
Reissner-Nordström black holes

Scattering Cross Section of Charged Black Holes

Massless Scalar Field





Scattering Cross Section of Charged Black Holes
Electromagnetic Field

$$\frac{d\sigma}{d\Omega} = \frac{1}{8\omega^2}\left\{\left|\sum_{l=1}^{\infty}\frac{2l+1}{l(l+1)}\left[e^{2i\delta_l^-(\omega)}T_l(\theta)+e^{2i\delta_l^+(\omega)}\pi_l(\theta)\right]\right|^2+\right.$$

$$\left.\left|\sum_{l=1}^{\infty}\frac{2l+1}{l(l+1)}\left[e^{2i\delta_l^-(\omega)}\pi_l(\theta)+e^{2i\delta_l^+(\omega)}T_l(\theta)\right]\right|^2\right\},$$

$$e^{2i\delta_l^{\mathcal{P}}(\omega)} = (-1)^{l+1}R_{\omega l}^{\mathcal{P}},$$

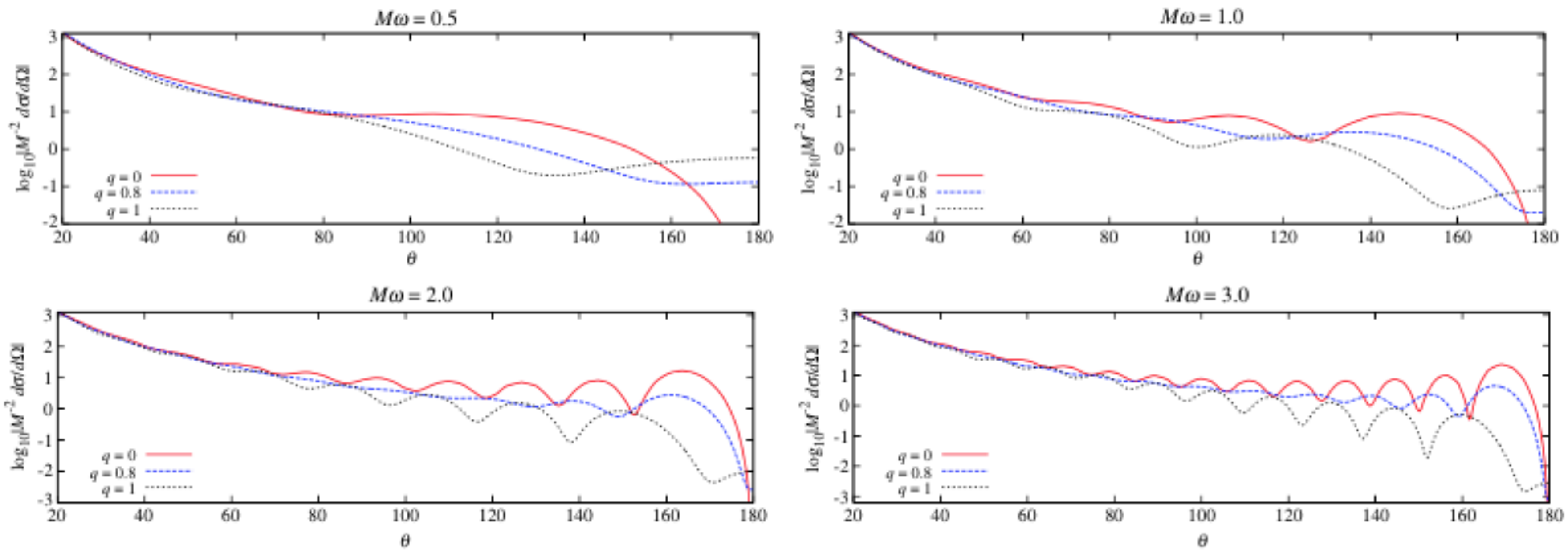
$$\pi_l(\theta) \equiv \frac{P_l^1(\cos\theta)}{\sin\theta}, \qquad T_l(\theta) \equiv \frac{d}{d\theta}P_l^1(\cos\theta)$$

