Perturbation and stability of higher dimensional black holes

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GR 100 years in Lisbon

TECNICO, LISBON, 19 Dec. 2015

Introduction

Perturbation analysis:

- GW emission from a particle
  plunging into or orbiting around a BH
- Stability problem
  - Stable → final state of gravitational collapse
  - Unstable → New branch of solutions
- Information about the geometry: Quasi-Normal Modes
- Insights into Uniqueness/non-uniqueness
- Attempt to find new, approximate solutions
  (by deforming an existing solution)

Purpose of this talk

A brief overview of linear perturbation theory of higher dimensional black holes

Two major issues when formulating perturbation theory

- Fixing gauge ambiguity
  - Imposing suitable gauge conditions

    or

  - Constructing manifestly gauge-invariant variables

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- Reduction of perturbation equations to a simple, tractable form (master equation)
  - Classifying perturbations into mutually decoupled groups
  - Separating variables

4D master equations

Static asymptotically flat vacuum case Regge-Wheeler 57

Zerilli 70

charge case Moncrief

-- Stability Regge-Wheeler 57, Veshveshwara 70 …

asymptotically AdS/dS case Cardoso-Lemos

--- set of decoupled *self-adjoint* ODEs

Stationary Rotating vacuum (Kerr) case Teukolsky 72

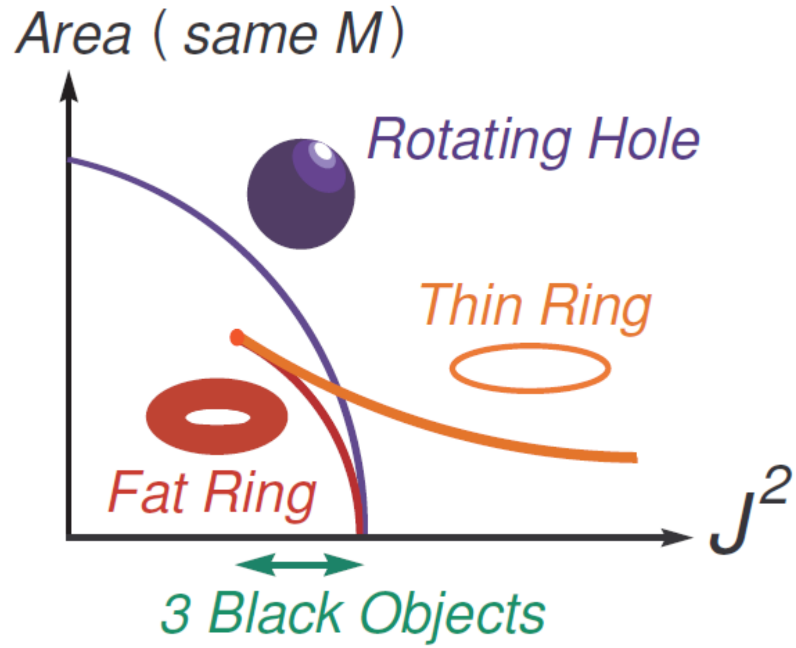
--- Stability Press-Teukolsky 73 --- Whihting 89 …

asymptotically AdS/dS case Chambers-Moss 94

Classification Problem in Higher Dimensions

- *D*>4 General Relativity

  No uniqueness like 4*D* GR



Many unstable black (rotating) objects

Dynamical uniqueness theorem

Uniqueness holds for “stable” black objects

Master equations for higher dimensional black holes

- Rotating BH case ➔ Not separable in general (e.g., Durkee-Godazgar-Reall)
  still a long way from having a complete perturbation theory

  Progress in some special cases

  Cohomogeneity-one (odd-dim.) Myers-Perry BH

  $D \geq 7$ Kunduri-Lucietti –Reall 07 (Tensor-modes)

  $5D$ Murata-Soda 08 (Tensor-Vector-Scalar modes)

  Single-spin (cohomogeneity-two) Myers-Perry

  $D \geq 7$ Kodama-Konoplya-Zhidenko 09

  Kundt spacetimes (e.g. Near-horizon geometry)

  Durkee-Reall 11

- Static BH case ➔ simpler and tractable:
  -- can reduce to a set of decoupled s.a. ODEs
  Kodama-AI 03

Background geometry

$$\mathcal{M}^D = \mathcal{N}^m \times \mathcal{K}^n \qquad ds^2 = g_{ab}(y)dy^a dy^b + r^2(y)d\sigma_n^2$$

$g_{ab}(y)$: *m* – dim spacetime metric

$d\sigma_n^2 = \gamma_{ij}(z)dz^i dz^j$: *n* – dim Einstein metric $\quad R_{ij} = (n-1)K\gamma_{ij}$

$K = \pm 1,\ 0$

-- corresponds to horizon-manifold

This metric can describe a fairly generic class of metrics

$m = 1 \quad y^a \to t$ FLRW universe $ds^2 = -dt^2 + r(t)^2 d\sigma_n^2$

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$m = 2$ $y^a \to (t, r)$ Static (Schwarzschild-type) black hole

