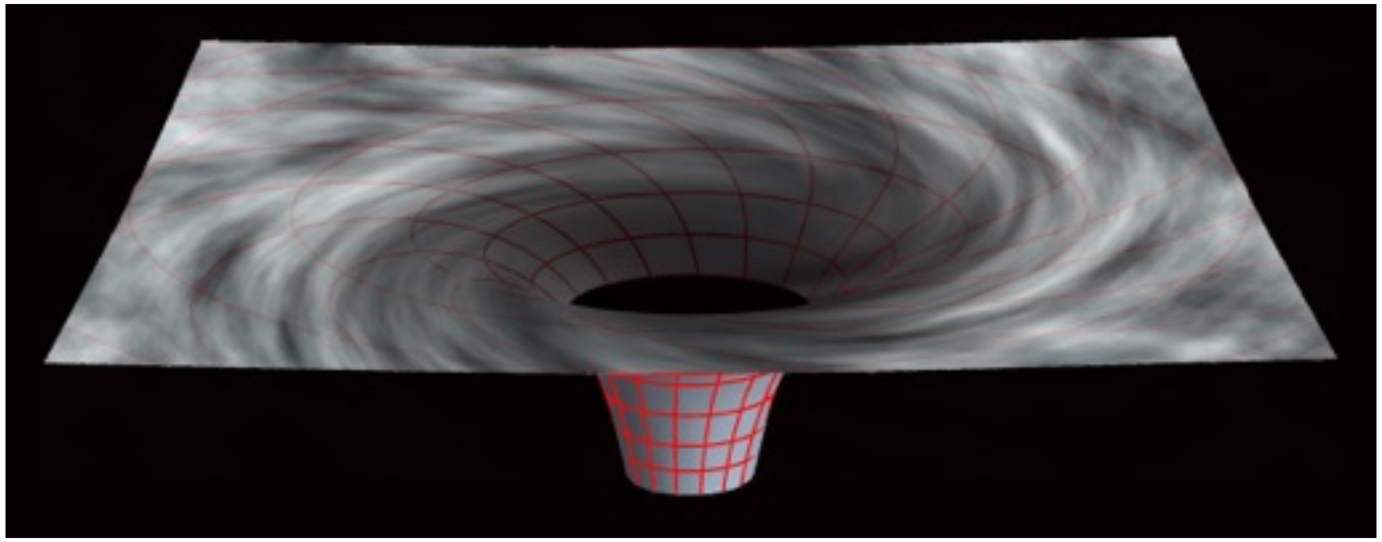
Progress on gravitational collapse with rotation

Jorge V. Rocha (Universitat de Barcelona)



IST, U. Lisboa GR 100 Years in Lisbon **Dec 18th, 2015**

based on:

❖ Phys. Rev. D89 (2014) 10, 104006	**1402.4161 [gr-qc]**
❖ Phys. Rev. D89 (2014) 121501(R)	**1405.1433 [gr-qc]**
❖ Int.J.Mod.Phys. D24 (2015) 09, 1542002	**1501.06724 [gr-qc]**
❖ ongoing work	

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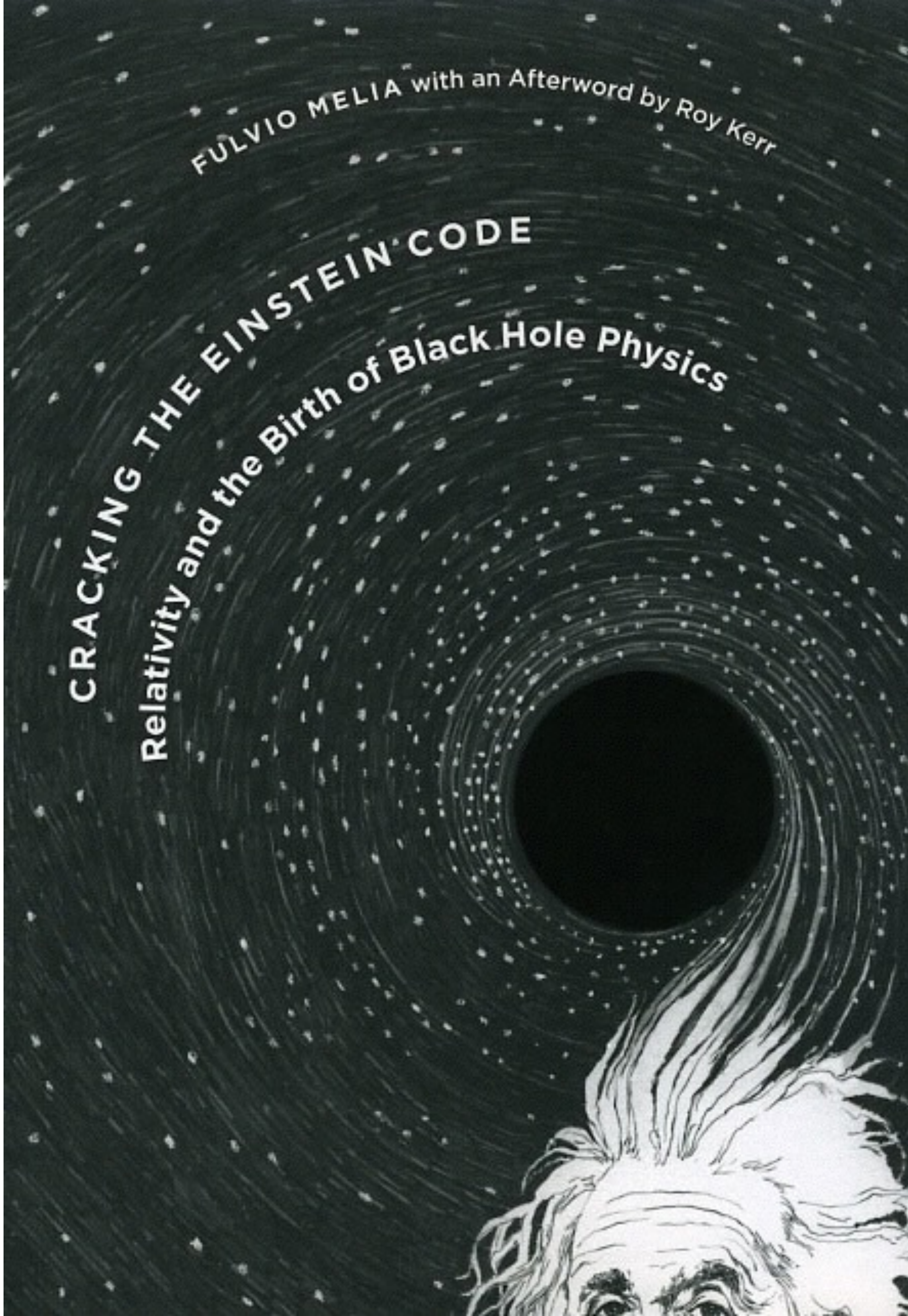

Introduction: Black holes and gravitational collapse

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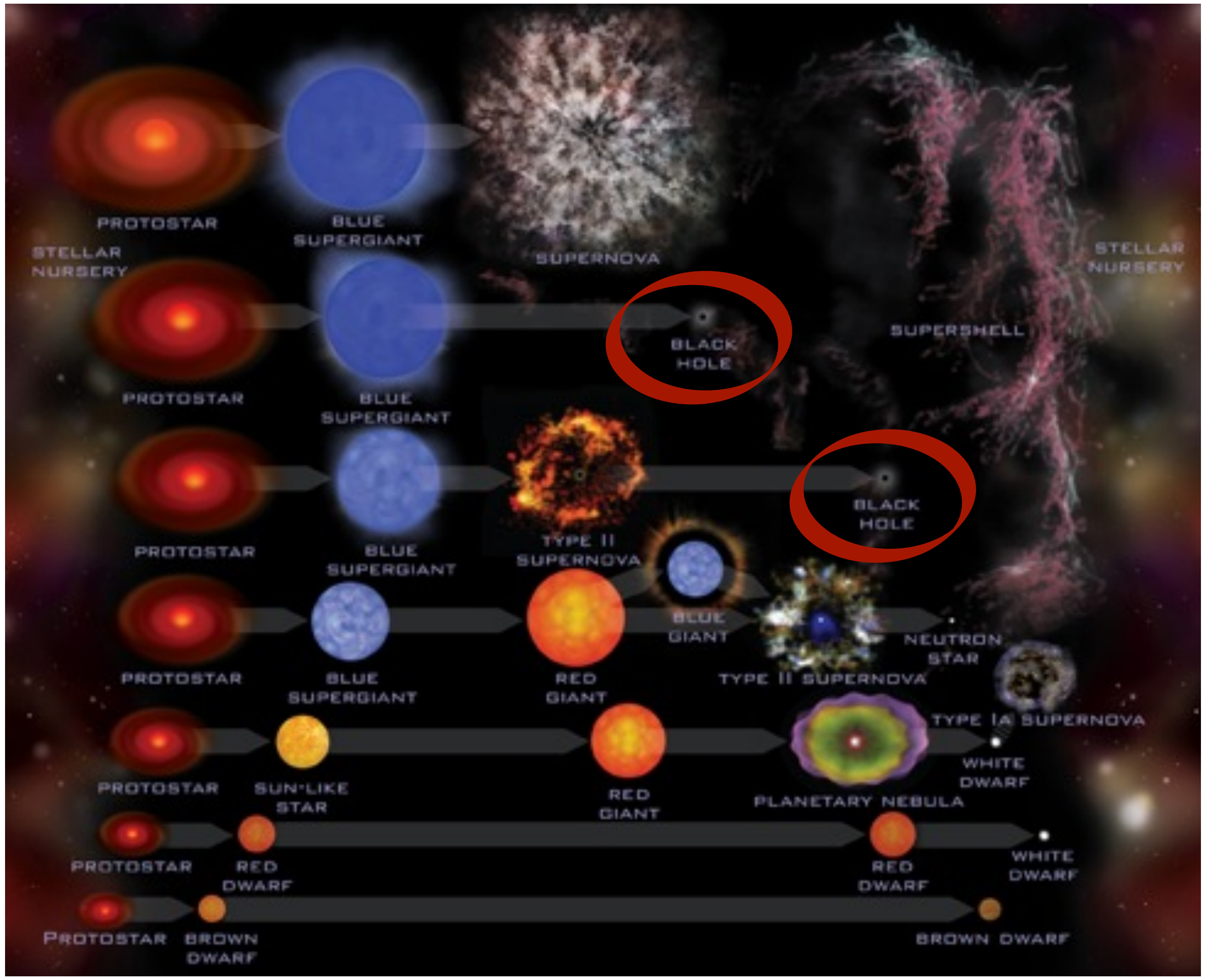

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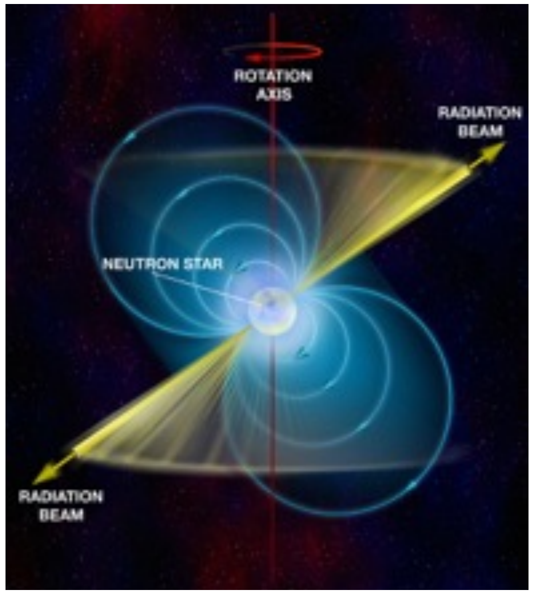
- The vast majority of celestial objects are rotating. Black holes are no exception.

