Beyond Einstein's general relativity: 100 years-on

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instituto de astrofísica e ciências do espaço

What is the status of General Relativity 100 years on?

Brief outline of the talk

- Basic criteria for the viability of a gravitation theory
- Foundations of gravitation theory
- Beyond General Relativity: Modified gravity
  - Extensions of $f(R)$ gravity and some applications;
  - Hybrid metric-Palatini gravity;
  - etc,etc.

- The perplexing fact of the late-time cosmic acceleration has forced theorists and experimentalists to pose the question: Is General Relativity (GR) the correct relativistic theory of gravitation?
- The fact GR is facing so many challenges:
  - Difficulty in explaining particular observations
  - Incompatibility with other well established theories
  - Lack of uniqueness

Is this indicative of a need for new gravitational physics?

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Experimental tests of GR

- In fact, Einstein, in deriving GR was not motivated to account for unexplained experimental results, but was driven by theoretical criteria of elegance and simplicity.
- Primary goal: produce a gravitation theory that incorporated the principle of equivalence and special relativity in a natural way.
- However, in the end the theory had to be confronted with experiment, in particular, the "three classical tests":
  - Accounts for the perihelion advance of Mercury;
  - Eddington's measurement of light deflection, in 1919;
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- Late 1950s, Schiff and Dicke suggested:
  - that the gravitational redshift was not a true test of GR.
  - It was purely a consequence of the equivalence principle, and did not test the field equations of gravitational theory.
- Note that by 1960, one may consider that the validity of GR rested upon the following empirical foundation:
  - One test of moderate precision (the perihelion shift of Mercury; precision: approx. 1%).
  - One test of low precision (the deflection of light; precision: approx. 50%).
  - One inconclusive test that was not a real test anyway (the gravitational redshift).
- In the meantime, alternative theories of gravitation were being constructed (Poincaré, Whitehead, Milne, Birkhoff, Belinfante, etc), that also laid claim to the viability of the above tests.

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Motivations

- Of course, cosmology is also an ideal testing ground for GR (in particular, late-time cosmic acceleration).
- Thus, a promising approach is to assume that at large scales GR breaks down, and a more general action describes the gravitational field.
- Generalizations of the Einstein-Hilbert Lagrangian: R^2, $R_{\mu\nu}R^{\mu\nu}$, $R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$, $\varepsilon^{\alpha\beta\mu\nu}R_{\alpha\beta\gamma\delta}R^{\gamma\delta}_{\mu\nu}$, $C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu}$, etc.
- Physical motivations for these modifications of gravity:
  - possibility of a more realistic representation of the gravitational fields near curvature singularities;
  - and to create some first order approximation for the quantum theory of gravitational fields.

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General Relativity (GR): Hilbert-Einstein action

- GR is a classical theory, therefore no reference to an action is required. Consider the Hilbert-Einstein action:

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} + L_m(g^{\mu\nu}, \psi) \right] . \tag{1}$$

- But, the Lagrangian formulation is elegant, and has merits:
  - Quantum level: the action acquires a physical meaning, and a more fundamental theory of gravity will provide an effective gravitational action at a suitable limit;
  - Easier to compare alternative gravitational theories through their actions rather than by their field equations;
  - In many cases one has a better grasp of the physics as described through the action, i.e., couplings, kinetic and dynamical terms, etc

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Foundations of Gravitation Theory

- Schiff and Dicke: gravitational experiments do not necessarily test GR, i.e., do not test the validity of specific field equations, experiments test the validity of principles;
- Triggered the development of powerful tools for distinguishing and testing theories, such as the Parametrized Post-Newtonian (PPN) expansion (pioneered by Nordvedt; extended by Nordvedt and Will)
- Indeed, the idea that experiments test principles and not specific theories, implies the need of exploring the conceptual basis of a gravitational theory.