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$m \geq 3$ $y^a \to (t, r, \mathbf{y})$ Black-brane

$$ds^2 = d\mathbf{y}^2 - f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma_n^2$$

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$m = 4$ $y^a \to (t, r, \theta, \phi)$ Myers-Perry black hole (w/ single rotation)

$r \to r \cos\theta$

$$ds^2 = \langle\langle \text{4-dim. Kerr type metric} \rangle\rangle + r^2 \cos^2\theta d\sigma_n^2$$

Kerr-brane

$r \to const.$ $ds^2 = \text{Kerr-metric} + d\sigma_n^2$

Cosmological perturbation theory

$$ds^2 = -dt^2 + r(t)^2 d\sigma_n^2$$: FLRW background metric

$r(t)$: scale factor

$d\sigma_n^2 = \gamma_{ij}(z) dz^i dz^j$: homogeneous isotropic time-slice

$n = 3$

Perturbations $\delta g_{\mu\nu}$ $\delta T_{\mu\nu}$ are decomposed into 3 types according to its tensorial behaviour on time-slice $(\mathcal{K}^n, \gamma_{ij})$

Tensor-type: transverse-traceless (possible only when $n \geq 3$)
➔ Gravitational Waves

Vectro-type: div-free vector ➔ couple to matter
e.g. velocity perturbations

Scalar-type: scalar ➔ couple to matter
e.g. density perturbations

Gauge-invariant formulation Bardeen 80 Kodama-Sasaki 84

Brane-world cosmology

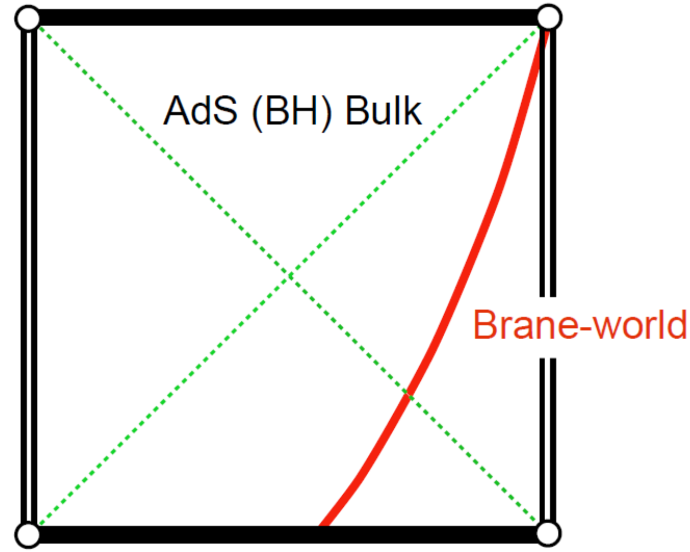
- AdS - (Black Hole)-Bulk spacetime

$$ds^2_{2+n} = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma_n^2$$

- Brane-world

$$f(r)\dot{t}^2 - \frac{1}{f(r)}\dot{r}^2 = 1$$

$$ds^2_{1+n} = -d\tau^2 + r^2(\tau)d\sigma_n^2$$



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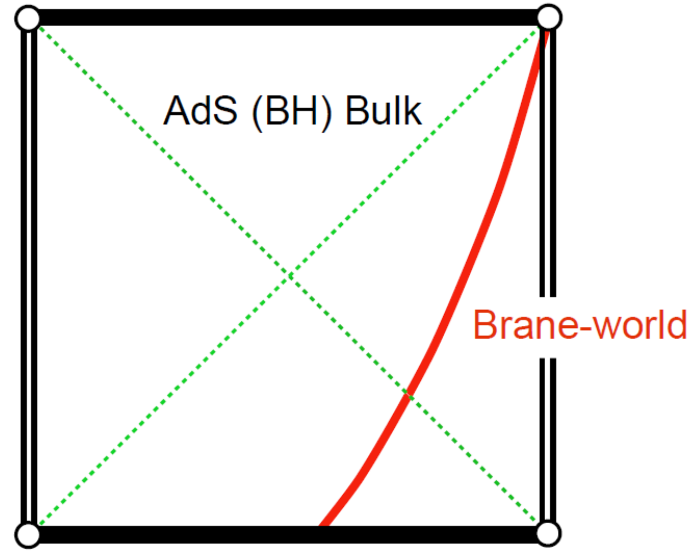