ESO / J. Pérez



ESO

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however see [Nakamura (1981)]
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non-spherical gravitational collapse

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non-spherical gravitational collapse

"the study of non-spherical collapse within exact solutions of Einstein field equations is a field where most of the work still needs to be done"

in "Recent developments in gravitational collapse and spacetime singularities",
P. S. Joshi and D. Malafarina, IJMP D (2012)

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1. realistic collapses should include rotation;
2. known 'violations' of the cosmic censorship conjecture (CCC) occur in non-rotating — thus non-generic — settings;
3. rotation introduces instabilities (e.g., superradiance);

Introduction: (Weak) Cosmic censorship conjecture

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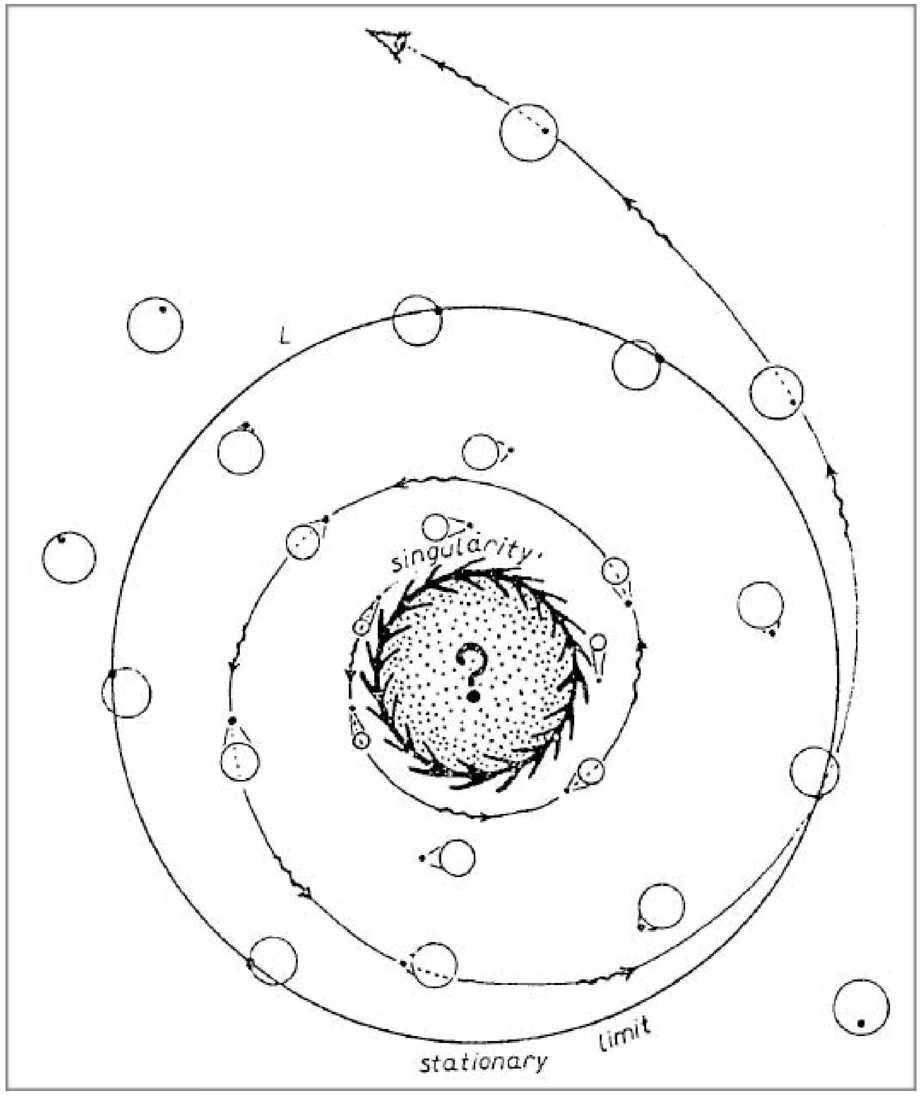

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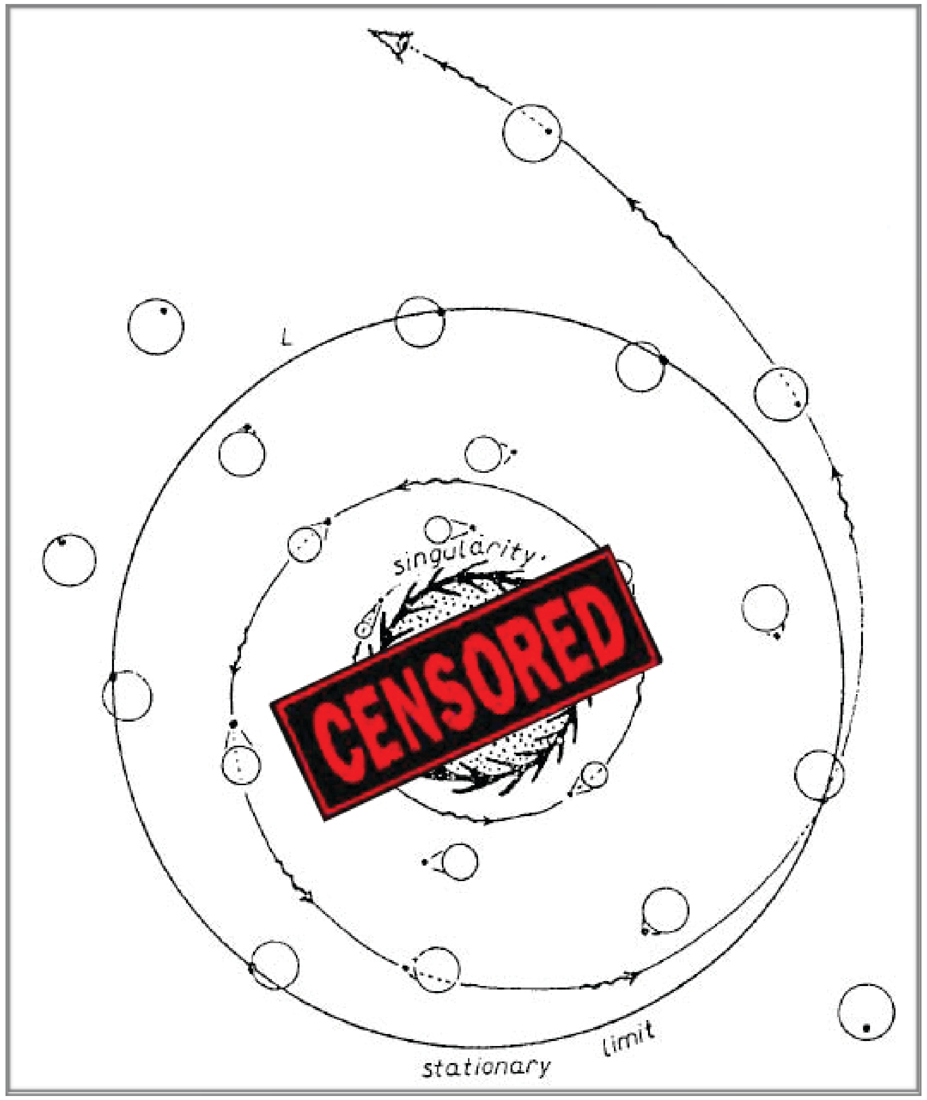

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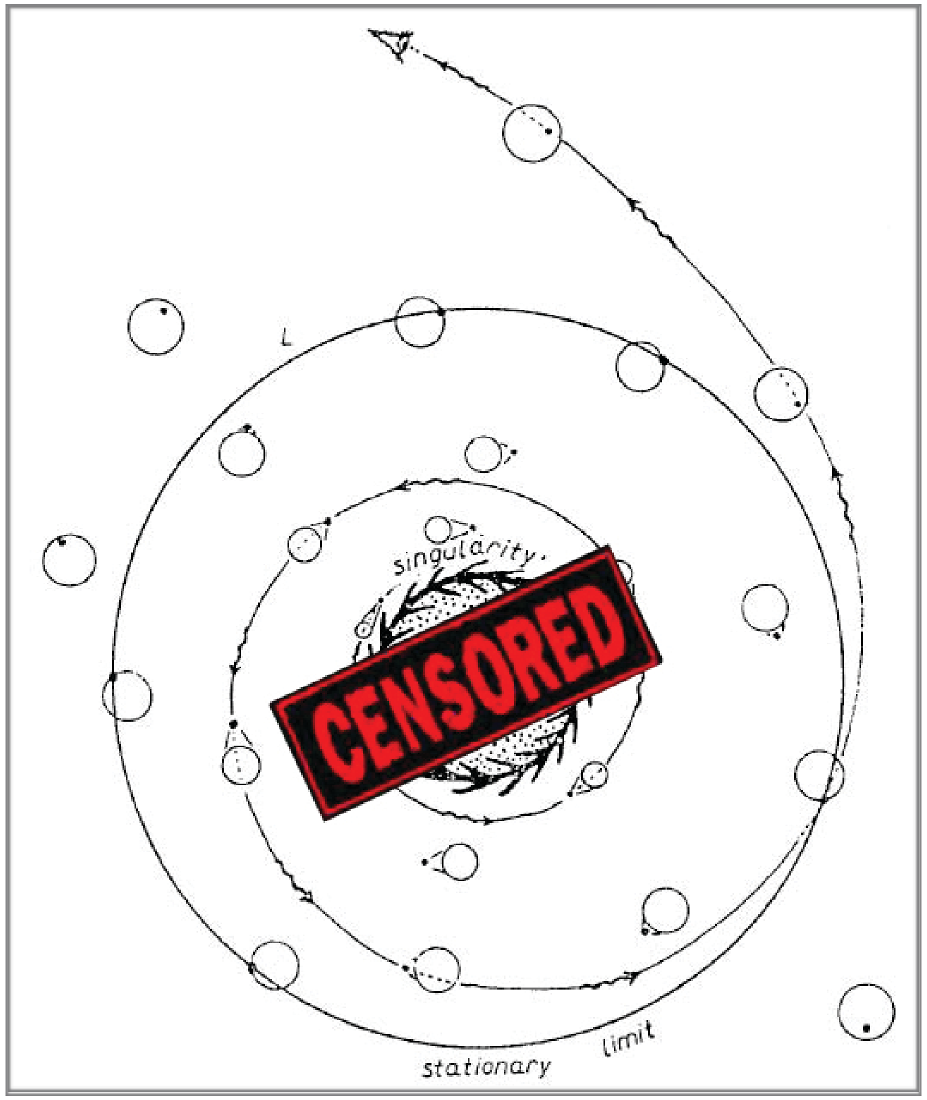
- Still no proof available or unquestionable counter-example (asymptotically flat).



