Strong cosmic censorship in spherical symmetry

José Natário

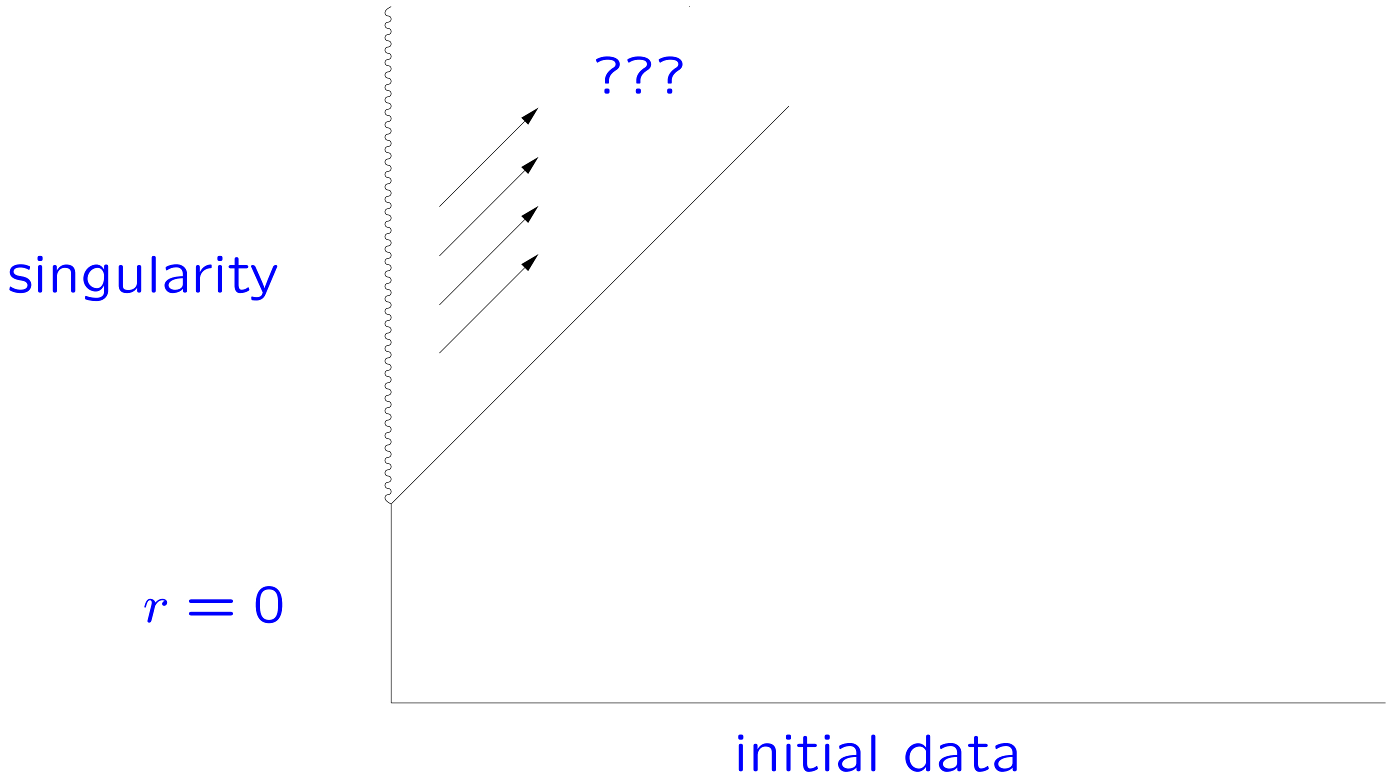
(Instituto Superior Técnico, Lisbon)

Lisbon, December 2015

Outline

- Strong cosmic censorship
- Einstein-Maxwell-scalar field equations in spherical symmetry
- Christodoulou's results
- Dafermos' results
- Our results