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Dicke framework

- **Dicke Framework.** Probably the most unbiased assumptions to start with, in developing a gravitation theory:
  - Spacetime is a 4-dim manifold, with each point in the manifold corresponding to a physical event (note that a metric and affine connection is not necessary at this stage);
  - The equations of gravity and the mathematical entities in them are to be expressed in a form that is independent of the coordinates used, i.e., in a covariant form.
- It is common to think of GR, or any other gravitation theory, as a set of field equations (or an action).
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Einstein Equivalence Principle (C. Will)

- Einstein Equivalence Principle (EEP): The EEP is at the heart of gravitation theory.
- Thus if the EEP is valid, then gravitation must be a curved spacetime phenomenon, i.e., it must obey the postulates of Metric Theories of Gravity:
  - Spacetime is endowed with a metric (second rank non-degenerate tensor);
  - The world lines of test bodies are geodesics of that metric;
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$f(R)$ gravity

- For instance, consider $f(R)$ gravity, for simplicity. Appealing feature: combines mathematical simplicity and a fair amount of generality!
- Ricci scalar is a dynamical degree of freedom: $FR - 2f + 3\Box F = \kappa T$ (where $F = df/dR$)
- Introduces a new light scalar degree of freedom
  - This produces a late-time cosmic acceleration
  - But, the light scalar strongly violates the Solar System constraints
  - Way out: 'chameleon' mechanism, i.e., the scalar field becomes massive in the Solar System (**very contrived!**)
- Approaches: metric, Palatini, metric-affine formalisms (and the hybrid metric-Palatini formalism)

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Action of hybrid metric-Palatini gravity

- Action:
(review: Capozziello, Harko, Koivisto, FL, Olmo, Universe 2015)
(Harko, Koivisto, FL, Olmo, PRD 2011)
(Capozziello, Harko, Koivisto, FL, Olmo, JCAP 2013)

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,[R + f(\mathcal{R})] + S_m \ , \tag{2}$$

where S_m is the matter action, $\kappa^2 \equiv 8\pi G$,
R is the Einstein-Hilbert term, $\mathcal{R} \equiv g^{\mu\nu}\mathcal{R}_{\mu\nu}$ is the Palatini curvature,
and $\mathcal{R}_{\mu\nu}$ is defined in terms of an independent connection $\hat{\Gamma}^\alpha_{\mu\nu}$ as
$\mathcal{R}_{\mu\nu} \equiv \hat{\Gamma}^\alpha_{\mu\nu,\alpha} - \hat{\Gamma}^\alpha_{\mu\alpha,\nu} + \hat{\Gamma}^\alpha_{\alpha\lambda}\hat{\Gamma}^\lambda_{\mu\nu} - \hat{\Gamma}^\alpha_{\mu\lambda}\hat{\Gamma}^\lambda_{\alpha\nu}$.

Scalar-tensor representation

- May be expressed as the following scalar-tensor theory

$$S=\int \frac{d^4x\sqrt{-g}}{2\kappa^2}\left[(1+\phi)R+\frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right]+S_m. \qquad (3)$$

  This action differs from the $w=-3/2$ Brans-Dicke theory in the coupling of the scalar to the curvature, which in the $w=-3/2$ theory is of the form ϕR.
- This simple modification will have important physical consequences.

Weak-field, slow-motion behaviour

- The effective Newton constant $G_{\rm eff}$ and the post-Newtonian parameter (PPN) γ are

$$G_{\rm eff} \equiv \frac{G}{1+\phi_0}\left[1-(\phi_0/3)\, e^{-m_\varphi r}\right] , \tag{4}$$

$$\gamma \equiv \frac{1+(\phi_0/3)\, e^{-m_\varphi r}}{1-(\phi_0/3)\, e^{-m_\varphi r}} . \tag{5}$$

- As is clear from the above expressions, the coupling of the scalar field to the local system depends on the amplitude of the background value ϕ_0.
- If ϕ_0 is small, then $G_{\rm eff} \approx G$ and $\gamma \approx 1$ regardless of the value of the effective mass m_φ^2.