from [Penrose (1969)]

However, see: [Lehner, Pretorius (2010)]
[Dias, Horowitz, Santos (2011)]
[Niehoff, Santos, Way (2015)]
[Green, Hollands, Ishibashi,, Wald (2015)]

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- The price to pay for the convenience provided by cohomogeneity-1 spacetimes is the restriction to higher (odd) dimensions, $D=2N+3$ with $N=1, 2, 3, \ldots$

Outline

✓ Introduction

✦ Background: cohomogeneity-1 black hole spacetimes

✦ Rotating thin shells & cosmic censorship

✦ Conclusion & outlook

Background: Cohomogeneity-1 black holes

- Myers-Perry(-AdS) BHs in $D=2N+3$ dims possess isometry group $\mathbb{R}\times U(1)^{N+1}$.

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- When all spin parameters are set equal, $a_i = a$, this symmetry is enhanced,

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and coordinates can be found that reflect this large amount of symmetry, so that the metric depends on just one (radial) coordinate.

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- n.b. Constant t and r sections are squashed $(2N+1)$-spheres.

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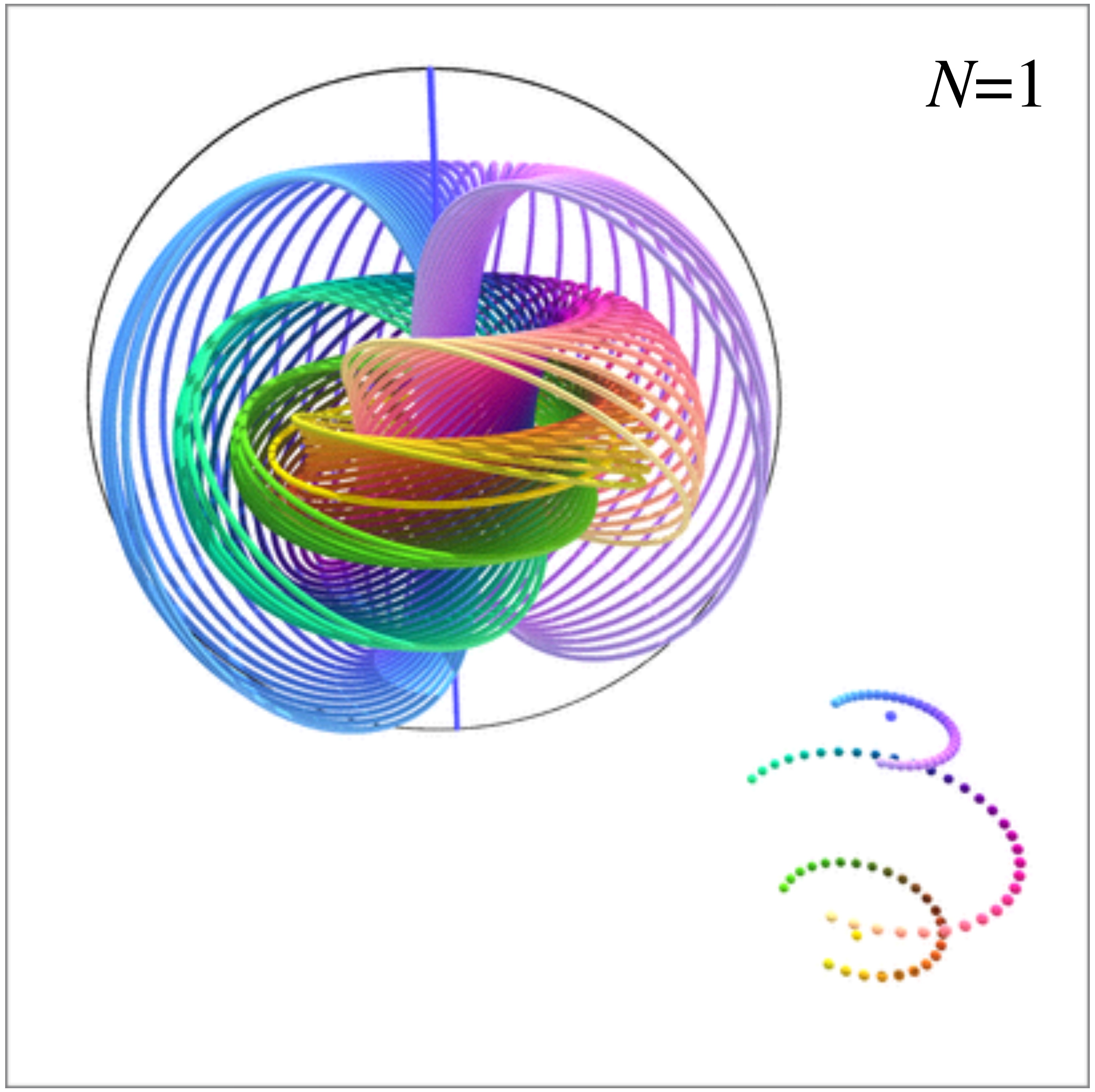

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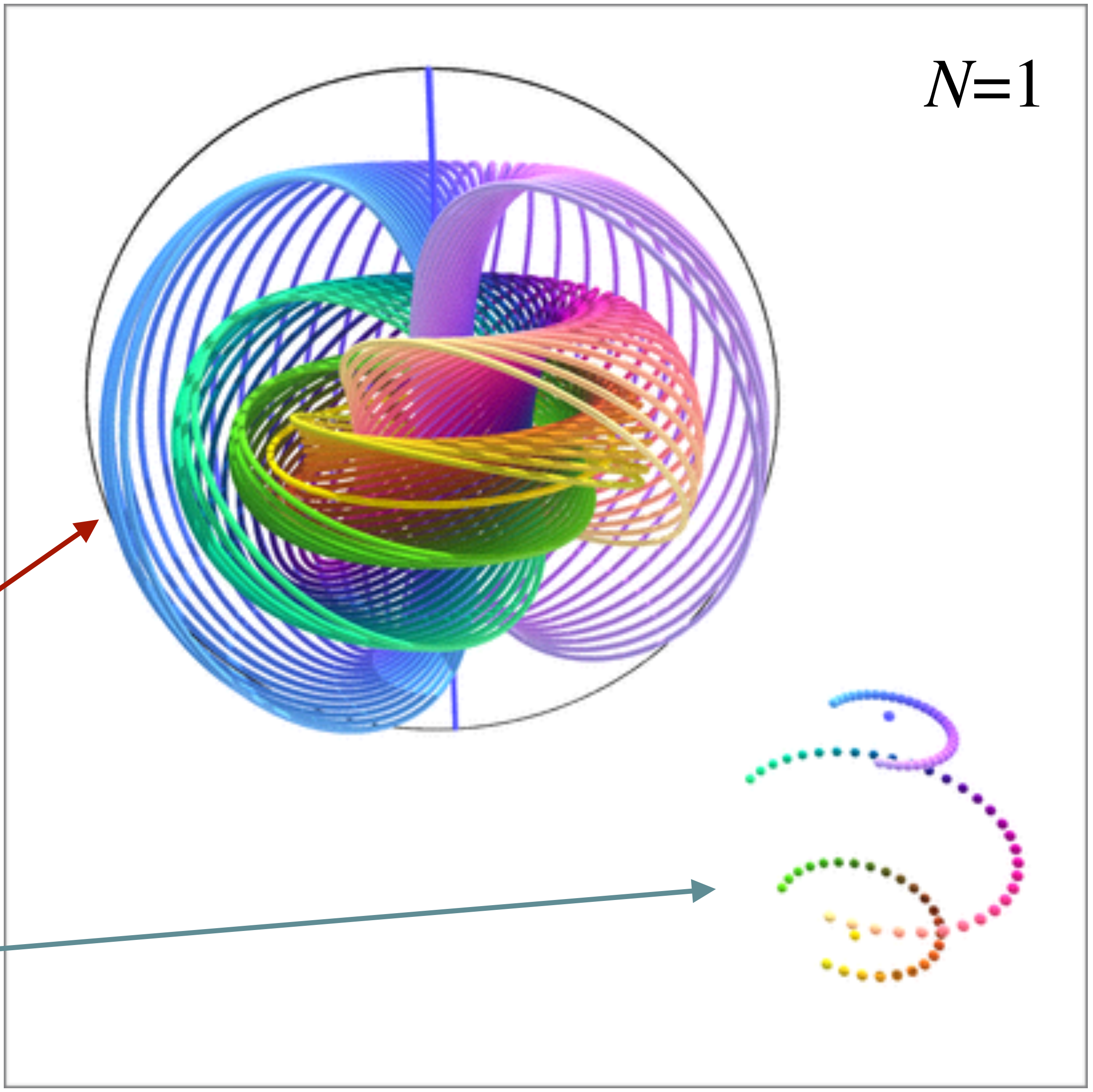
coordinate parametrising the S^1 fibers

$2N$ coordinates on CP^N



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where

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$\widehat{g}_{ab}$ denotes the Fubini-Study metric on CP^N and $A_a dx^a$ is its Kahler potential.