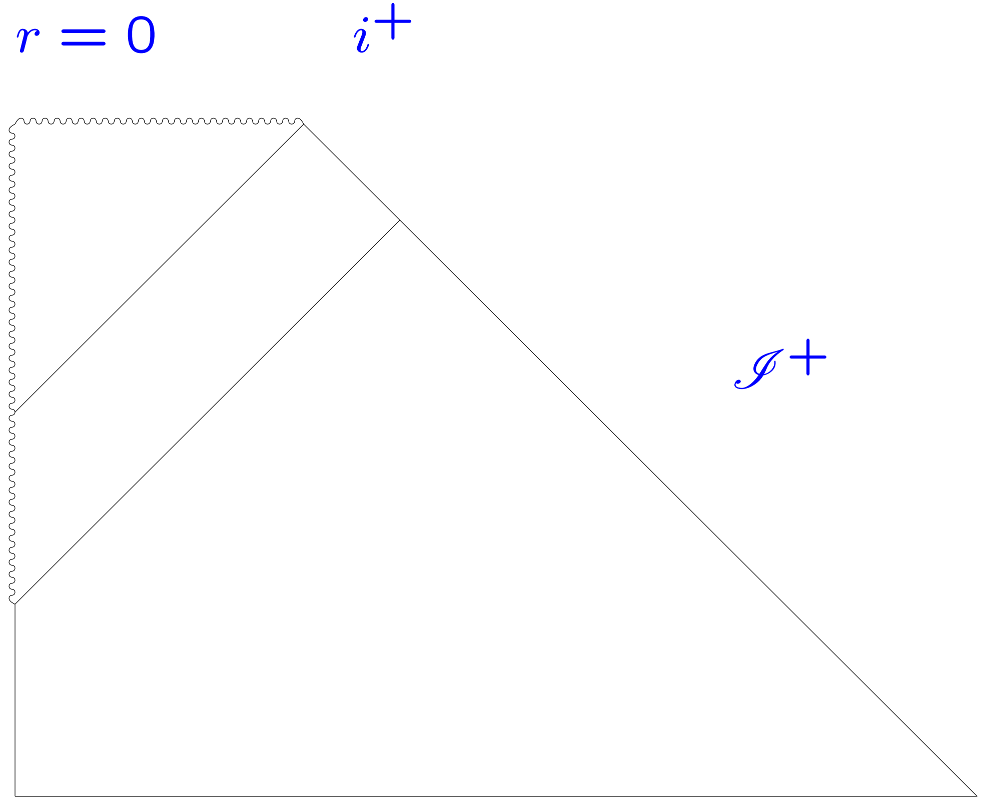
Strong cosmic censorship



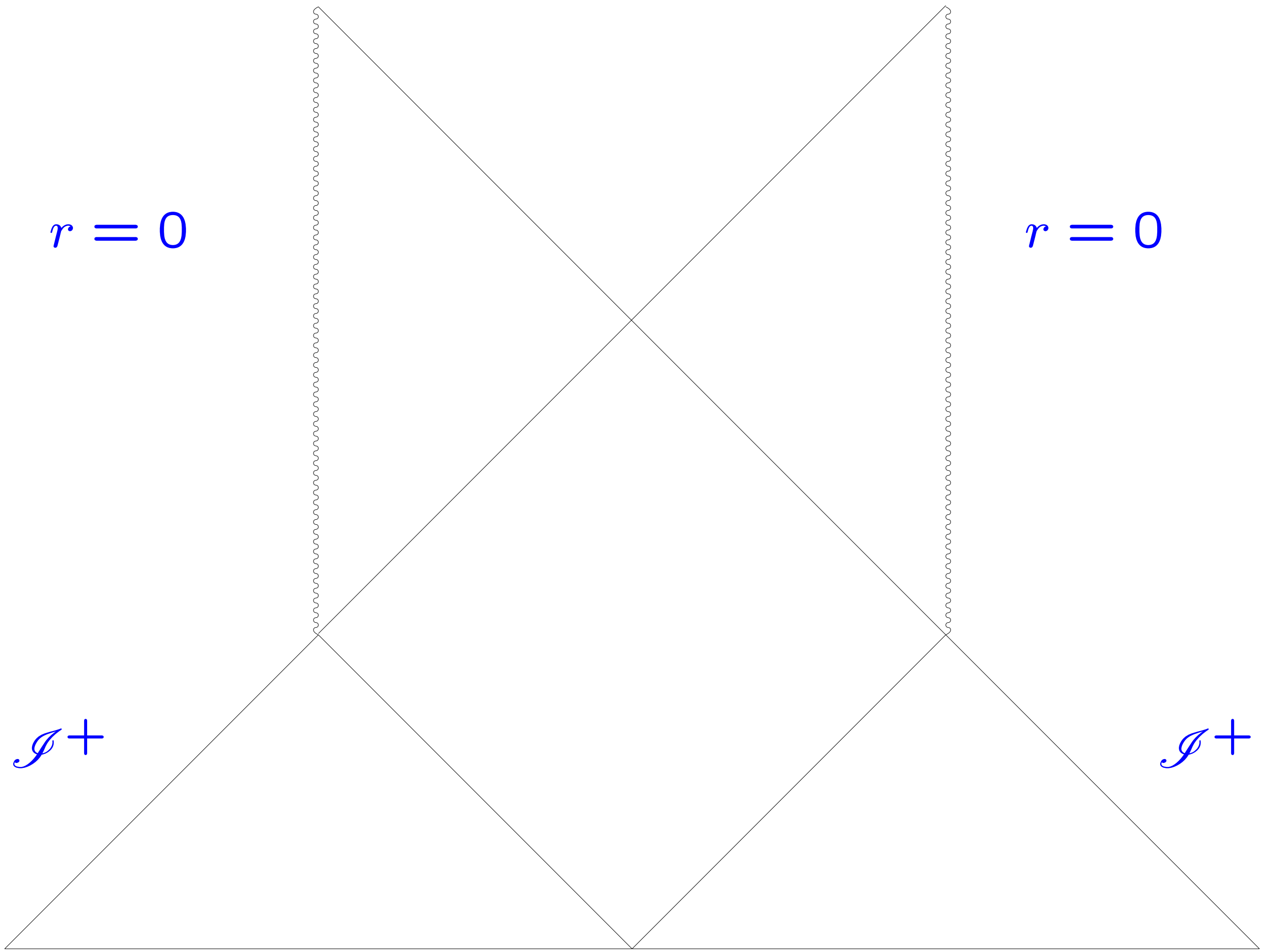
- The appearence of a visible singularity destroys determinism: the singularity can radiate gravitationally or otherwise (mathematically it is a singular boundary).

- Obvious conjecture: the singularities that form are never locally naked, that is, visible by some observer.

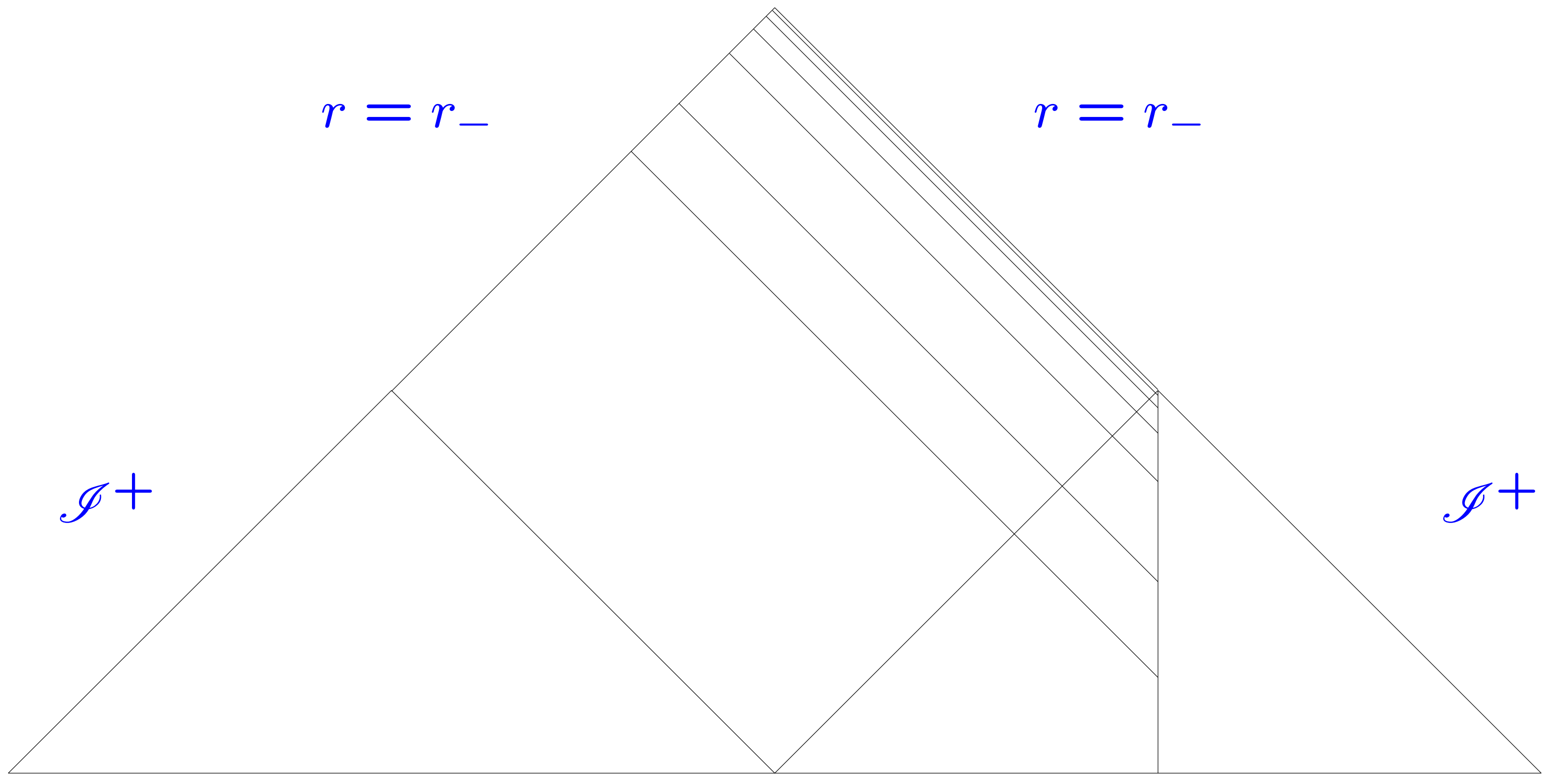
- Well known to be false: counter-examples are the dust cloud solutions of Christodoulou, or the Reissner-Nordström solution.