Scattering Cross Section of Charged Black Holes [s=1,2]
Electromagnetic Field

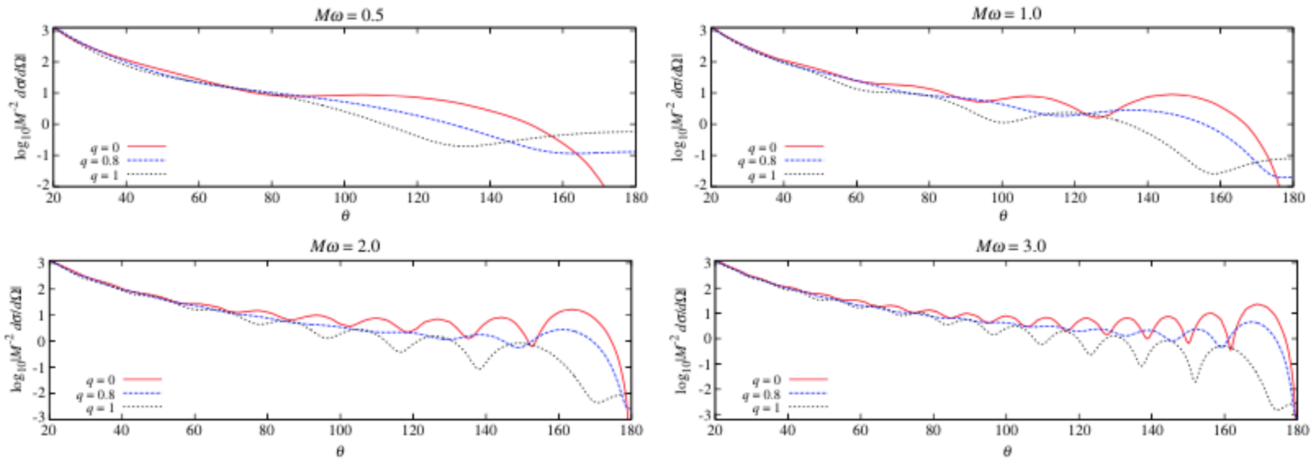
$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(|\mathcal{F}+\mathcal{G}|^2 + |\mathcal{F}-\mathcal{G}|^2\right) = |\mathcal{F}|^2 + |\mathcal{G}|^2,$$