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- n.b. These solutions accommodate a non-vanishing cosmological constant:

$$R_{\mu\nu} = -(D-1)\ell^{-2} g_{\mu\nu}$$

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Rotating thin shells: Shells in cohomogeneity-1 spacetimes

exact approach: Darmois-Israel junction conditions

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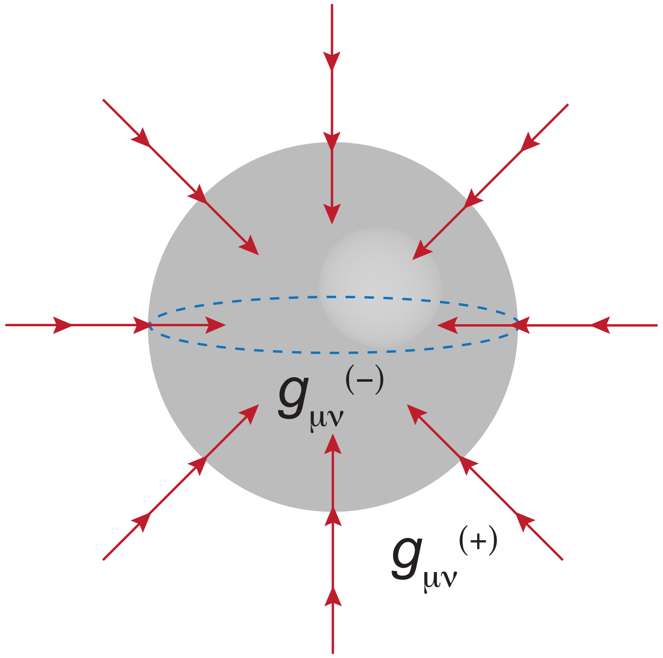

[Israel (1966)]
[Boulware (1973)]
[Peleg, Steiff (1995)]
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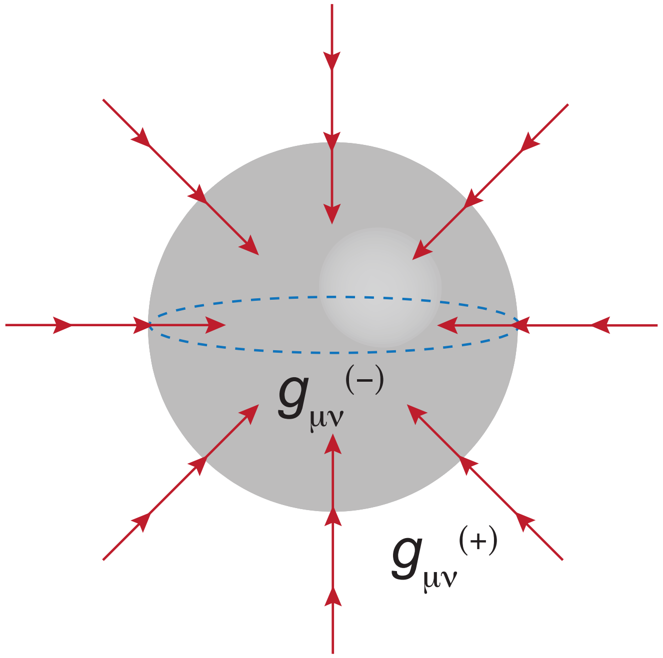

- Previous attempts with rotation have been successful only in the slowly rotating regime or in 2+1 dimensions.

[de la Cruz, Israel (1968)]
[Lindblom, Brill (1974)]
[Musgrave, Lake (1996)]
[Crisóstomo, Olea (2004)]
[Mann, Oh, Park (2009)]

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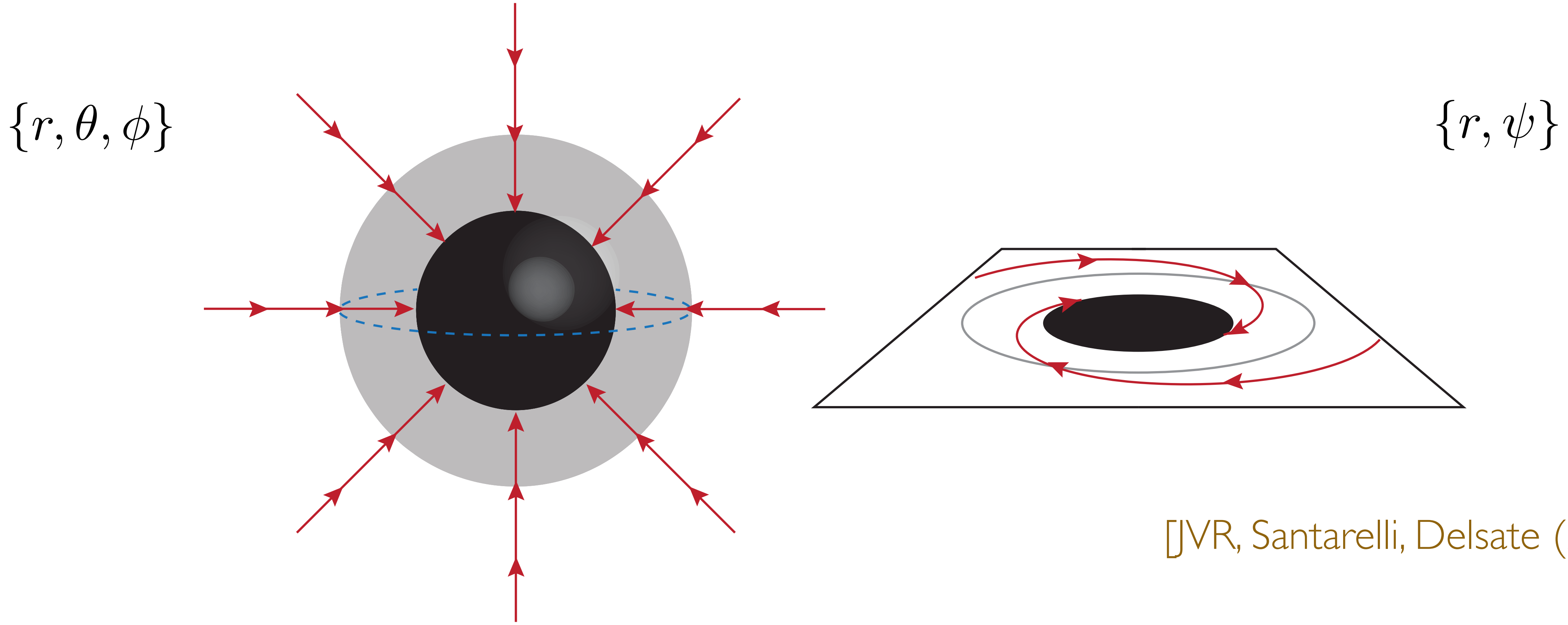
- n.b. The dynamics on the $CP^1 \cong S^2$ and on the S^1 **separate**.
  All traces of the rotation show up in the $\{r, \psi\}$ plane.



[JVR, Santarelli, Delsate (2014)]

Rotating thin shells: Junction conditions I

- Use junction conditions along a timelike hypersurface, $t = \mathcal{T}(\tau),\, r = \mathcal{R}(\tau)$:

[Israel (1966)]

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If we wish to include rotation, we must have a BH (or a star) in the interior geometry.

Rotating thin shells: Junction conditions II

- The 2nd junction condition requires the shell stress-energy tensor to take the form of an imperfect fluid:

$$\mathcal{S}_{ij} = (\rho + P)u_i u_j + P\,\mathfrak{g}_{ij} + 2\varphi\, u_{(i}\xi_{j)} + \Delta P\, \mathcal{R}^2 \widehat{g}_{ij}$$

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energy density

pressure

heat flux / intrinsic momentum

pressure anisotropy

where $u = \partial_\tau$ and $\xi = h(\mathcal{R})^{-1}\partial_\psi$.

Rotating thin shells: Equation of state and shell equation of motion

- The stress-energy tensor components are dictated by the metric components:

$$\rho = -\frac{(\beta_+ - \beta_-)(\mathcal{R}^2 h)'}{8\pi\mathcal{R}^3}$$

$$\varphi = -\frac{(\mathcal{J}_+ - \mathcal{J}_-)(\mathcal{R}h)'}{4\pi^2\mathcal{R}^4 h}$$

$$P = \frac{h}{8\pi\mathcal{R}^3}\left[\mathcal{R}^2(\beta_+ - \beta_-)\right]'$$

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- These equations can be integrated, yielding the shell's equation of motion:

$$\boxed{\dot{\mathcal{R}}^2 + V_{\text{eff}}(\mathcal{R}) = 0}$$

Rotating thin shells: Effective potential for shell equation of motion

- For generic values of N, and a linear equation of state:

$$\boxed{\dot{\mathcal{R}}^2 + V_{\text{eff}}(\mathcal{R}) = 0}$$

$$\begin{aligned} V_{\text{eff}}(\mathcal{R}) &= 1 + \frac{\mathcal{R}^2}{\ell^2} + \frac{2Ma^2}{\ell^2 \mathcal{R}^{2N}} + \frac{2Ma^2}{\mathcal{R}^{2N+2}} - \frac{M_+ + M_-}{\mathcal{R}^{2N}} \\ &\quad - \left(\frac{M_+ - M_-}{m_0}\right)^2 \left(\frac{\mathcal{R}^{2N}}{m_0}\right)^{\frac{2N+1}{N}w} \left(1 + \frac{2Ma^2}{\mathcal{R}^{2N+2}}\right)^{w-1} \\ &\quad - \frac{1}{4}\left(\frac{m_0}{\mathcal{R}^{2N}}\right)^{2+\frac{2N+1}{N}w} \left(1 + \frac{2Ma^2}{\mathcal{R}^{2N+2}}\right)^{1-w} . \end{aligned}$$