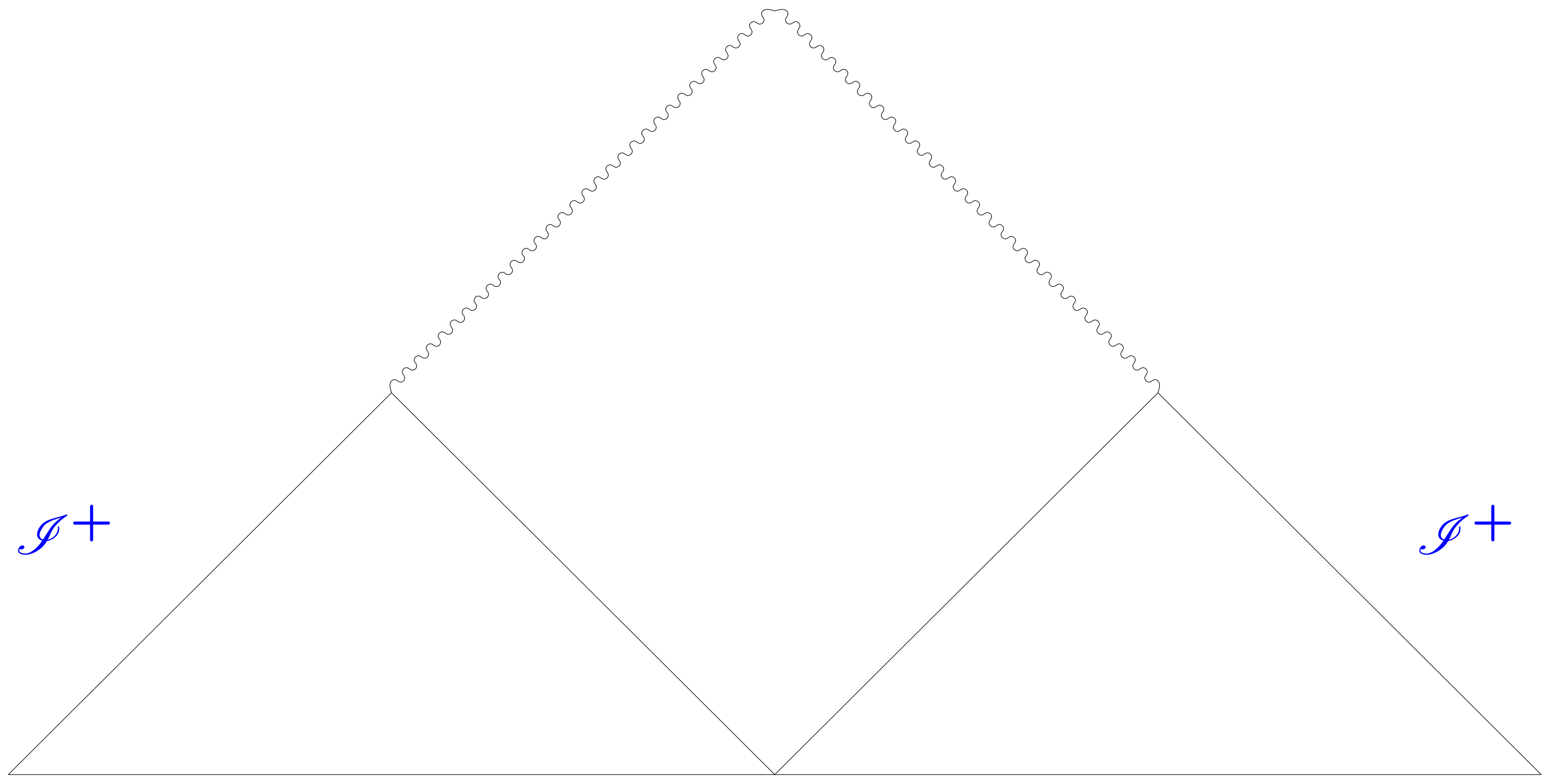


$r=0$
i^+
$\mathscr{I}^+$


$r = 0$
$r = 0$
$\mathscr{I}^+$
$\mathscr{I}^+$

- However, the blueshift effect should make the Cauchy horizon unstable:





$\mathscr{I}^+$
$\mathscr{I}^+$

- Corrected conjecture: For generic solutions of the Einstein field equations with reasonable matter models, the singularities that form are never locally naked.

- PDE version: For generic asymptotically flat or compact initial data the future maximal globally hyperbolic development is inextendible.

Einstein-Maxwell-scalar field equations in spherical symmetry

- Birkhoff's theorem: there are no gravitational degrees of freedom in spherical symmetry.

- Simplest hyperbolic matter model: massless scalar field.

- Simplest spherically symmetric solution containing a Cauchy horizon: Reissner-Nordström (electromagnetic field).