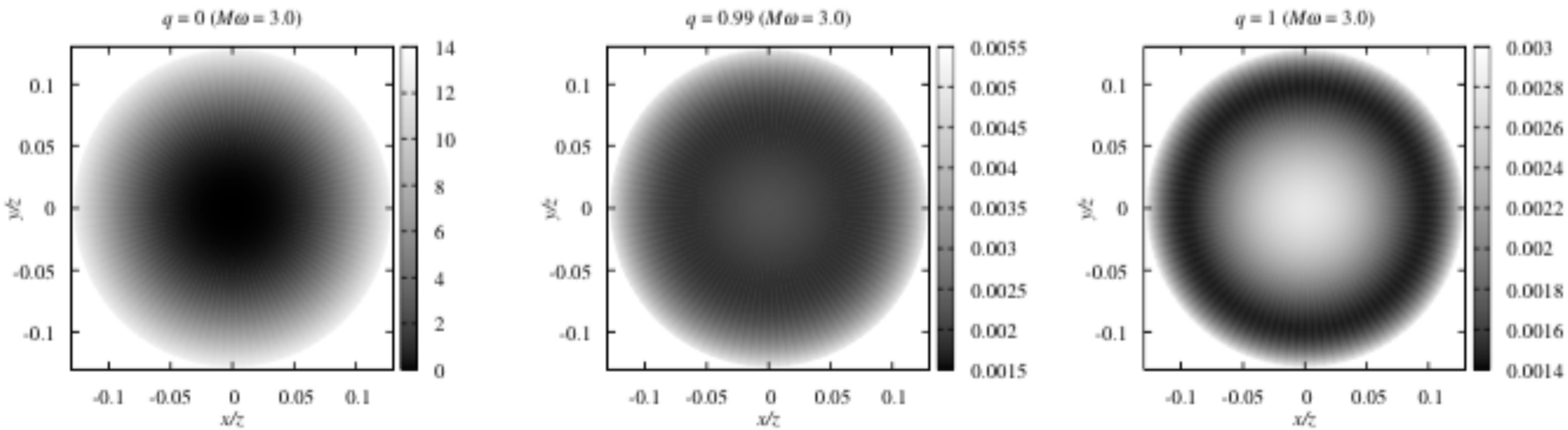
$$\begin{aligned}
\mathcal{F}(\theta) &= \frac{\pi}{i\omega}\sum_{l=1}^{\infty}\sum_{\mathcal{P}=\pm}\left[\exp\left(2i\delta_l^{\mathcal{P}}\right)-1\right] {}_{-1}Y_l^1(1)\,{}_{-1}Y_l^1(\cos\theta), \\
\mathcal{G}(\theta) &= \frac{\pi}{i\omega}\sum_{l=1}^{\infty}\sum_{\mathcal{P}=\pm}\left[\exp\left(2i\delta_l^{\mathcal{P}}\right)-1\right] \mathcal{P}(-1)^l\,{}_{-1}Y_l^1(1)\,{}_{-1}Y_l^1(-\cos\theta).
\end{aligned}$$