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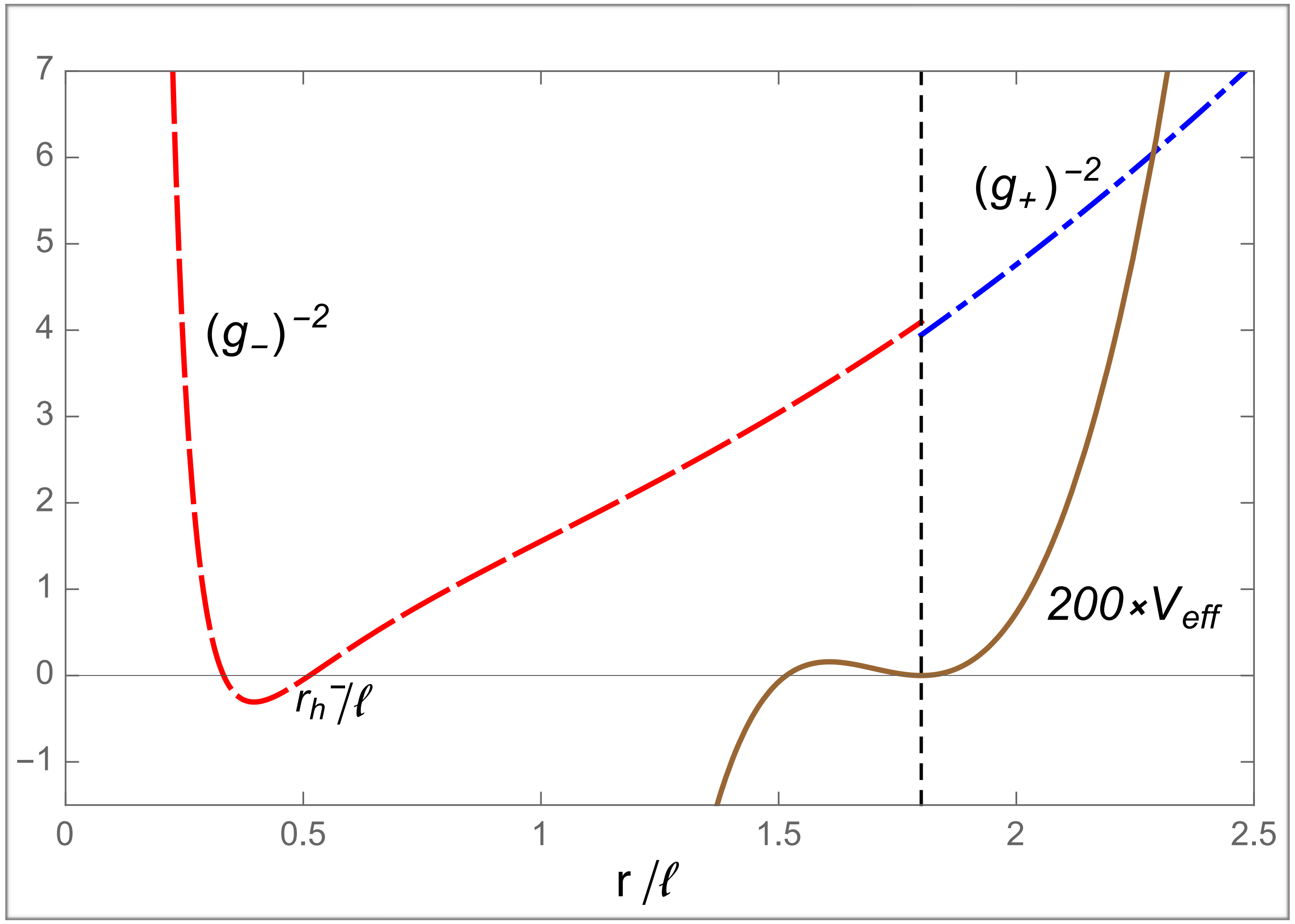
$m_0/\ell^2 = 0.324,\ w = 0.285,\ Ma^2/\ell^4 = 0.02,\ \mathcal{R}_*/\ell = 1.8$



[Delsate, JVR, Santarelli (2014)]

Rotating thin shells: Full collapse in asymptotically flat spacetime

- Take asymptotically flat limit, $\ell \to \infty$.
- Collapse starting from rest at infinity imposes:

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i.e., matter on the shell has EoS of dust

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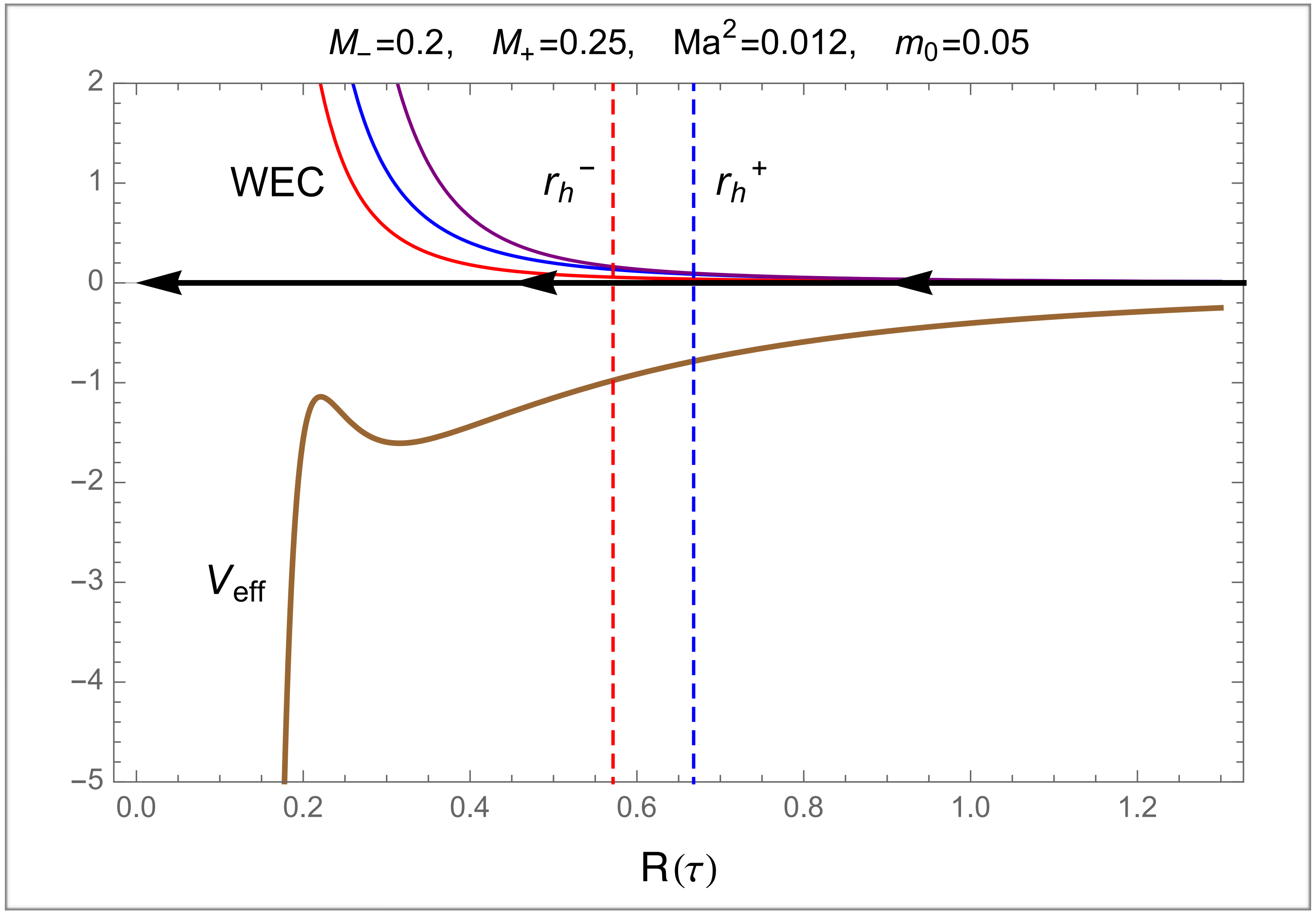

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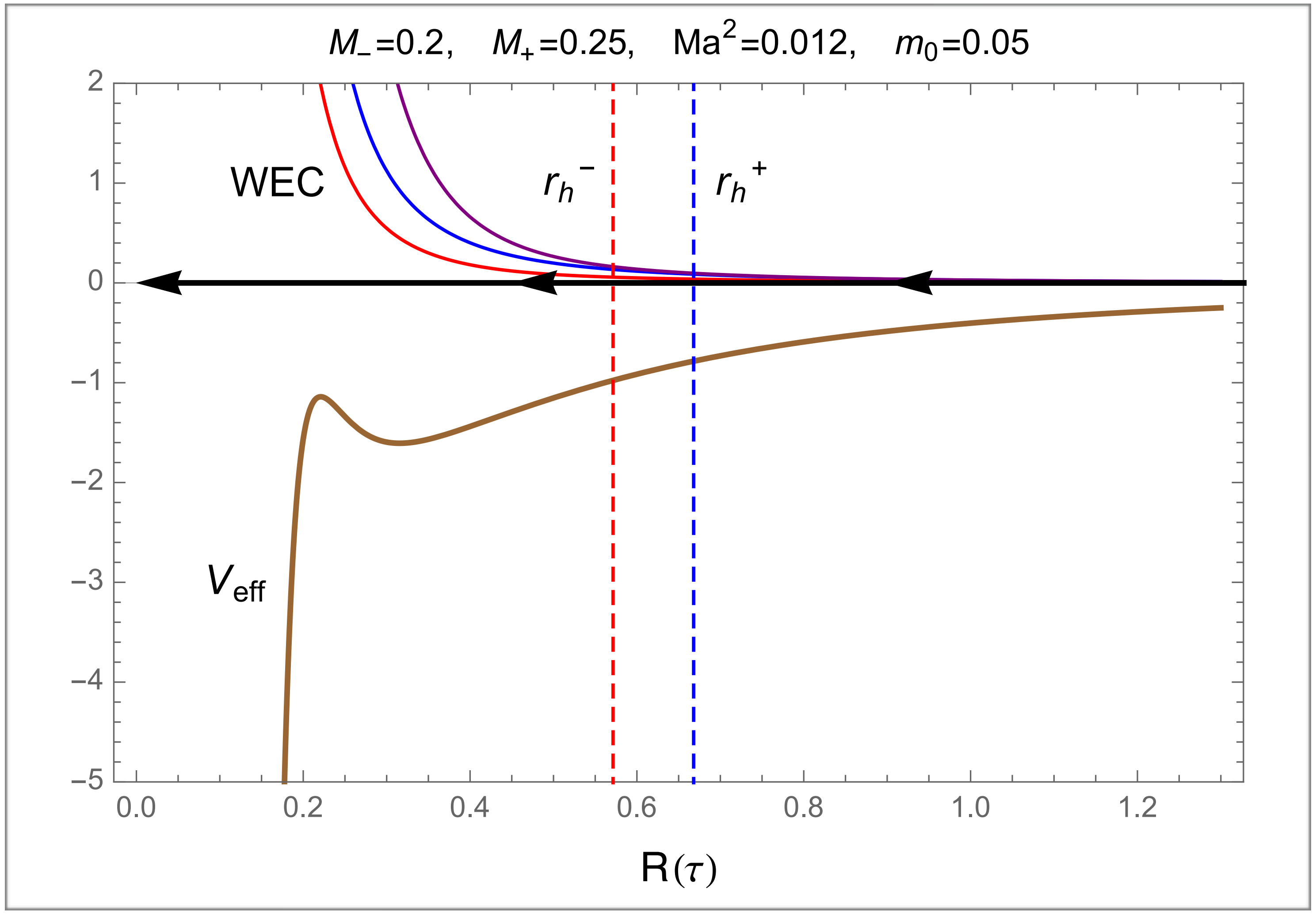