- The equations for a gravitating massless scalar field ϕ in a sourceless electromagnetic field F with a cosmological constant Λ are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 2T_{\mu\nu}$$

$$T_{\mu\nu} = \partial_\mu \phi \, \partial_\nu \phi - \frac{1}{2} \partial_\alpha \phi \, \partial^\alpha \phi \, g_{\mu\nu} + F_{\mu\alpha} F_\nu{}^\alpha - \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} g_{\mu\nu}$$

$$\Box \phi = 0$$

$$dF = d^* F = 0$$

(using units for which $c = 4\pi G = \varepsilon_0 = 1$)

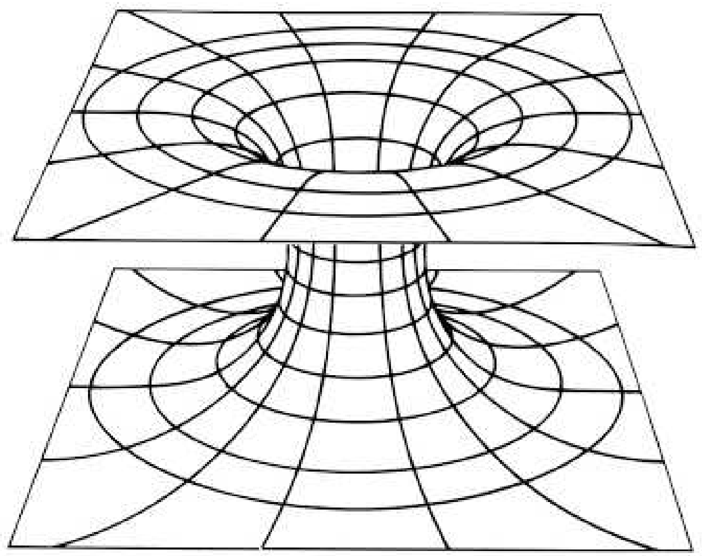
- If we impose spherical symmetry then the metric and the fields become

$$ds^2 = -\Omega^2(u,v)du\,dv + r^2(u,v)\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right)$$
$$\phi = \phi(u,v)$$
$$F = -E(u,v)\frac{\Omega^2(u,v)}{2}du \wedge dv$$

- In particular the electromagnetic field completely decouples:

$$^\star F = E(u,v)r^2(u,v)\sin\theta\,d\theta\wedge d\varphi \;\Rightarrow\; E(u,v) = \frac{e}{r^2(u,v)}$$

- The total charge $4\pi e$ is topological: initial surface $t = 0$ in Reissner-Nordström is



- Introducing the renormalized Hawking mass ϖ through

$$1-\frac{2\varpi}{r}+\frac{e^2}{r^2}-\frac{\Lambda}{3}r^2=-\frac{4\partial_u r\partial_v r}{\Omega^2}=(\mathrm{grad}\,r)^2$$

the Einstein-Maxwell-scalar field equations become

$$\partial_u\partial_v\phi=-\frac{\partial_u r\,\partial_v\phi}{r}-\frac{\partial_v r\,\partial_u\phi}{r}$$

$$\partial_u\partial_v r=\partial_u r\,\partial_v r\,\frac{\frac{2\varpi}{r^2}-\frac{2e^2}{r^3}-\frac{2\Lambda}{3}r}{1-\frac{2\varpi}{r}+\frac{e^2}{r^2}-\frac{\Lambda}{3}r^2}$$

$$\partial_u\varpi=\left(1-\frac{2\varpi}{r}+\frac{e^2}{r^2}-\frac{\Lambda}{3}r^2\right)\frac{(r\partial_u\phi)^2}{2\partial_u r}$$

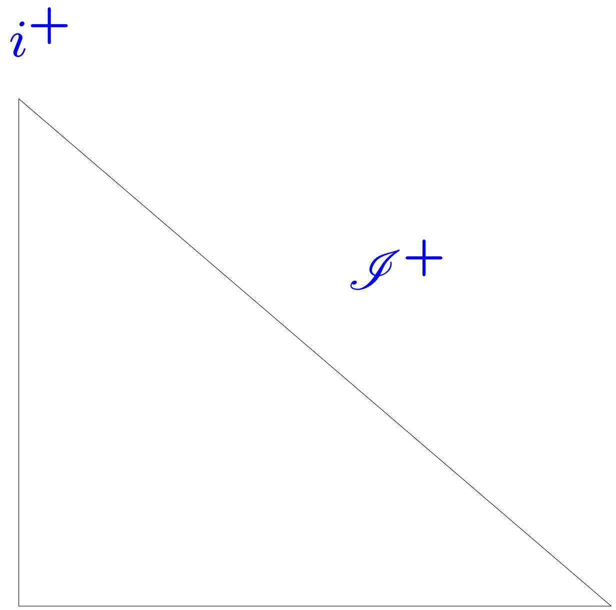
$$\partial_v\varpi=\left(1-\frac{2\varpi}{r}+\frac{e^2}{r^2}-\frac{\Lambda}{3}r^2\right)\frac{(r\partial_v\phi)^2}{2\partial_v r}$$

Christodoulou's results

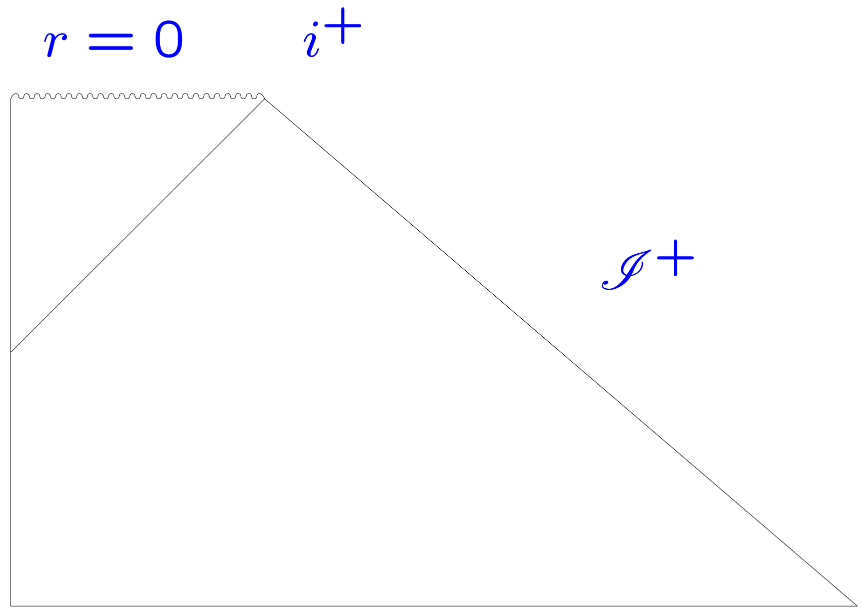
- Christodoulou (1999): Strong cosmic censorship is true for the Einstein-scalar field system ($e = \Lambda = 0$) in the C^0 formulation.