Bulk perturbations induce brane-world cosmological perturbations

--- need to develop a formula for AdS-Black Hole perturbations

--- convenient to decompose bulk perturbations into

Tensor-, Vector-, Scalar-type wrt $d\sigma_n^2 = \gamma_{ij}(z)dz^i dz^j$

Kodama – AI – Seto '00

Black hole background geometry

Static solutions of Einstein-Maxwell + cosmological constant in $D = 2 + n$

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma_n^2$$

$$f(r) = K - \frac{2M}{r^{n-1}} + \frac{Q^2}{r^{2(n-1)}} - \lambda r^2$$

$K = \pm 1,\ 0$

M ADM-mass

Q charge

$\lambda \propto \Lambda$ Cosmological constant

Basic strategy to derive master equations

(1) Mode-decompose $\delta g_{\mu\nu}$ as

- Tensor-type ↔ new component in $D > 4$ case
- Vector-type ↔ axial - mode in $D = 4$ case
- Scalar-type ↔ polar - mode in $D = 4$ case

(2) Expand $\delta g_{\mu\nu}$ by tensor harmonics $\mathbb{T}_{ij}$ $\mathbb{V}_i$ $\mathbb{S}$ defined on $\mathcal{K}^n$

(3) Write the Einstein equations in terms of the expansion coefficients in *2*-dim. spacetime $\mathcal{N}^2$ spanned by $y^a = (t, r)$

Tensor-type perturbations

$$\delta g_{\mu\nu} = \left(\begin{array}{c|c} \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & r^{(4-n)/2}\,\Phi(t,r)\ \mathbb{T}_{ij} \end{array} \right) \quad \begin{array}{l} \}\ y^a = (t,r) \\ \}\ z^i \end{array}$$

- $\mathbb{T}_{ij}$: Transverse-Traceless harmonic tensor on $\mathcal{K}^n$

$$(\hat{\triangle}_n + k_T^2)\mathbb{T}_{ij} = 0 \qquad \mathbb{T}^i{}_i = 0, \quad \hat{D}_j \mathbb{T}^j{}_i = 0$$

- $\Phi(t,r)$ is a gauge-invariant variable

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- $\Phi(t,r)$ is a gauge-invariant variable

- Einstein's equations reduce to Master equation $\mathcal{N}^2$

$$\left(\Box - \frac{V_T}{f}\right)\Phi = 0$$

$$V_T \equiv \frac{f}{r^2}\left[\frac{n(n+2)}{4}f + \frac{n(n+1)M}{r^{n-1}} + k_T^2 - (n-2)K\right]$$

Vector-type perturbations

$$\delta g_{\mu\nu} = \left(\begin{array}{c|c} 0 & h_a(t,r)\ \mathbb{V}_i \\ \hline * & H(t,r)\ D_{(i}\mathbb{V}_{j)} \end{array} \right) \begin{array}{l} \} \ y^a = (t,r) \\ \} \ z^i \end{array}$$

- $\mathbb{V}_i$: Div.-free vector harmonics on $\mathcal{K}^n$: $(\hat{\Delta}_n + k_V^2)\mathbb{V}_i = 0\,, \quad \hat{D}_i \mathbb{V}^i = 0$
- Gauge-invariant variable: $F^a := r^{n-2} h^a - \frac{r^n}{2} D^a \left(\frac{H}{r^2} \right)$
- Einstein's equations reduce to $\left\{ \begin{array}{ll} D_a F^a = 0 & \cdots \quad (1) \\ \Box F^a + \cdots = 0 & \cdots \quad (2) \end{array} \right.$

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(1) ➡ There exists $\Phi(t,r)$ such that $F^a = \epsilon^{ab}D_b\left(r^{n/2}\Phi\right)$

(2) ➡ Einstein's equation reduces to Master equation

$$\left(\Box - \frac{V_V}{f}\right)\Phi = 0 \qquad V_V \equiv \frac{f}{r^2}\left[k_V^2 - (n-1)K + \frac{n(n+2)}{4}f - \frac{n}{2}r\frac{df}{dr}\right]$$

-- corresponds to the Regge-Wheeler equation in *4D*

Scalar-type perturbations

- Expand $\delta g_{\mu\nu}$ by scalar harmonics $\mathbb{S}$ on $\mathcal{K}^n$: $(\hat{\Delta}_n + k_S^2)\mathbb{S} = 0$

- Construct gauge-invariant variables: X , Y , Z on $\mathcal{N}^2$

- After Fourier transf. wrt 't' Einstein's equations reduce to

  - Set of 1st –order ODEs for X , Y , Z
  - A linear algebraic relation among them

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--- such a system can be reduced to a single wave equation

- For a certain linear combination $\Phi(t,r)$ of X , Y , Z

Einstein's equations reduce to

$$\left(\Box - \frac{V_S}{f}\right)\Phi = 0$$

-- corresponds to the Zerilli equation in *4D*

Stability analysis

- Master equation takes the form:

$$\frac{\partial^2}{\partial t^2}\Phi = -A\Phi \qquad A = -\frac{d^2}{dr_*^2} + V$$

Stability analysis

- Master equation takes the form:

$$\frac{\partial^2}{\partial t^2}\Phi = -A\Phi \qquad A = -\frac{d^2}{dr_*^2} + V$$