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- If ϕ_0 is small, then $G_{\rm eff} \approx G$ and $\gamma \approx 1$ regardless of the value of the effective mass m_φ^2.

- This contrasts with the result obtained in the metric version of $f(R)$ theories:

$$\varphi = \frac{2GM}{3r} e^{-m_f r}, \tag{6}$$

$$G_{\text{eff}} \equiv G\left(1 + e^{-m_f r}/3\right)/\phi_0 \tag{7}$$

$$\gamma \equiv \left(1 - \frac{e^{-m_f r}}{3}\right) \Big/ \left(1 + \frac{e^{-m_f r}}{3}\right). \tag{8}$$

which requires a large mass $m_f^2 \equiv (\phi V_{\phi\phi} - V_\phi)/3$ to make the Yukawa-type corrections negligible in local experiments.

Late-time cosmic speedup

- Consider the FRW metric ($k = 0$): $ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$.
- Modified Friedmann equations:

$$3H^2 = \frac{1}{1+\phi}\left[\kappa^2\rho + \frac{V}{2} - 3\dot{\phi}\left(H + \frac{\dot{\phi}}{4\phi}\right)\right], \quad (9)$$

$$2\dot{H} = \frac{1}{1+\phi}\left[-\kappa^2(\rho + P) + H\dot{\phi} + \frac{3}{2}\frac{\dot{\phi}^2}{\phi} - \ddot{\phi}\right]. \quad (10)$$

- Scalar field equation:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\dot{\phi}^2}{2\phi} + \frac{\phi}{3}[2V - (1+\phi)V_\phi] = -\frac{\phi\kappa^2}{3}(\rho - 3P). \quad (11)$$

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Consistency at Solar System and cosmological scales

- Consider for mathematical simplicity:

$$V(\phi) = V_0 + V_1\phi^2\,. \tag{12}$$

- Trace of the field eq. automatically implies $R = -\kappa^2 T + 2V_0$.
- As $T \to 0$ with the cosmic expansion, naturally evolves into a de Sitter phase ($V_0 \sim \Lambda$) for consistency with observations.
- If V_1 is positive, the de Sitter regime represents the minimum of the potential.
- The effective mass for local experiments, $m_\varphi^2 = 2(V_0 - 2V_1\phi)/3$, is positive if $\phi < V_0/V_1$.
  For $V_1 \gg V_0$, the amplitude small enough to pass SS tests.
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Further cosmological solutions

- (Capozziello, Harko, Koivisto, FL, Olmo, JCAP 2013)
- (João Luís Rosa, José Lemos, FL, work in progress)

Dynamical System Analysis

- Cosmological dynamics can be addressed by taking into account a suitable dynamical system. Let us introduce the dimensionless variables

$$\Omega_m \equiv \frac{\kappa^2 \rho_m}{3H^2}\,, \quad x \equiv \phi\,, \quad y = x_{,N}\,, \quad z = \frac{\kappa^2 V}{3H^2}\,, \tag{13}$$

  where $N = \log a$ is the e-folding time.

- The Friedmann equation can then be rewritten as

$$1 + x + y - z + \frac{y^2}{4x} = \Omega_m\,. \tag{14}$$

Dynamical System Analysis

- Cosmological dynamics can be addressed by taking into account a suitable dynamical system. Let us introduce the dimensionless variables

$$\Omega_m \equiv \frac{\kappa^2 \rho_m}{3H^2}\,, \quad x \equiv \phi\,, \quad y = x_{,N}\,, \quad z = \frac{\kappa^2 V}{3H^2}\,, \tag{13}$$

  where $N = \log a$ is the e-folding time.
- The Friedmann equation can then be rewritten as