- Generic: dispersive or black hole.

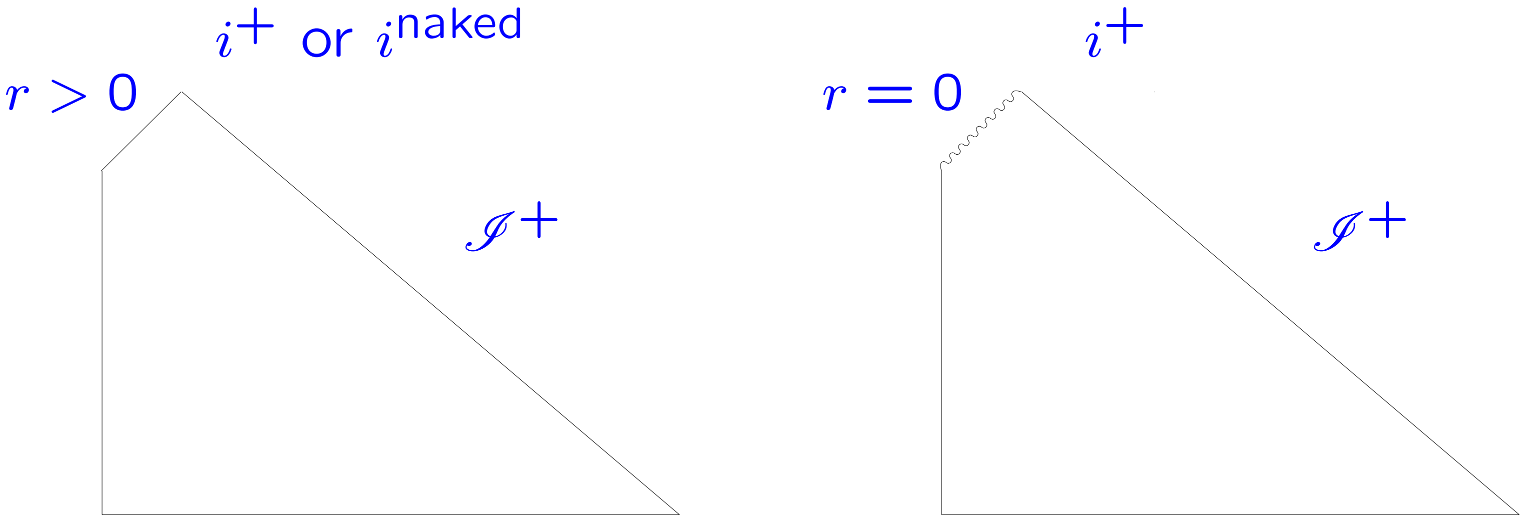
- Non-generic: light cone singularity (possibly naked), collapsed light cone singularity, black hole with light cone singularity, black hole with collapsed light cone singularity.