$\Phi \propto \exp(-i\omega t)$

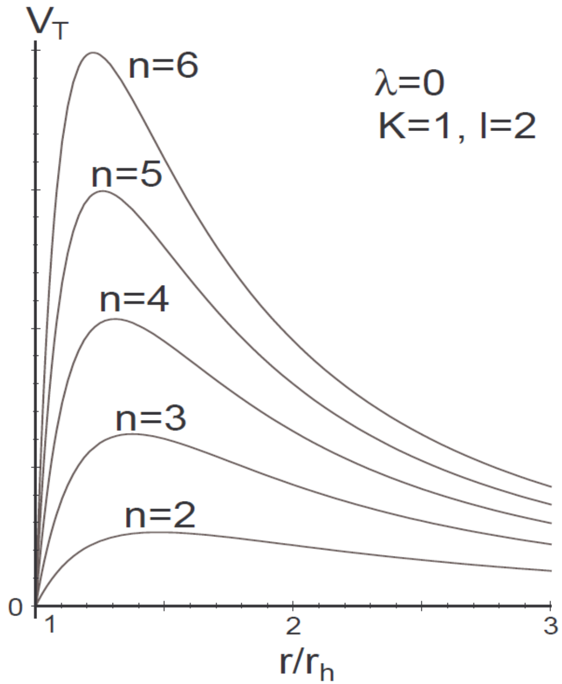
$$\omega^2 \int dr_* |\Phi|^2 = \int dr_* \Phi^* A \Phi$$

If "A" is a *positive* self-adjoint operator, the master equation does *not* admit *"unstable"* solutions

--- The black hole is stable

Stability wrt Tensor-type

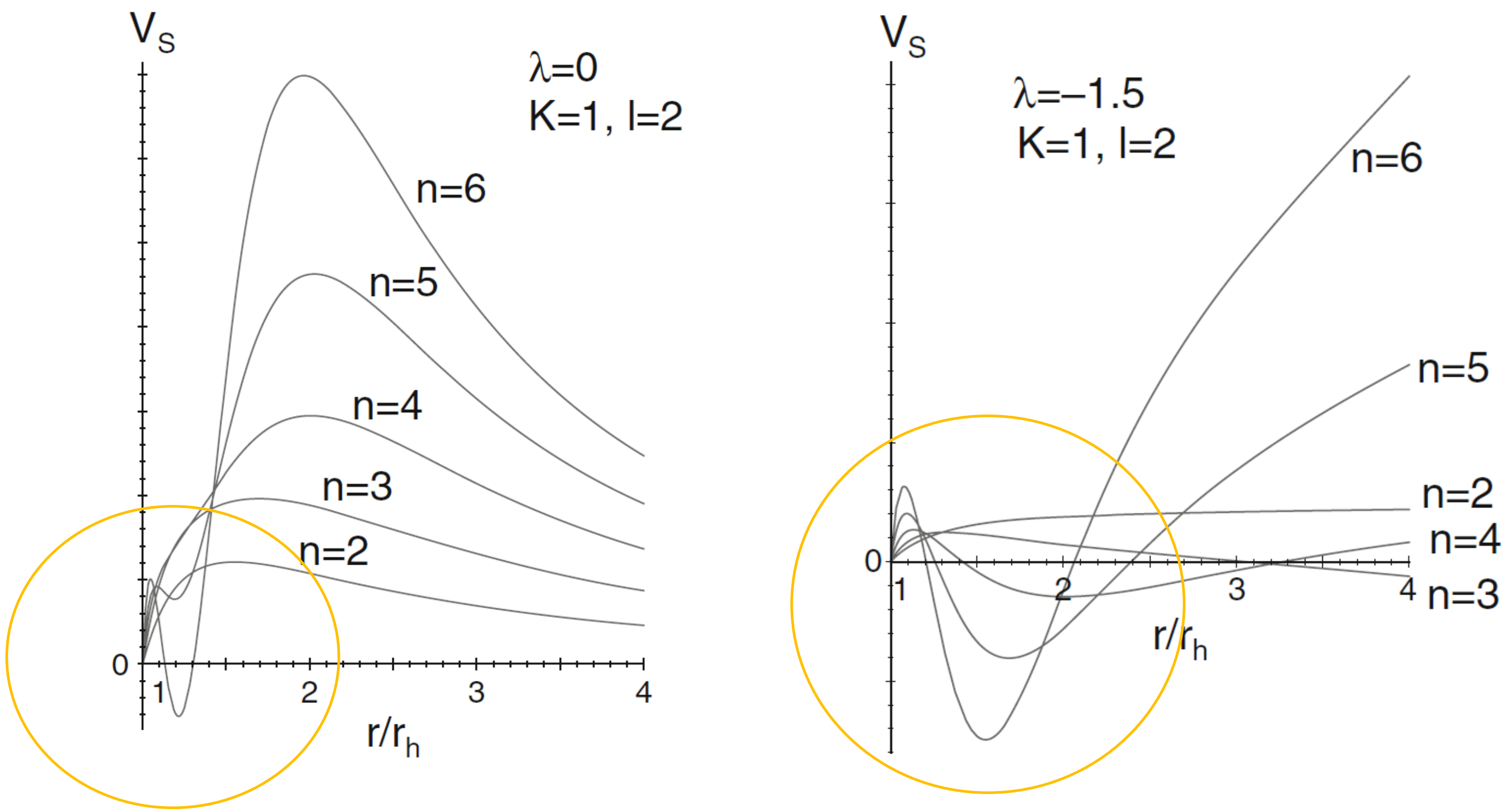
$$V_T \equiv \frac{f}{r^2}\left[\frac{n(n+2)}{4}f + \frac{n(n+1)M}{r^{n-1}} + k_T^2 - (n-2)K\right] > 0$$