$$1 + x + y - z + \frac{y^2}{4x} = \Omega_m\,. \tag{14}$$

The autonomous system of equations reads

$$
\begin{aligned}
x_{,N} &= y\,, \\
y_{,N} &= \frac{2x+y}{8x}\Big\{(3w_m-1)y^2+4x[(3w_m-1)y-3(1+w_m)z] \\
&\quad -4x^2[1-3w_m-2u(x)z]+4\left[3x(w_m-1)y+y^2-x^2(2-6w_m-4u(x)z)\right]\Big\} \\
z_{,N} &= \frac{z}{4x}\Big\{(3w_m-1)y^2+4x[(3w_m-1)y-3(1+w_m)z] \\
&\quad +4x[3+3w_m+u(x)y]+4x^2(3w_m-1+2u(x)z)\Big\}\,,
\end{aligned}
$$

where we have defined $u(x)\equiv V'(\phi)/V(\phi)$.

- To construct a viable model, the potential should be such that we meet the two requirements:
  - The matter dominated fixed point should be a saddle point, the de Sitter fixed point an attractor. Then we naturally obtain a transition to acceleration following standard cosmological evolution.
  - At the present epoch the field value should be sufficiently close to zero. Then we avoid conflict with the Solar System tests of gravity.
- **See (Carloni, Koivisto, FL, PRD 2015 [arXiv:1507.04306 [gr-qc]]) for more details.**

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Cosmological Perturbations

- To understand the cosmological structure formation, deduce the perturbation equations and analyse them for specific cases.
- This paves the way for a detailed comparison of the predictions with the cosmological data on large scale structure and the cosmic microwave background.
- Consider the Newtonian gauge, which can be parameterized by the two gravitational potentials Φ and Ψ,

$$\mathrm{d}s^2 = -(1+2\Psi)\mathrm{d}t^2 + a^2(t)(1+2\Phi)\mathrm{d}\vec{x}^2 \,. \tag{15}$$

- As matter source we consider a perfect fluid, with the background equation of state w and with density perturbation $\delta = \delta\rho_m/\rho_m$, pressure perturbation $\delta p_m = c_s^2 \delta\rho_m$ and velocity perturbation v.
- One can show:

$$\Psi + \Phi = -\frac{\varphi}{1+\phi} \,. \tag{16}$$

where we have denoted $\varphi = \delta\phi$.

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- Assuming a perfect fluid, the continuity and Euler equations for the matter component are

$$\dot{\delta} + 3H(c_s^2 - w)\delta = -(1+w)\left(3\dot{\Phi} - \frac{k^2}{a}v\right)\,, \qquad (17)$$

$$\ddot{v} + (1-3c_s^2)Hv = \frac{1}{a}\left(\Psi + \frac{c_s^2}{1+w}\delta\right)\,, \qquad (18)$$

respectively.

- Linear part of the Klein-Gordon equation is

$$\ddot{\varphi} + \left(3H + \frac{1}{\phi}\right)\dot{\varphi} + \left(\frac{k^2}{a^2} + \frac{\dot{\phi}^2}{2\phi^2} - \frac{2}{3}V''(\phi)\right)\varphi =$$

$$\left(2\ddot{\phi} + 6H\dot{\phi} - \frac{3}{2\phi}\dot{\phi}^2\right)\Psi + \dot{\phi}\left(\dot{\Psi} - 3\dot{\Phi}\right) - \frac{\phi}{3}\delta R\,. \qquad (19)$$

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Matter dominated cosmology

- Consider the formation of structure in the matter-dominated universe, where $w = c_s^2 = 0$ (assume scales deep inside the Hubble radius: so called quasi-static approximation).
- Combining the continuity and the Euler equation in this approximation, one finally obtains:

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\text{eff}}\,\rho_m \delta\,, \tag{20}$$

with

$$G_{\text{eff}} \equiv \frac{1 - \frac{1}{3}\phi}{1+\phi} G\,. \tag{21}$$

- Equation (20) provides a very simple approximation to track the growth of structure accurately within the linear regime during matter dominated cosmology.
- **Work to be done**: Confrontations of specific model predictions with the present large scale structure data and forecasts for the constraints from future experiments (ex. EUCLID mission).