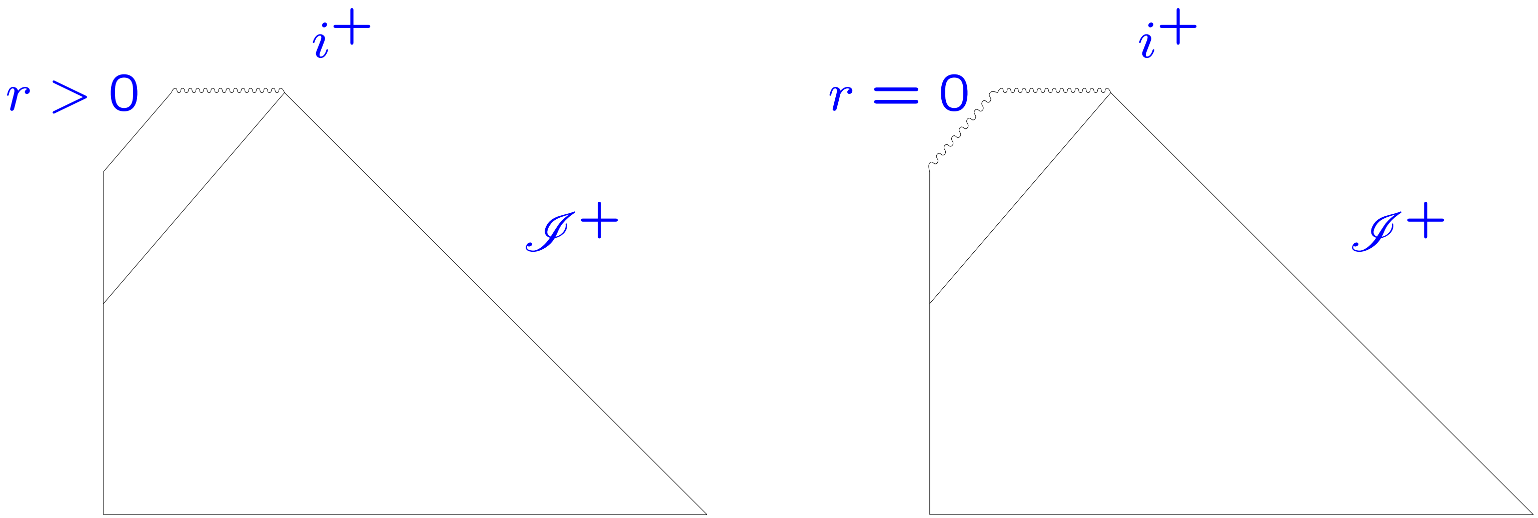


i^+
$\mathscr{I}^+$



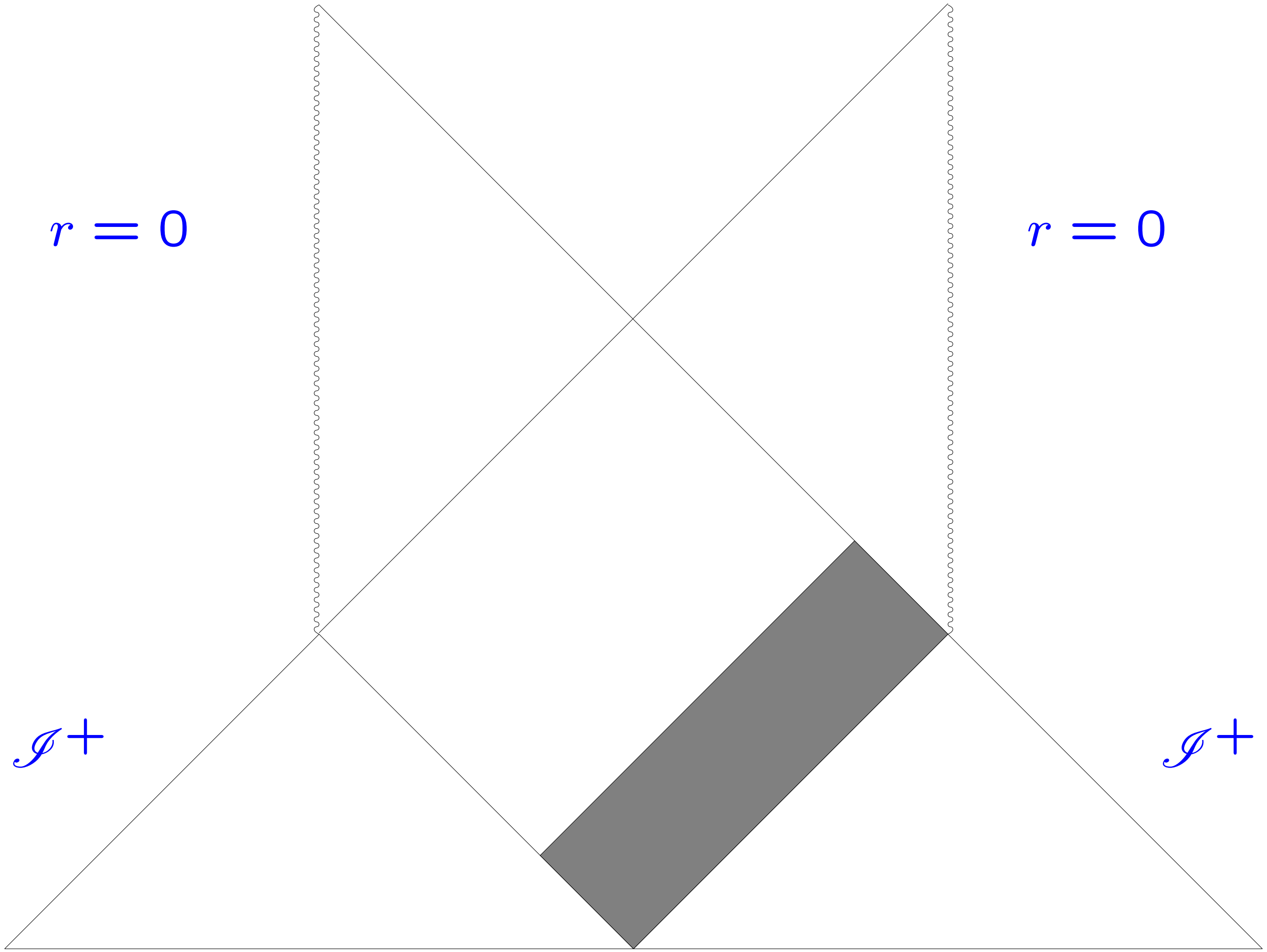
$r = 0$
i^+
$\mathscr{I}^+$


i^+ or i^{naked}
$r > 0$
$\mathscr{I}^+$
i^+
$r = 0$
$\mathscr{I}^+$

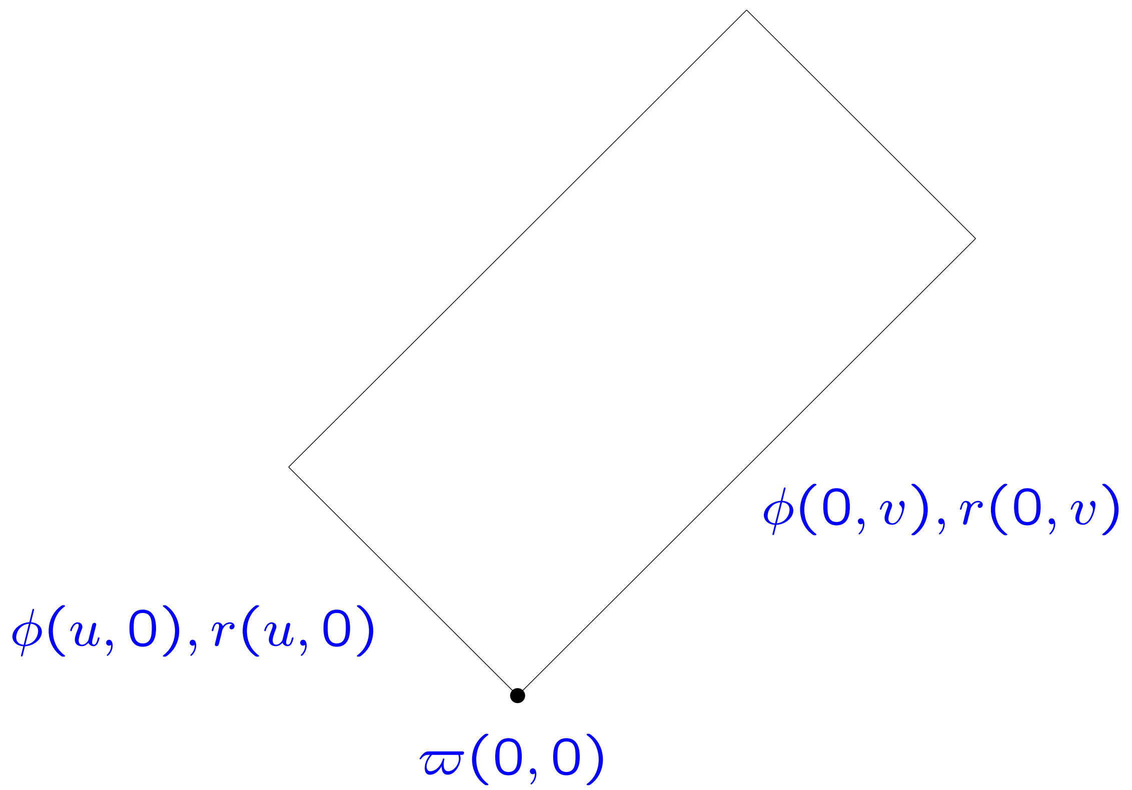


i^+
$r > 0$
$\mathscr{I}^+$
i^+
$r = 0$
$\mathscr{I}^+$

Dafermos' results



- Characteristic initial data:



- Poisson and Israel (1989) gave a nonlinear heuristic analysis suggesting that for $\Lambda = 0$ the Cauchy horizon of generic solutions ($\phi \neq 0$) still has $r \sim r_-$, but $\varpi \to +\infty$ (mass inflation).