Scattering Cross Section of Charged Black Holes
Electromagnetic Field

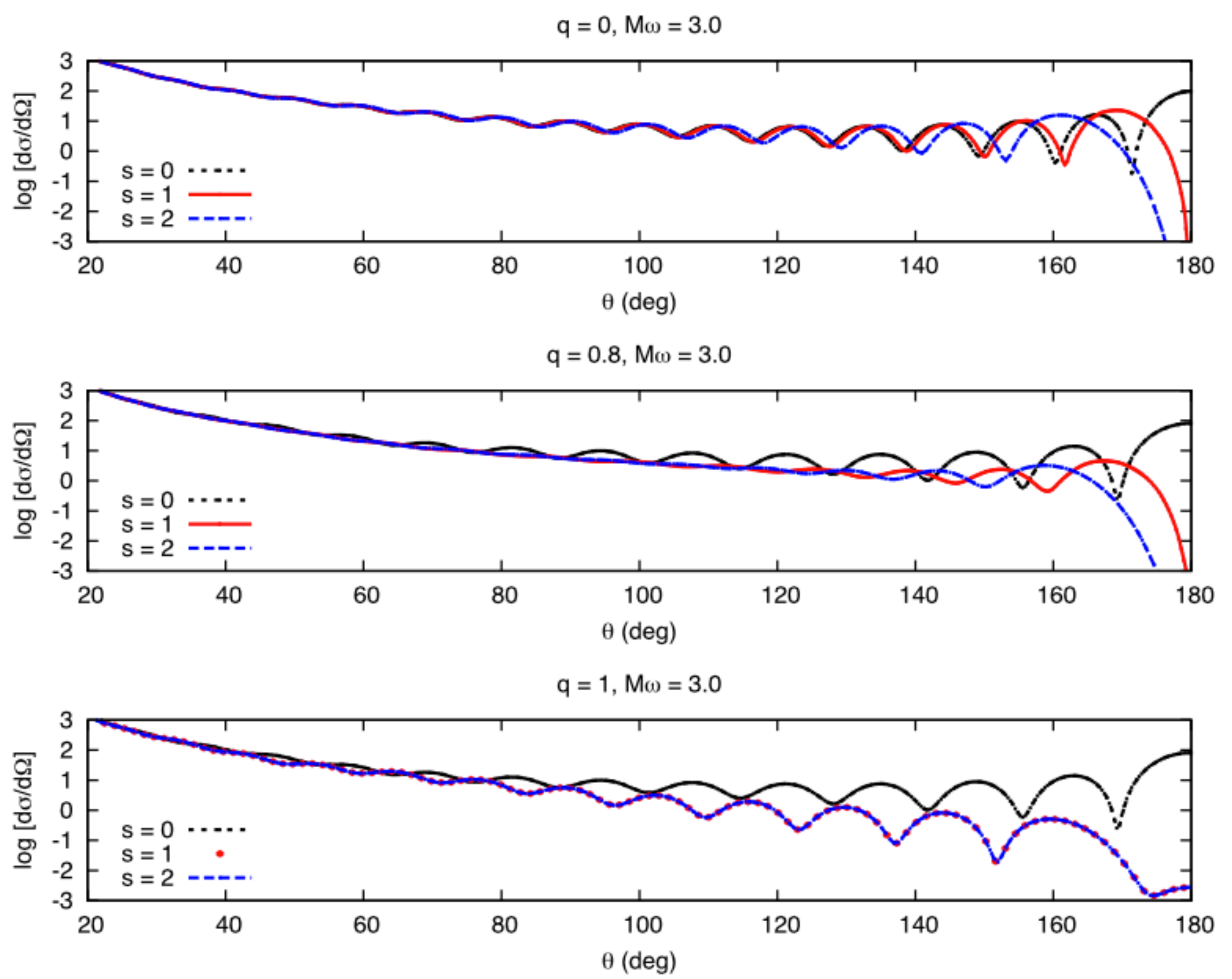


Scattering Cross Section of Charged Black Holes
Electromagnetic Field





Scattering Cross Section of Charged Black Holes [s = 0, 1, 2]



Scattering Cross Section of Charged Black Holes [s = 0, 1, 2]

N=2 supergravity

$$\mathcal{L} = -\frac{\sqrt{-g}}{2}\left[\hat{R} + i\overline{\psi_\mu^{(I)}}\gamma^{[\mu}\gamma^\rho\gamma^{\sigma]}\hat{D}_\rho\psi_\rho^{(I)} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$
$$+\frac{\sqrt{-g}}{4\sqrt{2}}\overline{\psi_\mu^{(I)}}\left[F^{\mu\nu} + \hat{F}^{\mu\nu} + \frac{1}{2}\gamma_5(\tilde{F}^{\mu\nu} + \tilde{\hat{F}}^{\mu\nu})\right]\psi_\nu^{(J)}\epsilon^{IJ},$$

where I, J take values $1, 2$ and $\epsilon^{12} = -\epsilon^{21} = 1$, $\epsilon^{11} = \epsilon^{22} = 0$.

$$\hat{F}_{\mu\nu} = \left(\partial_\mu A_\nu - \frac{1}{2\sqrt{2}}\overline{\psi_\mu^{(I)}}\psi_\nu^{(J)}\epsilon^{IJ}\right) - (\mu \leftrightarrow \nu),$$
$$\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\lambda\sigma}F_{\lambda\sigma}.$$