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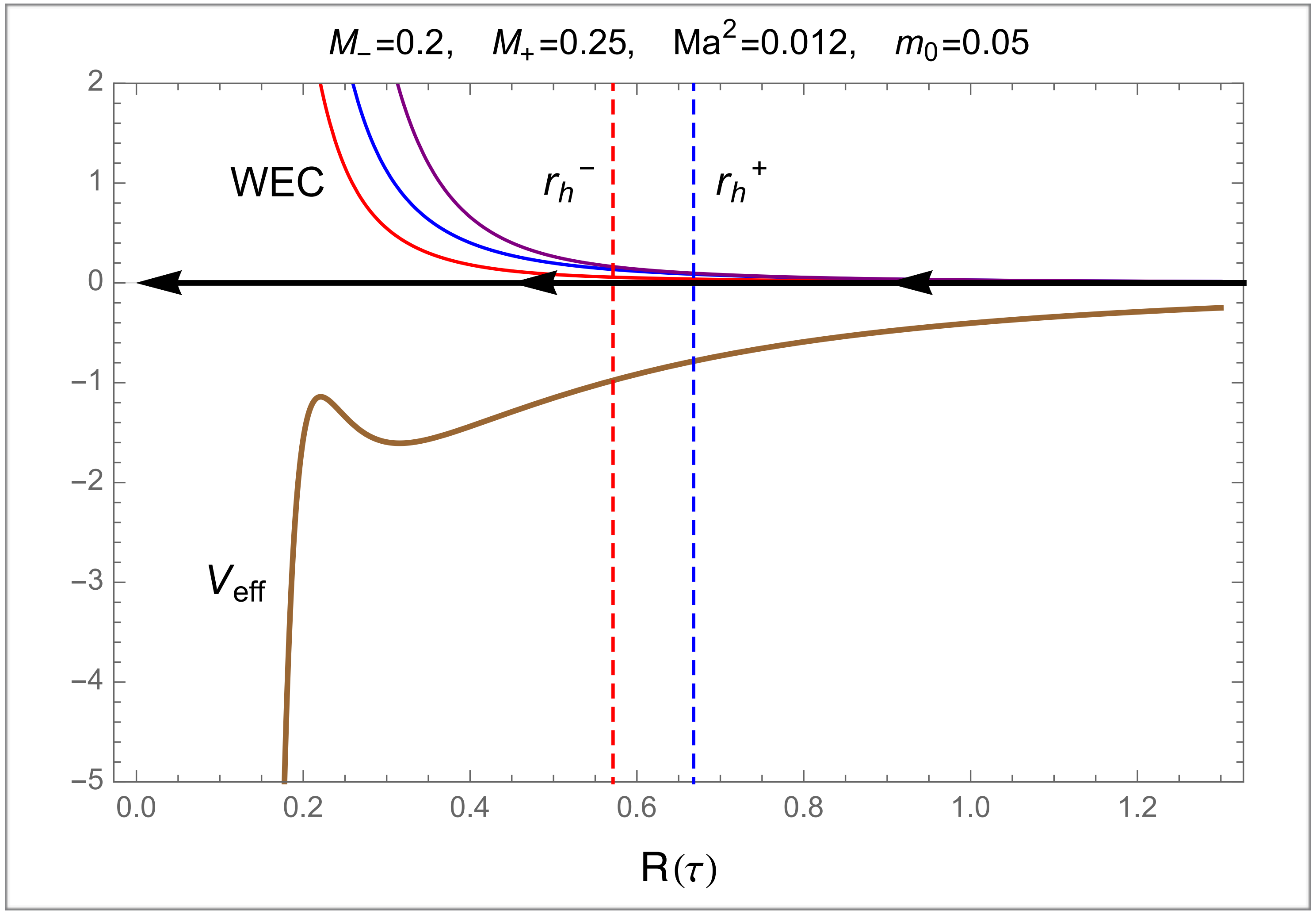

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Weak energy conditions (WEC) are satisfied

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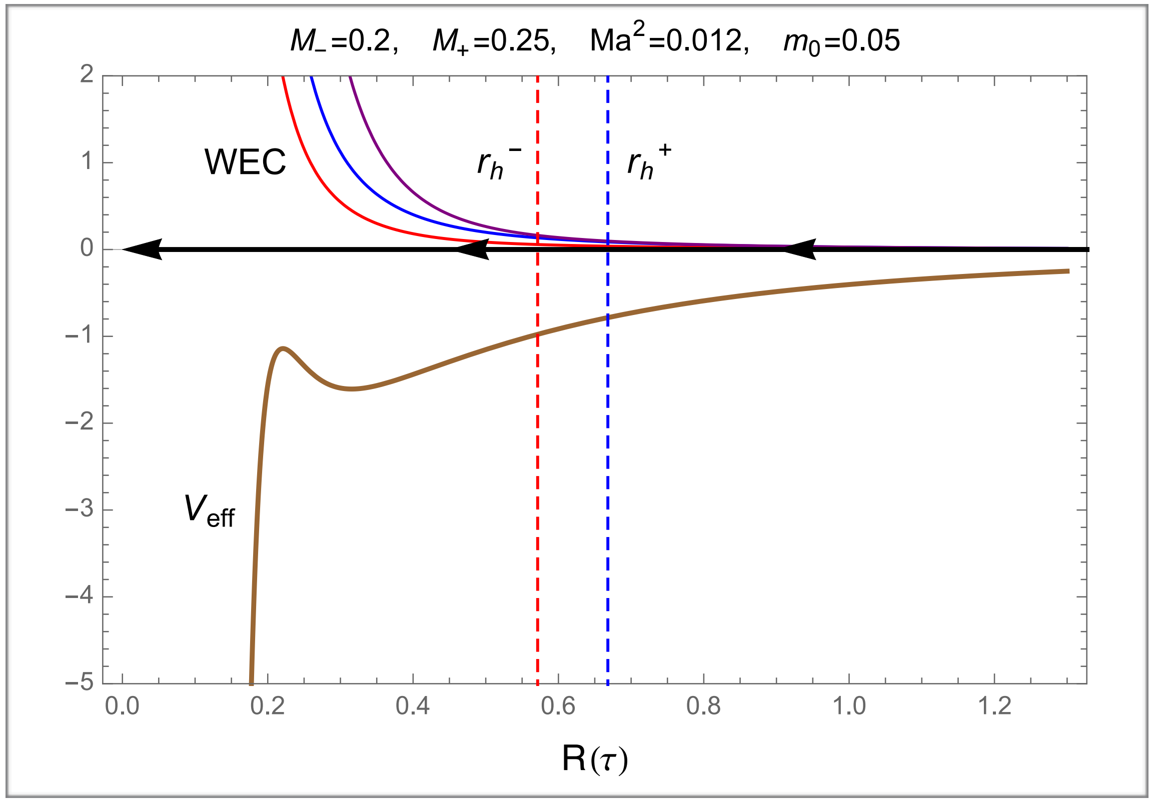
Rotating thin shells: Diverse scenarios



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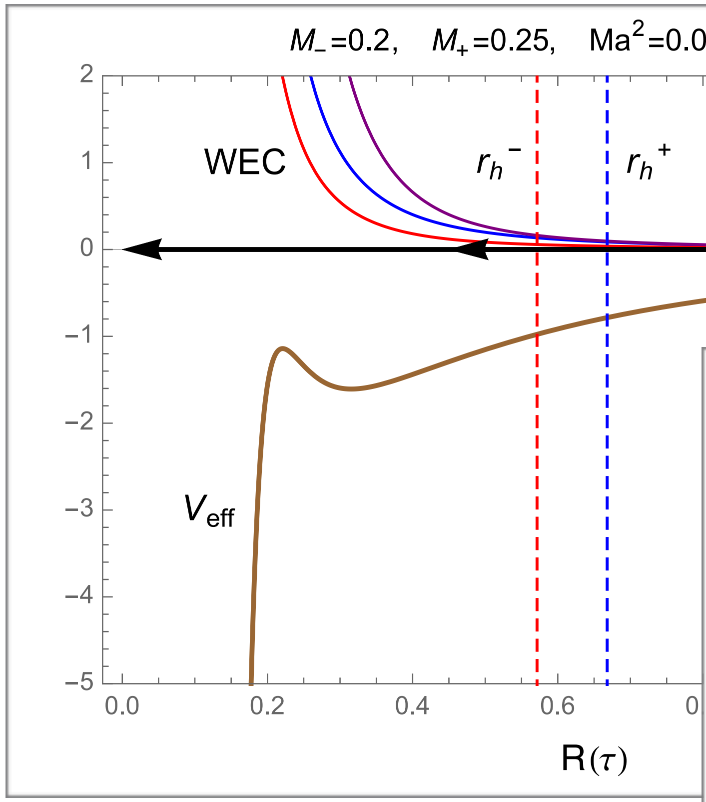