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Vacuum fluctuations

- The propagation of the scalar degree of freedom in vacuum is also a crucial consistency check on the theory.
- Set $\rho_m = 0$, and consider the curvature perturbation in the uniform-field gauge ζ. In terms of the Newtonian gauge perturbations this is

$$\zeta = \Phi - \frac{H}{\dot{\phi}}\varphi\,. \tag{22}$$

- We obtain the exact (linear) evolution equation (tedious algebra):

$$\ddot{\zeta} + \left[3H - 2\frac{\ddot{\phi} + 2\dot{H}(1+\phi) - \frac{\dot{\phi}^2}{1+\phi}}{\dot{\phi} + 2H(1+\phi)} + \frac{\phi(1+\phi)}{\dot{\phi}^2} \times \right.$$
$$\left. \times \left(\frac{2\ddot{\phi}\dot{\phi}}{\phi(1+\phi)} + \frac{\dot{\phi}^3(1+\phi)^2\phi}{1-\phi^3(1+\phi)^3}\right)\right]\dot{\zeta} = -\frac{k^2}{a^2}\zeta\,. \tag{23}$$

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- Equation (23) can be used to study generation of fluctuations in hybrid-gravity-inflation.
- **Work to be done –** Construction of specific models and their observational tests:
  - the Einstein-frame formulation might present a convenient starting point for that as it, given the function $f(\mathcal{R})$, presents directly the relevant inflationary potential in terms of the canonic field.

Consistency at galactic scales

- "The virial theorem and the dark matter problem in hybrid metric-Palatini gravity,"
  (Capozziello, Harko, Koivisto, FL, Olmo, JCAP 2013)
- "Galactic rotation curves in hybrid metric-Palatini gravity,"
  (Capozziello, Harko, Koivisto, FL, Olmo, Astroparticle Physics 2013)

More General Hybrid Metric-Palatini Theories

- The “hybrid” theory space is a priori large. In addition to the metric and its Levi-Civita connection, one also has an additional independent connection as a building block to construct curvature invariants from.
- Thus one can consider various new terms such as

$$\hat{R}^{\mu\nu}\hat{R}_{\mu\nu}\,, \quad R^{\mu\nu}\hat{R}_{\mu\nu}\,, \quad \hat{R}^{\mu\nu\alpha\beta}\hat{R}_{\mu\nu\alpha\beta}\,, \quad R^{\mu\nu\alpha\beta}\hat{R}_{\mu\nu\alpha\beta}\,, \quad \mathcal{R}R\,, \quad \textit{etc}. \tag{24}$$

- Though an exhaustive analysis of such hybrid theories has not been performed, there is some evidence that the so called hybrid class of theories presented here is a unique class of viable higher order hybrid gravity theories.
- Verify that these theories are not inhabited by ghosts, superluminalities or other unphysical degrees of freedom.

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Summary and Conclusion

1. While dark gravity offers an alternative explanation to the standard cosmological model for the expansion history of the universe, it offers a paradigm for nature fundamentally distinct from dark energy models of cosmic acceleration, even those that perfectly mimic the same expansion history.
2. It is fundamental to understand how one may differentiate these modified theories of gravity from dark energy models: Through structure formation.
3. Tests from the solar system, large scale structure and lensing essentially restrict the range of allowed modified gravity models.

1. Surveys such as the EUCLID space telescope, the Square Kilometre Array (SKA) radio telescope, the Dark Energy Survey (DES), and the Extended Baryon Oscillation Spectroscopic Survey (eBOSS) as part of the Sloan Digital Sky Survey III (SDSS) will provide new opportunities to test the different cosmological models.
2. Indeed, with the wealth of unprecedented high precision observational data that will become available by these upcoming and planned surveys, we are dawning in a golden age of cosmology, which offers a window into understanding the perplexing nature of the cosmic acceleration, dark matter and of gravity itself.

THANK YOU FOR YOUR TIME AND ATTENTION!