The Lagrangian $\mathcal{L}$ is invariant up to a total derivative under

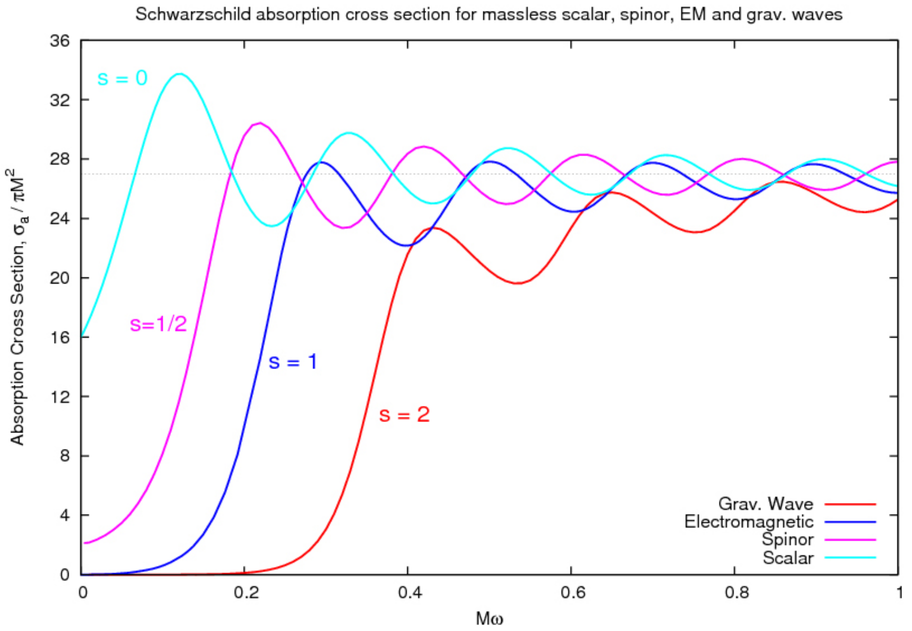
$$\delta g_{\mu\nu} = \frac{i}{\sqrt{2}}\overline{\alpha^{(I)}}\gamma_{(\mu}\psi_{\nu)}^{(I)}, \quad \delta A_\mu = i\overline{\alpha^{(I)}}\psi_\mu^{(J)}\epsilon^{IJ},$$
$$\delta\psi_\mu^{(I)} = \hat{D}_\mu\alpha^{(I)} + \frac{1}{2}\epsilon^{IJ}\left(\hat{F}_{\mu\lambda}\gamma^\lambda + \frac{1}{2}\hat{\tilde{F}}_{\mu\lambda}\gamma^\lambda\gamma_5\right)\alpha^{(J)}.$$

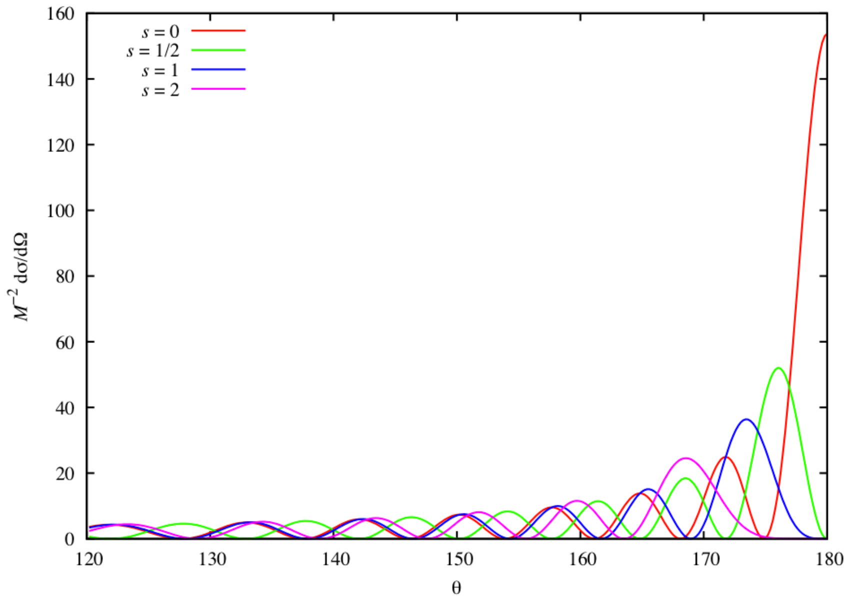
Conversion Cross Section of Charged Black Holes

• The gravitational to electromagnetic conversion cross section of the Reissner-Nordström black hole are currently under investigation.