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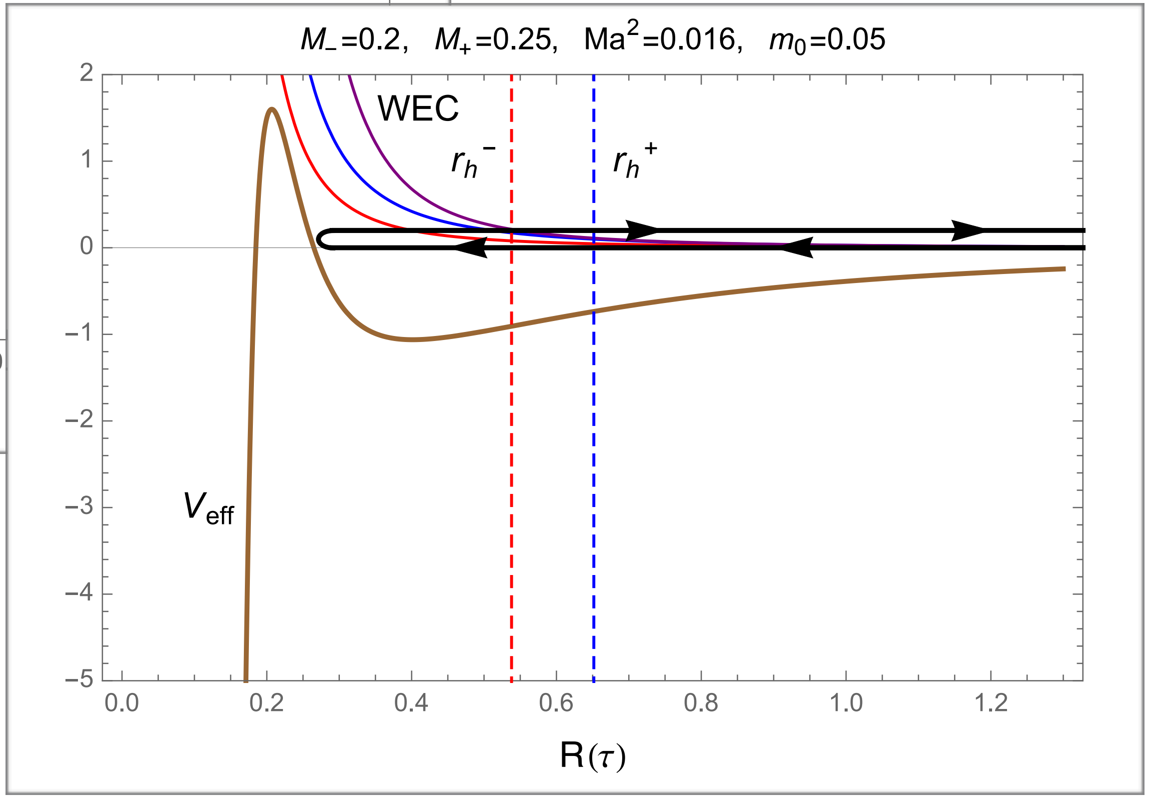

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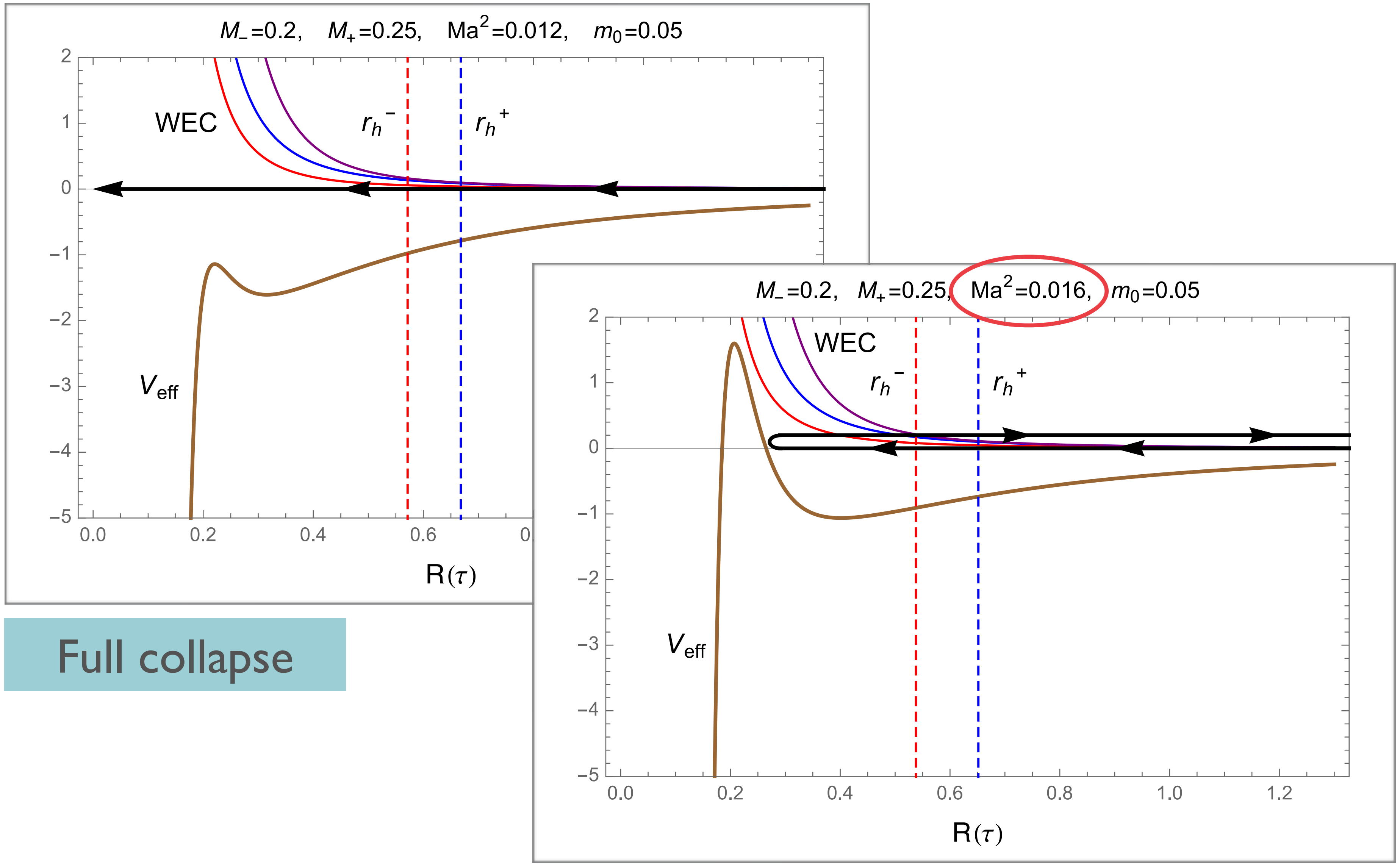

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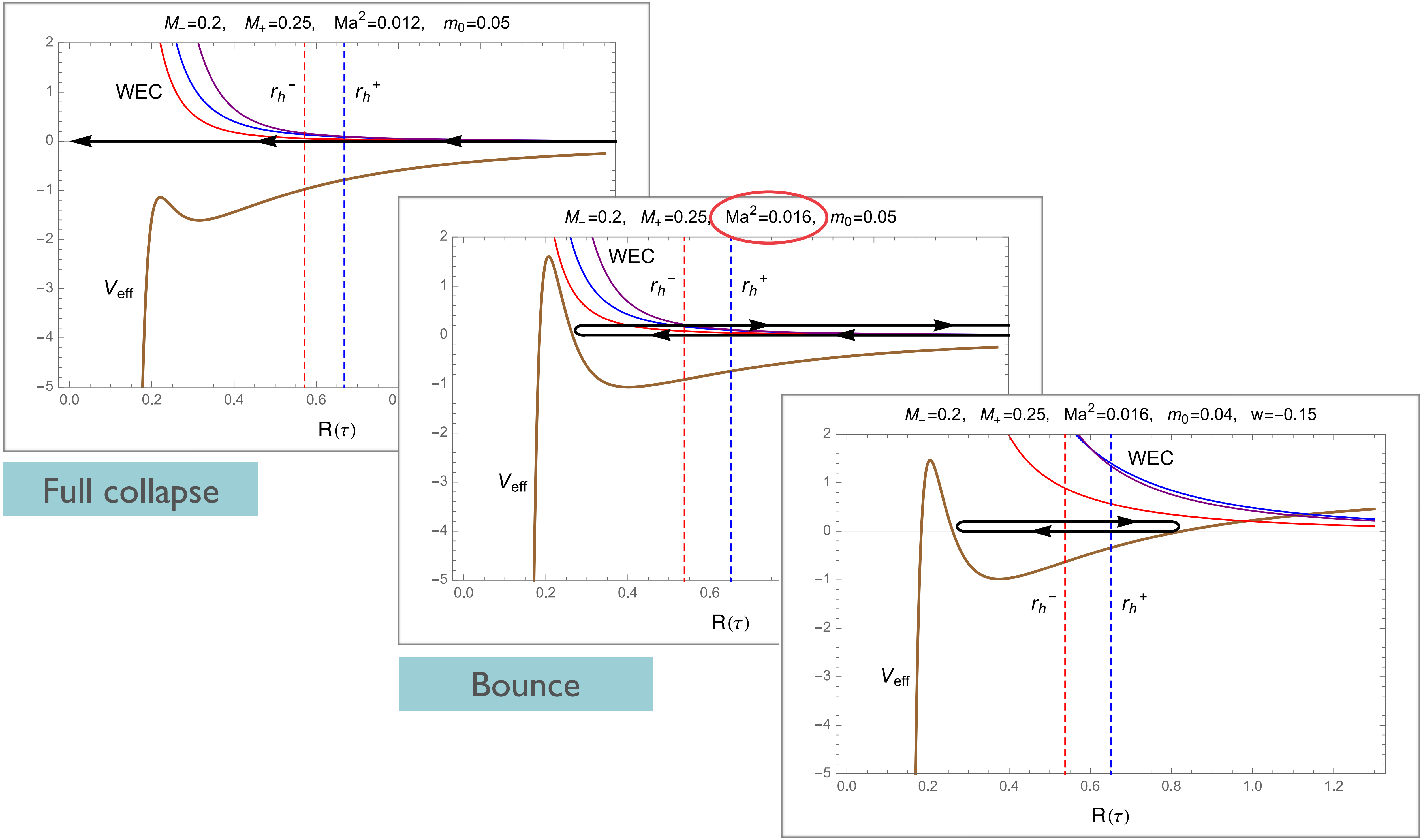