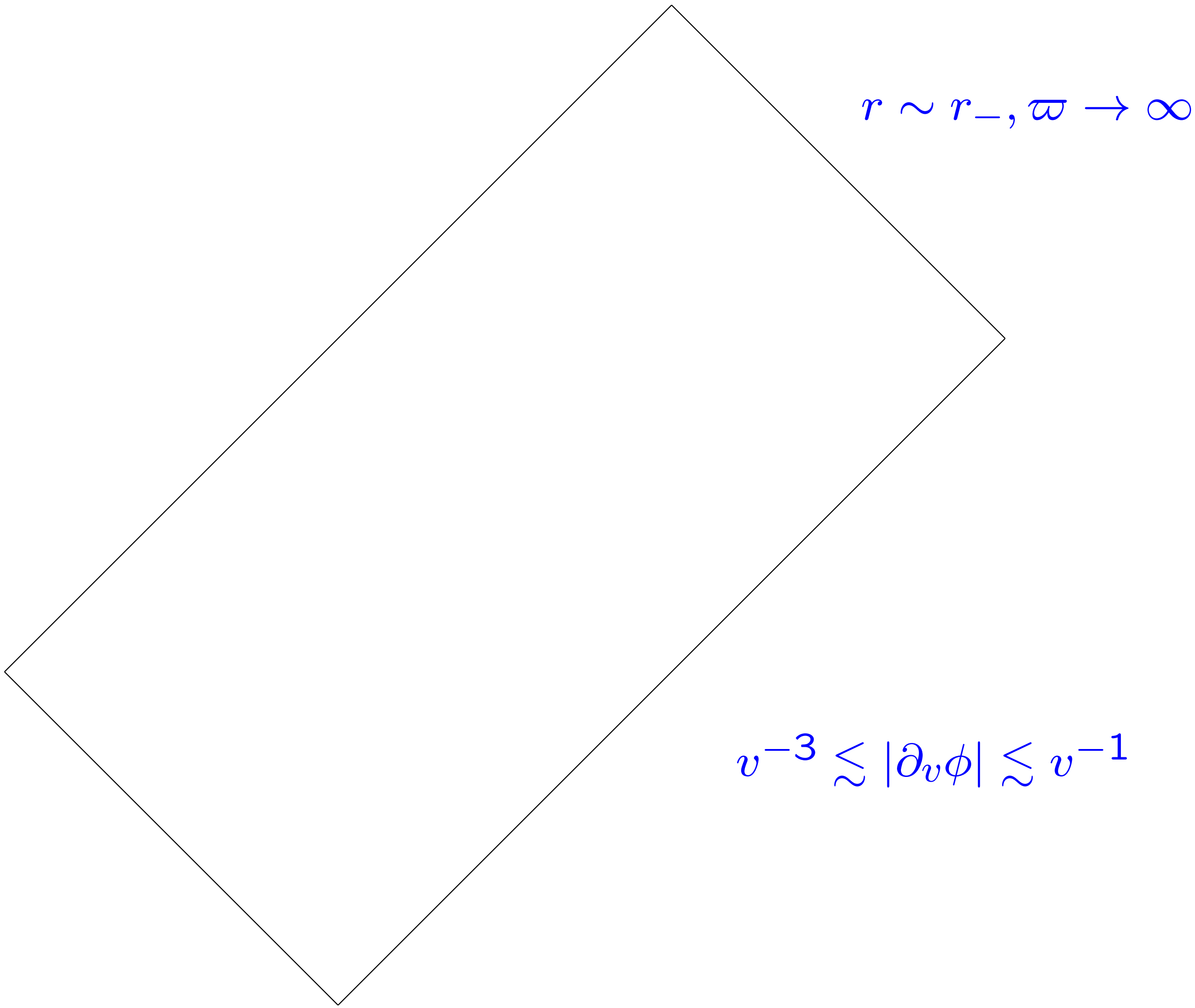
- Brady, Moss and Myers (1998) performed a linear analysis suggesting that mass inflation might not occur for $\Lambda > 0$ near extremality (but the curvature still blows up at the Cauchy horizon).

- Dafermos (2005) proved the following two results for the spherically symmetric Einstein-Maxwell-scalar field system.

1. If $|\partial_v \phi| \lesssim v^{-1}$ along the event horizon then r can be extended to a nonvanishing continuous function on the Cauchy horizon, and so the metric can be extended as a C^0 metric.

2. If $v^{-3} \lesssim |\partial_v \phi| \lesssim v^{-1}$ along the event horizon then the Hawking mass blows up identically along the Cauchy horizon, and so the metric is inextendible as a C^1 metric.

- The first hypothesis (Price's law) was subsequently proved to occur by Dafermos and Rodnianski (2005).