Final Remarks

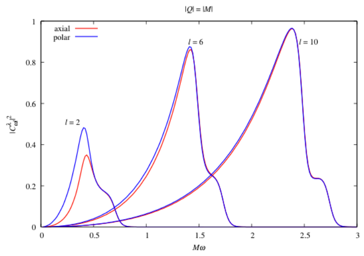
- **Absorption and scattering by black holes have been recently revisited in the literature using numerical techniques.**

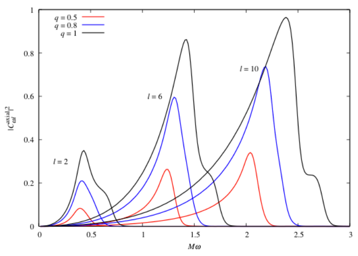




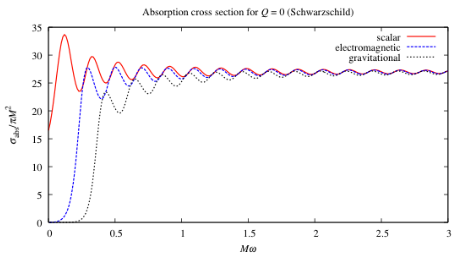
Final Remarks

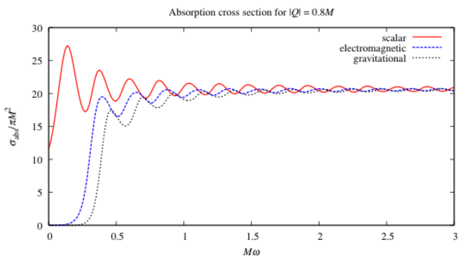
- **Conversion between electromagnetic and gravitational radiation.**

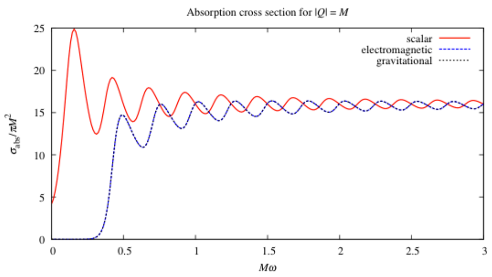




- **Equality between gravitational and electromagnetic absorption cross sections of extreme Reissner-Nordström black holes.**







Final Remarks

- **It is possible to infer the black hole charge from backscattered and electromagnetic radiation.**