$$A = -\frac{d^2}{dr_*^2} + V > 0$$

Stable

Stability wrt Scalar-type



The potential is *NOT* positive definite in $D > 4$

- Not obvious to see whether $A = -\frac{d^2}{dr_*^2} + V$ is positive or not …

Stability proof

- Define $D := \frac{d}{dr_*} + S$ w. some function $S(r)$

$$(\Phi, A\Phi) = -\Phi^* D\Phi|_{\text{bndry}} + \int dr_* \, |D\Phi|^2 + \tilde{V} \, |\Phi|^2$$

where $\tilde{V} := V + \frac{dS}{dr_*} - S^2$

Boundary terms vanish under the Dirichlet conditions $\Phi = 0$

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Boundary terms vanish under the Dirichlet conditions $\Phi = 0$

Task: Find $S(r)$ that makes $\tilde{V}$ positive definite

Then, A is uniquely extended to be a positive self-adjoint operator

When the horizon manifold $\mathcal{K}^n$ is maximally symmetric

		Tensor		Vector		Scalar	
		$Q=0$	$Q\neq 0$	$Q=0$	$Q\neq 0$	$Q=0$	$Q\neq 0$
$K=1$	$\lambda=0$	OK	OK	OK	OK	OK	$D=4,5$ OK $D\geq 6$?
	$\lambda>0$	OK	OK	OK	OK	$D\leq 6$ OK $D\geq 7$?	$D=4,5$ OK $D\geq 6$?
	$\lambda<0$	OK	OK	OK	OK	$D=4$ OK $D\geq 5$?	$D=4$ OK $D\geq 5$?
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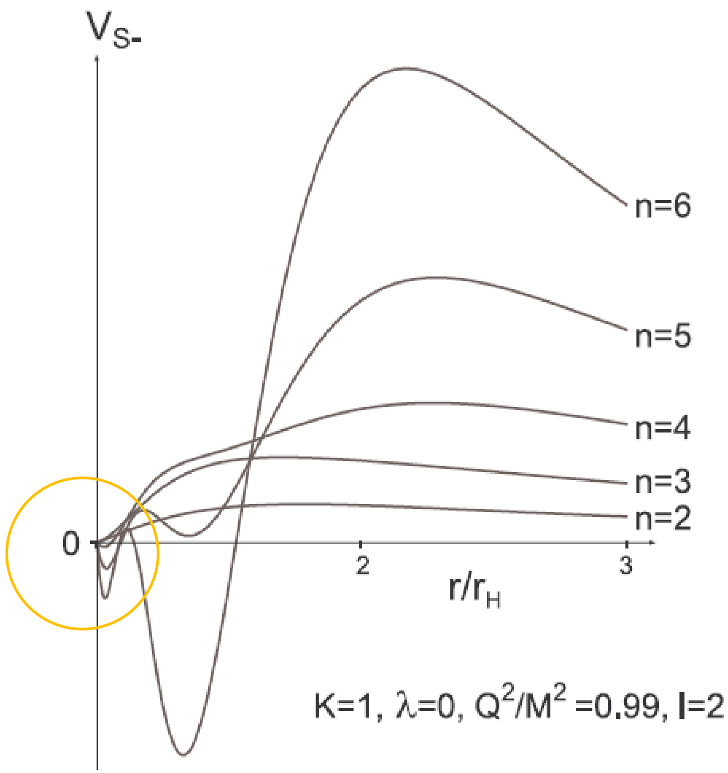
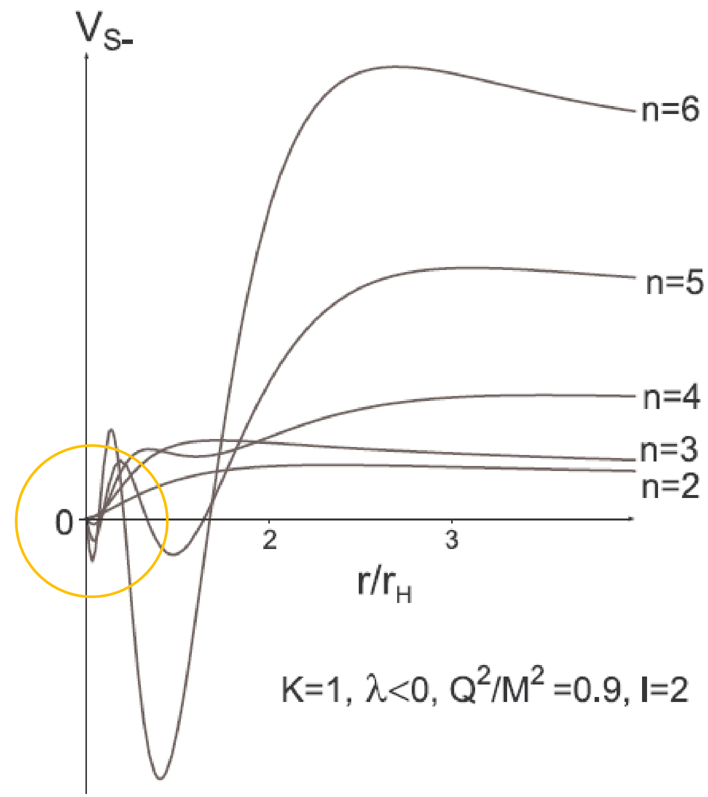
"OK" ➔ "Stable"