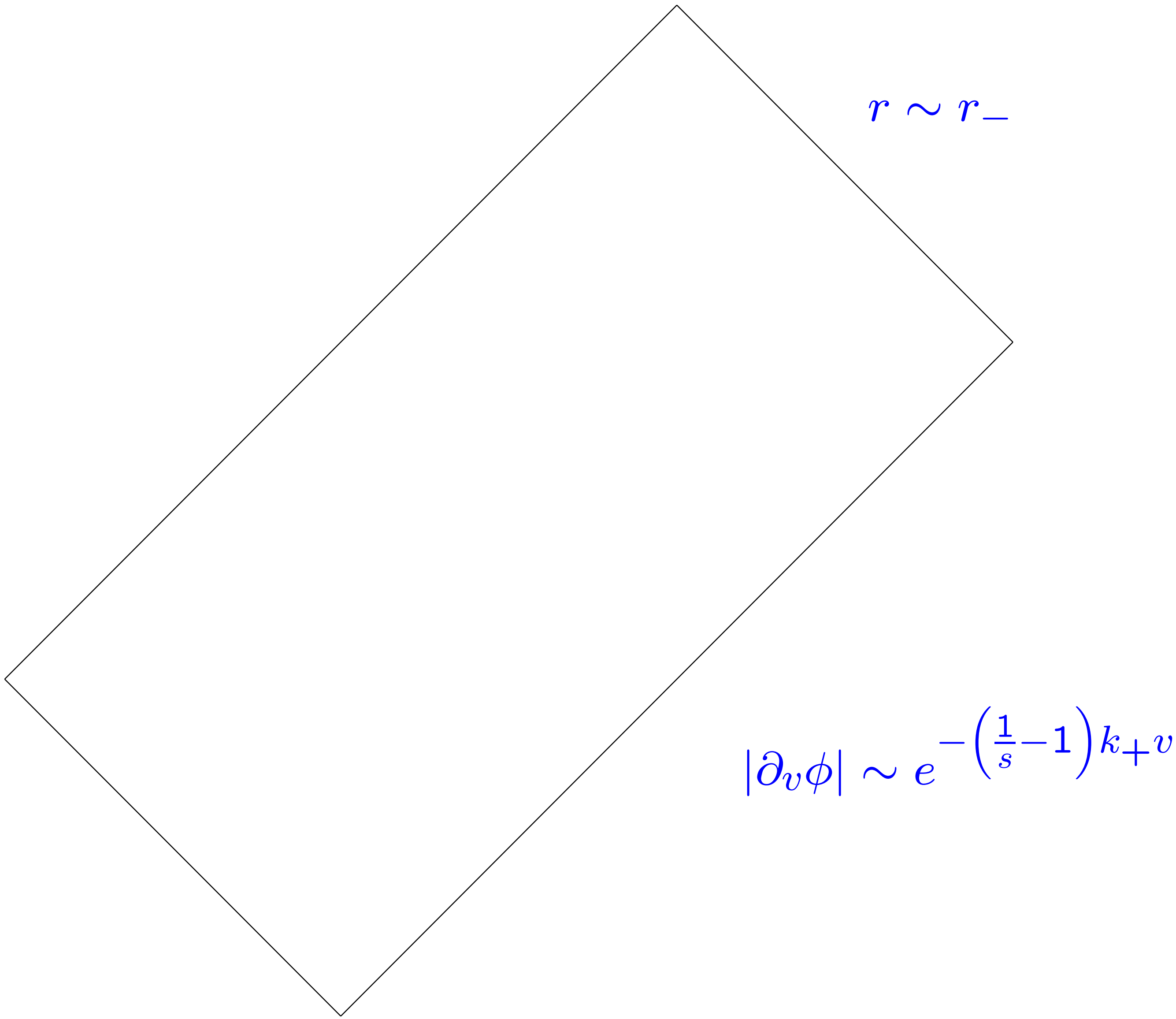

$r \sim r_-, \varpi \to \infty$
$v^{-3} \lesssim |\partial_v \phi| \lesssim v^{-1}$

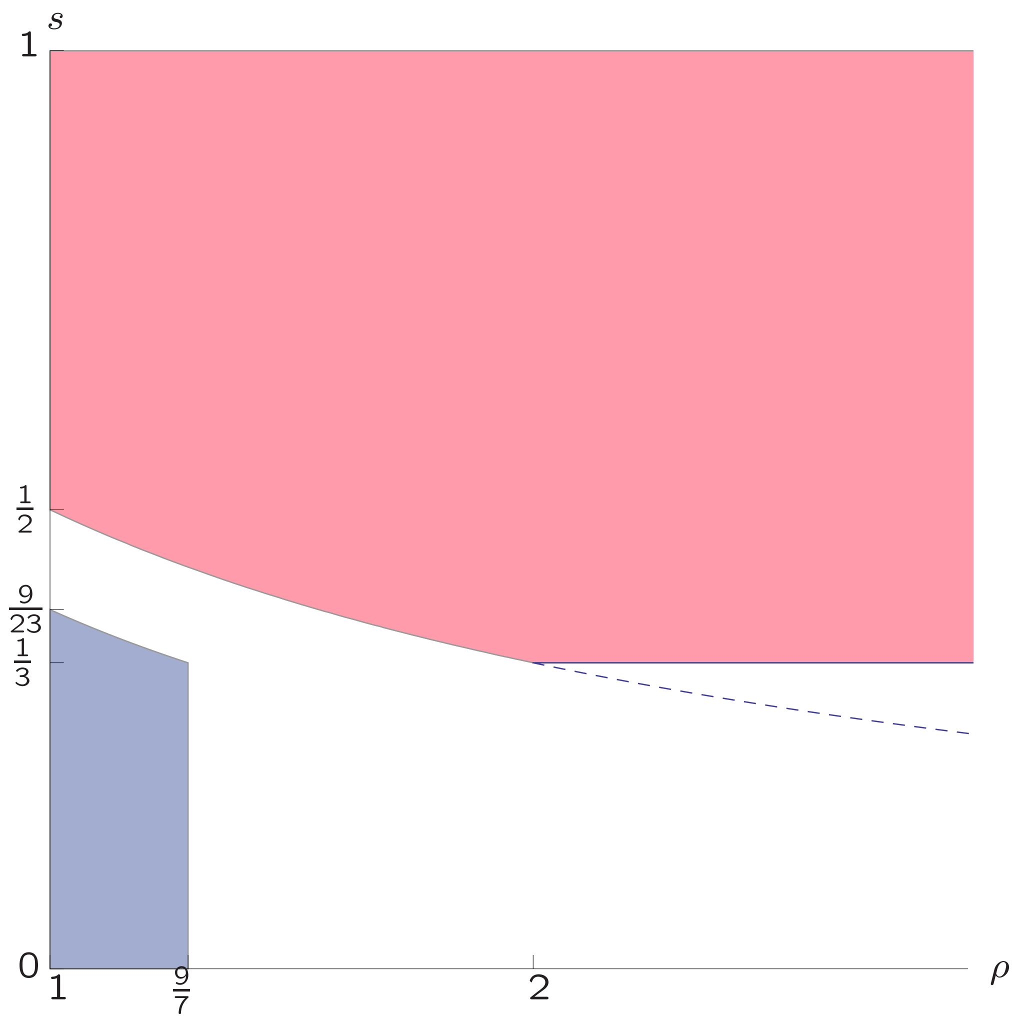
Our results

- We (João Costa, Pedro Girão, J. N., Jorge Drumond Silva) consider the case $|\partial_v \phi| \sim e^{-\left(\frac{1}{s}-1\right)k_+ v}, 0 < s < 1$, for any Λ.

- r can always be extended to a nonvanishing continuous function on the Cauchy horizon, and so the metric can be extended as a C^0 metric.

- Mass inflation depends on s and $\rho = \frac{k_-}{k_+} > 1$ (we exclude the extremal case $\rho = 1$).


$r \sim r_-$
$|\partial_v \phi| \sim e^{-\left(\frac{1}{s}-1\right)k_+ v}$



s
1
1/2
9/23
1/3
0
1
9/7
2
ρ

- For $\Lambda > 0$ one expects an exponential decay in Price's law, which can be as fast as $|\partial_v \phi| \sim e^{-2k+v}$, that is, $s = \frac{1}{3}$.

- So it is likely that there is no mass inflation near extremality.

- However, the Kretschmann scalar blows up for $s > \frac{1}{3}$, and so the metric is inextendible as a C^2 metric.