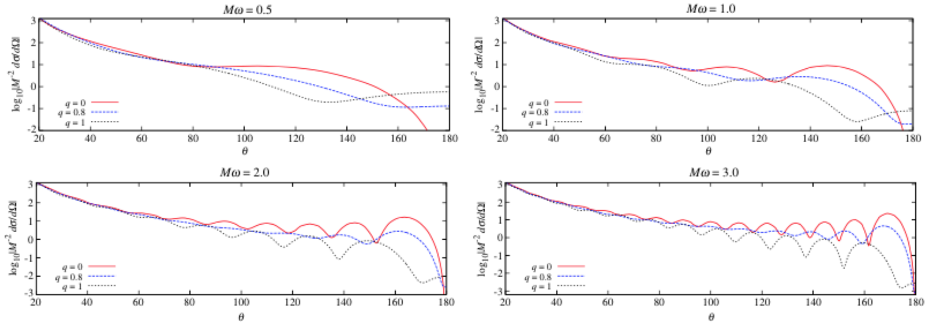
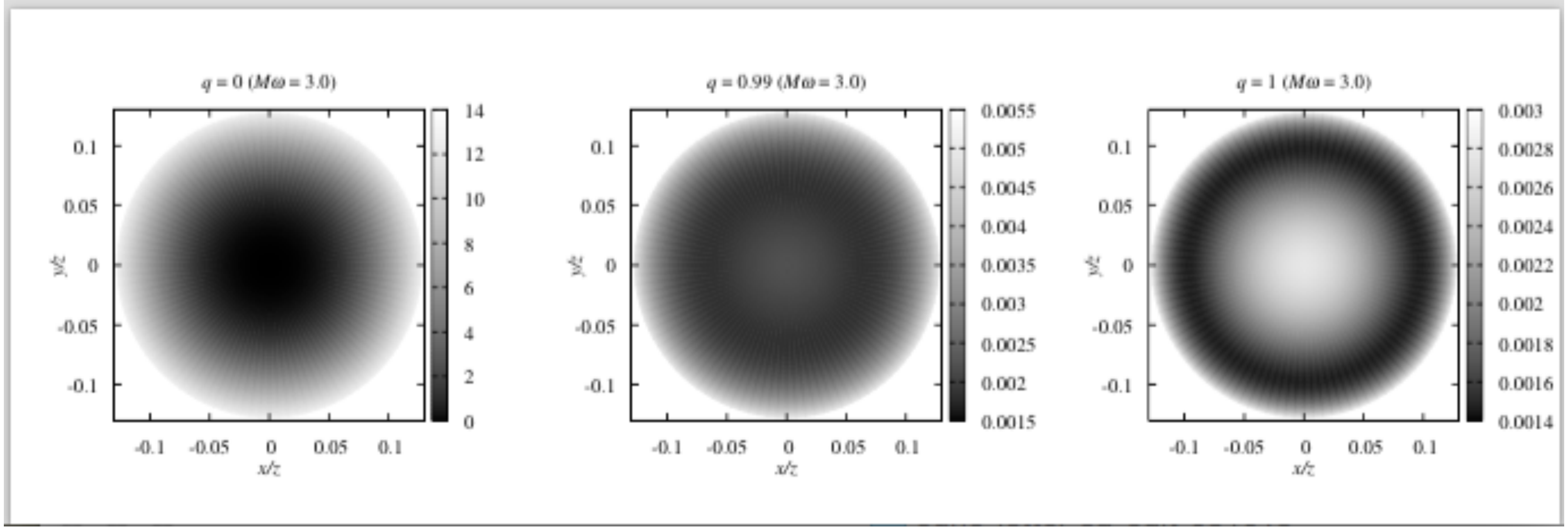
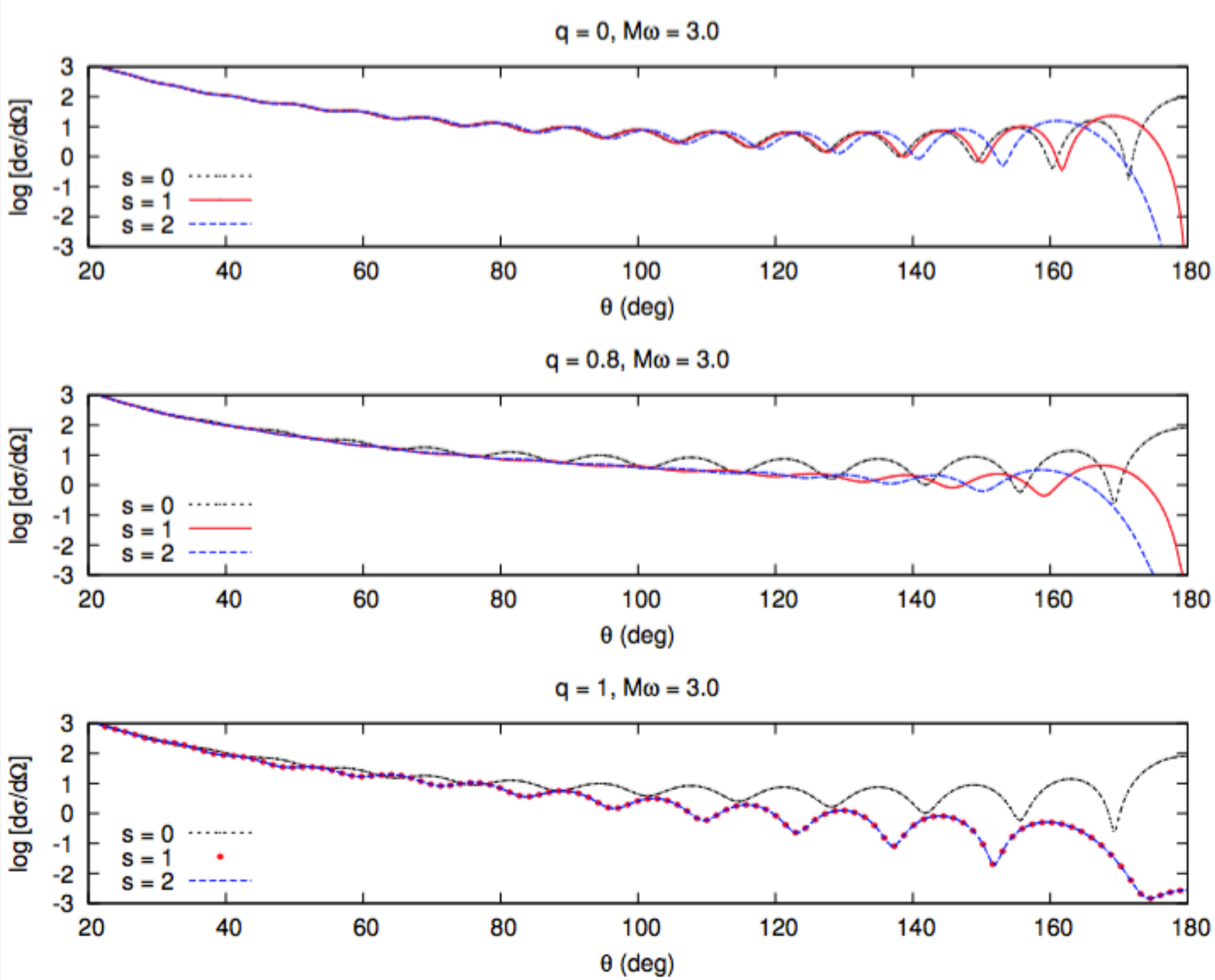