WRT Tensor- and Vector-perturbations ➔ Stable over entire parameter range

WRT Scalar-perturbations ➔ ??? when $Q\neq 0$ $\Lambda\neq 0$

Potential for Scalar-type pert. w. non-vanishing Q , Λ





For extremal and near-extremal case, the potential becomes *negative* in the *immediate vicinity* of the horizon

Numerical study for charged-AdS/dS case Konoplya-Zhidenko 07, 08, 09

Some generalizations and open problems

Static black holes in Lovelock theory

Higher curvature terms involved

$$L = \sum_{m=0}^{k} c_m \mathcal{L}_m \qquad \mathcal{L}_m = \frac{1}{2^m} \delta^{\lambda_1\sigma_1\cdots\lambda_m\sigma_m}_{\rho_1\kappa_1\cdots\rho_m\kappa_m} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1} \cdots R_{\lambda_m\sigma_m}{}^{\rho_m\kappa_m}$$

Equations of motion contain only up to 2nd-order derivatives

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\sigma_n^2$$

$$f(r) = K - X(r)r^2$$

- Master equations in generic Lovelock theory Takahashi – Soda 10
  in Gauss-Bonnet theory Dotti – Gleiser 05

- Asymptotically flat, small mass BHs are unstable wrt
  Tensor-type perturbations (in even-dim.)
  Scalar-type perturbations (in odd-dim.)

- Instability is stronger in higher multipoles rather than low-multipoles

$$(\Phi, A\Phi) = \int dr_* \, |D\Phi|^2 + \ell(\ell+n-1) \int dr_* N(r) \, |\Phi|^2$$

If $N(r) < 0$, then $(\Phi, A\Phi) < 0$ for sufficiently large ℓ

Rotating case: Cohomogeneity-2 Myers-Perry BHs

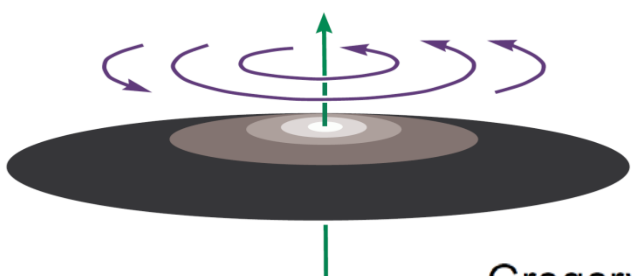
$$m = 4 \qquad ds^2 = \underbrace{\langle\langle \text{4-dim. Kerr type metric} \rangle\rangle}_{\mathcal{N}^4} + \underbrace{r^2 \cos^2\theta d\sigma_n^2}_{\mathcal{K}^n}$$

symmetry enhance $U(1)^N \Rightarrow U(1) \times SO(D-3)$

Numerical approach to stability analysis

5D bar-mode Shibata-Yoshino 10

--- include the *ultra-spinning* case



Gregory-Laflamme modes

Axisymmetric perturbation Dias-et. al. 09

Cohomogeneity-2 MP case: Analytic formulation?

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Tensor-type perturbations: A single master scalar variable Φ on $\mathcal{N}^4$

satisfy the same equation for a massless Klein-Gordon field

$n \geq 3$

How about **vector-type** and **scalar-type** perturbations?

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Kerr-brane: 4-dim. Kerr-metric + Ricci flat space

KK-reduction along the Ricci flat space $\mathcal{K}^n$

➔ Equations for *massive vector/ tensor fields*

on $\mathcal{N}^4$: 4-dim. Kerr metric

Pani, Gualtieri, Cardoso, AI 15

c.f. Cohomogeneity-1 Myers-Perry BHs

$$D = \text{odd},\ J_1 = J_2 = \cdots J_{[(D-1)/2]}$$

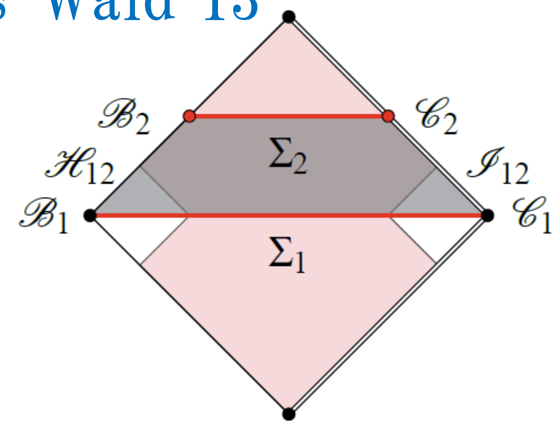
enhanced symmetry: $\mathbb{R} \times U((D-1)/2)$

Perturbation equations reduce to *ODEs*

Kunduri-Lucietti –Reall 07, Murata-Soda 08

Canonical energy method for initial data

Hollands-Wald 13



Symplectic current

$$w^a = \frac{1}{16\pi} P^{abcdef}(\gamma_{2\,bc}\nabla_d \gamma_{1\,ef} - \gamma_{1\,bc}\nabla_d \gamma_{2\,ef})$$

Symplectic form $$W(\Sigma;\gamma_1,\gamma_2) \equiv \int_\Sigma \star w(g;\gamma_1,\gamma_2)$$

Canonical energy $$\mathscr{E}(\Sigma,\gamma) \equiv W(\Sigma;\gamma,\pounds_K\gamma) - B(\mathscr{B},\gamma) - C(\mathscr{C},\gamma)$$

$$B(\mathscr{B},\gamma) = \frac{1}{32\pi}\int_{\mathscr{B}} \gamma^{ab}\delta\sigma_{ab}$$

$$C(\mathscr{C},\gamma) = -\frac{1}{32\pi}\int_{\mathscr{C}} \tilde{\gamma}^{ab}\delta\tilde{N}_{ab}$$

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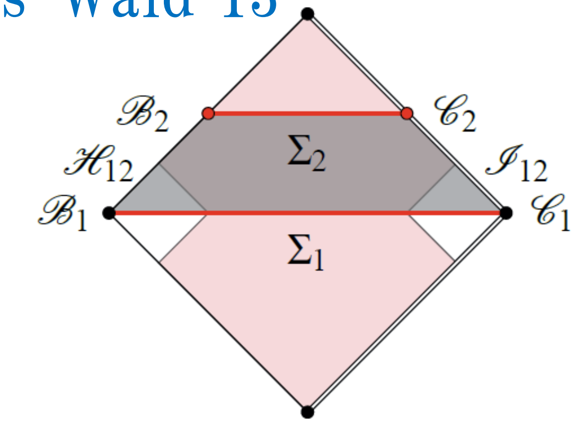

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1) $\mathscr{E}$ is gauge invariant

$$B(\mathscr{B}, \gamma) = \frac{1}{32\pi} \int_{\mathscr{B}} \gamma^{ab} \delta\sigma_{ab}$$

2) $\mathscr{E}$ is monotonically decreasing
for any axi-symmetric perturbation

$$C(\mathscr{C}, \gamma) = -\frac{1}{32\pi} \int_{\mathscr{C}} \tilde{\gamma}^{ab} \delta\tilde{N}_{ab}$$

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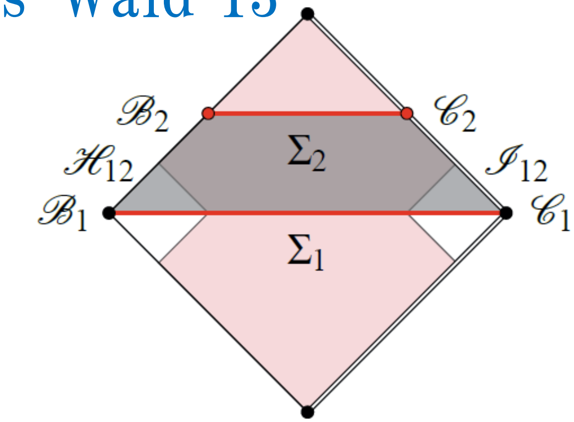

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This method relates Dynamic and Thermodynamic stability criterion and proves Gubser-Mitra conjecture

Role of symmetry in Stability problem

- Stability of extremal black holes

Examine perturbations of the near-horizon geometry that respect the symmetry (axisymmetry) of the full BH solution   Conjectured by Durkee - Reall 11

When axi-symmetric perturbations on the NHG violate AdS_2 -BF-bound on the NHG, then the original extremal BH is unstable $e^{im_I\phi^I}$ $m_I N^I(x) = 0$.