FIG. 1 (color online). Electromagnetic scattering by Reissner-Nordström black holes for $q = 0, 0.8, 1$ and $M\omega = 0.5, 1.0, 2.0, 3.0$. For $0 < q \leq 1$, the flux of EM radiation in the backward direction is nonzero; it diminishes as $M\omega$ increases.



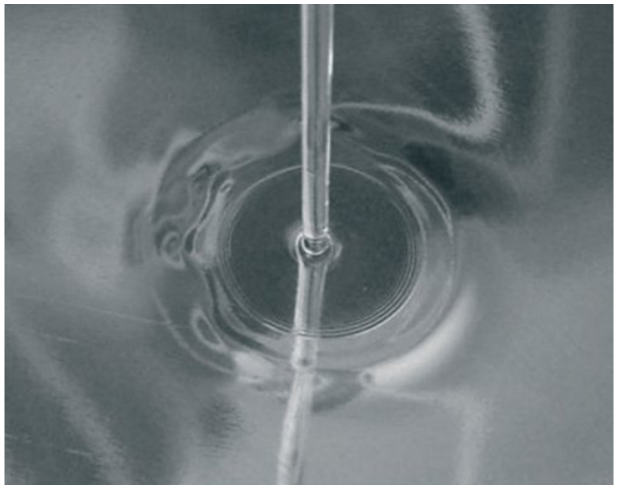
Final Remarks

- **Equality between gravitational and electromagnetic scattering cross sections of extreme Reissner-Nordström black holes.**



Final Remarks

• **Analogue models (fluids, optics, Bose-Einstein condensates, etc.) of gravity presents as a possibility of verifying black hole physics in the laboratory.**



Thanks!

Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

Fapespa
Fundação Amazônia de Amparo a Estudos e Pesquisas do Pará

SEVENTH FRAMEWORK PROGRAMME

MARIE CURIE ACTIONS

(crispino@ufpa.br)

Collaborators: Atsushi Higuchi, Ednilton Santos de Oliveira, Samuel Richard Dolan, …