… supportd by numerical results. Dias et al

Proven by use of Canonical energy method Hollands-AI 14

Role of symmetry in Stability problem

- Stability of extremal black holes

Examine perturbations of the near-horizon geometry that respect the symmetry (axisymmetry) of the full BH solution   Conjectured by Durkee - Reall 11

When axi-symmetric perturbations on the NHG violate AdS_2 *-BF-bound on the NHG, then the original extremal BH is unstable* $e^{im_I \phi^I}$ $m_I N^I(x) = 0.$

… supportd by numerical results. Dias et al

Proven by use of Canonical energy method Hollands-AI 14

Another application of Canonical energy method

→ Superradiant instability of rotating AdS black holes

Green-Hollands-AI-Wald 15

Summary

- Static HDBHs: Complete formulation for perturbations

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  -- Still a long way from having a complete formulation

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- Interplay between

  Exact solutions + Perturbation analysis

  Numerical Analysis

  Mathematical Theorems

Interplay between Exact solutions + Perturbation, Numerical Analysis, Mathematical Theorems

1915	Einstein equations
1915	Schwarzschild Solution
1939	Oppenheimer-Snyder
1957	Regge-Wheeler equation
1963	Kerr solution
1965	Singularity Theorems
1970	Zerilli eqution
1973	Teukolsky equation
	BH Thermodynamcis laws
1975	Hawking radiation
1982	Uniqueness Theorem
1983	Positive Energy Theorem
1985	Accurate method to BH QNMs

Exact Solutions + Perturbation analysis

Mathematica Theorems

Numerical Approach

Exact solutions + Perturbation

1986	Myers-Perry Solution
1993	BTZ Solution
	Gregory-Laflamme Instability
	Choptuick's critical collapse in Numerical GR
	BSSN system in Numerical GR
1997	AdS-CFT correspondence
1998	Brane-world scenario
2001	Emparan-Reall black ring
	HD BH Perturbation theory: This talk
	Doubly spinning black ring
	Black saturn
	Multiple- black rings
2015	Black-lens Kunduri-Lucietti

Numerical Analysis

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High energy collisions of BHs Sperhake et al

Axisymmetric perturbation of MP BH – Dias et. al

Bar-mode instability of MP BH Shibata-Yoshino

Black-String final fate Lehner-Pretorius

2015 Instability of AdS spacetimes – Bizon-Rostworowsky

Mathematical Theorems

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HD generalization of BH Topology Theorem

HD generalization of BH rigidity (Symmetry) Theorem

HD Uniqueness/Non-uniqueness Theorems

2015

Interplay between Exact solutions + Perturbation
Numerical Analysis
Mathematical Theorems

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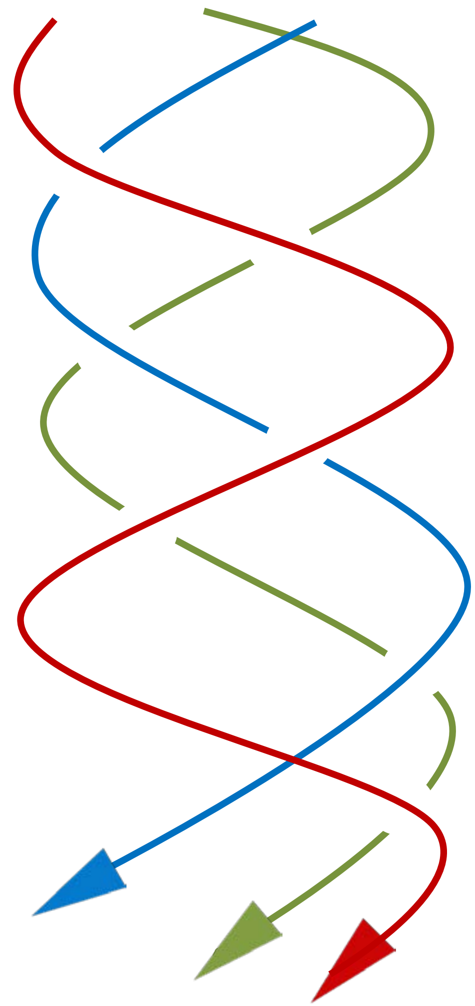
Higher dimensional General Relativity

2015



At GR Centenary

- **Perturbation theory** has played a major role in understanding basic properties—e.g. stability—of **exact solutions** at hand.

- **Numerical Approach** has become more important to reveal interesting properties of complicated systems and/or to deal with more realistic models

- **Mathematical theorems** as guide lines

- Interplay between

  Numerical Approach

  Mathematical Theorems and

  Exact solution + Perturbation analysis

  will be getting more and more important .