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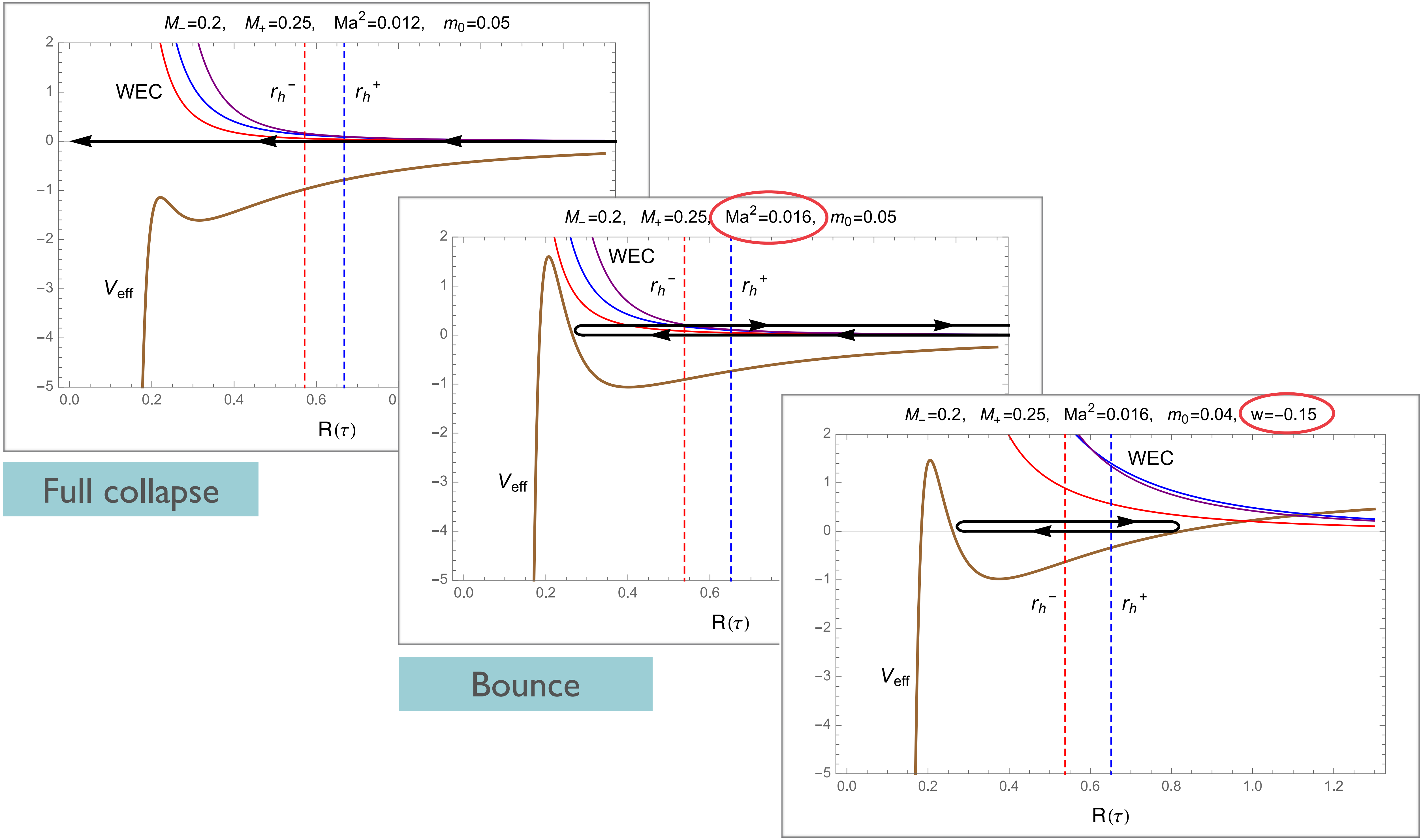

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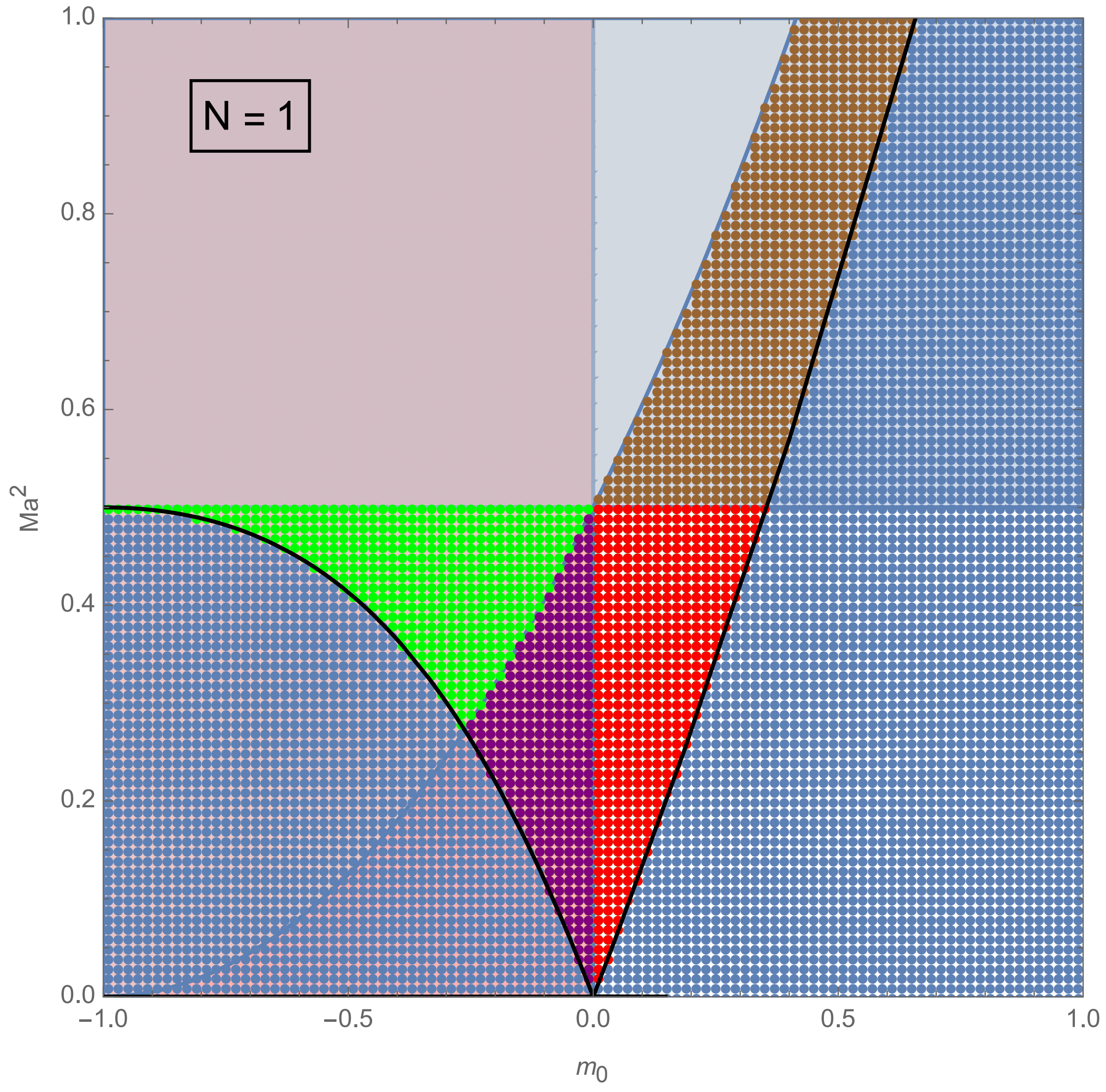

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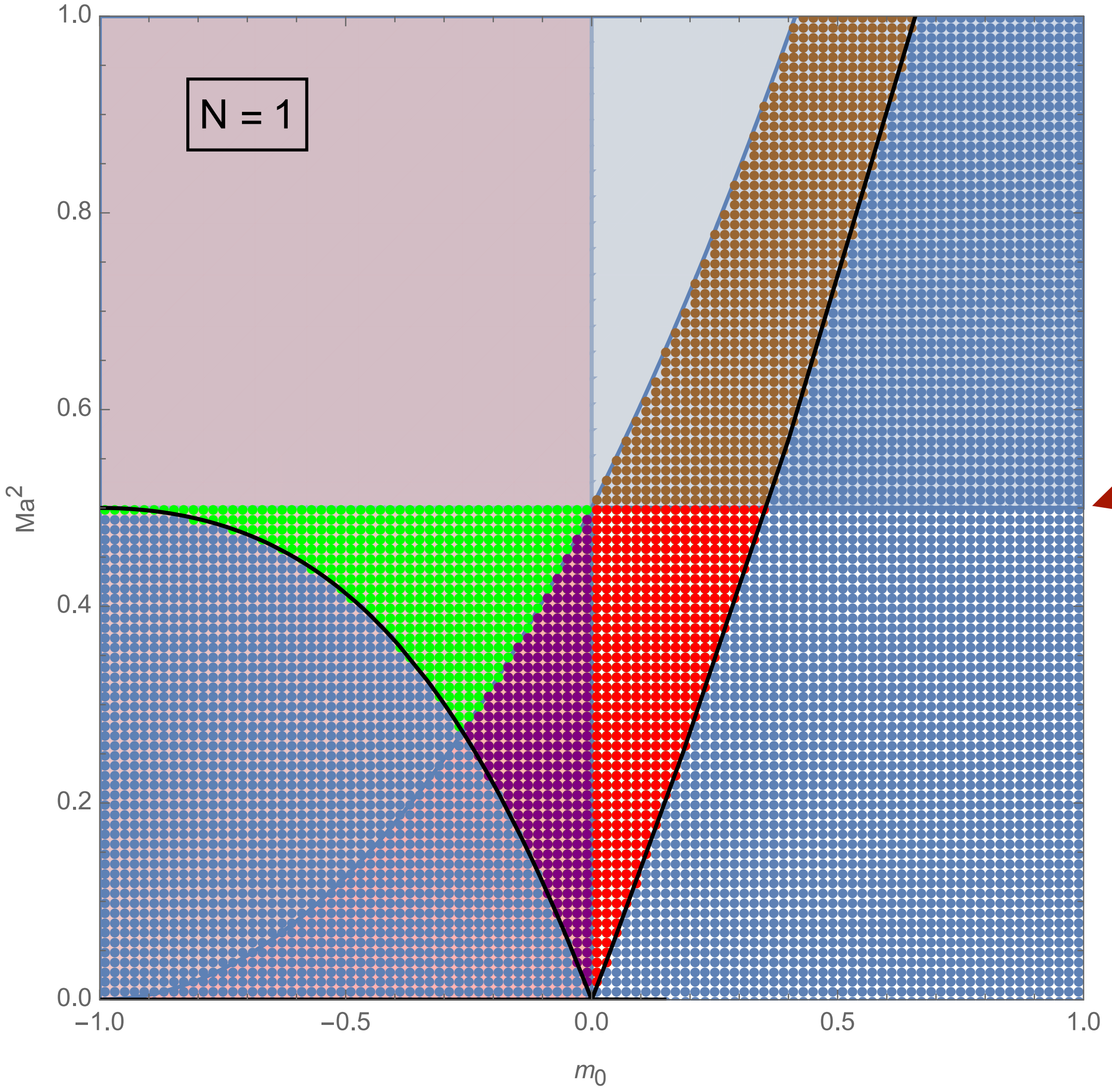

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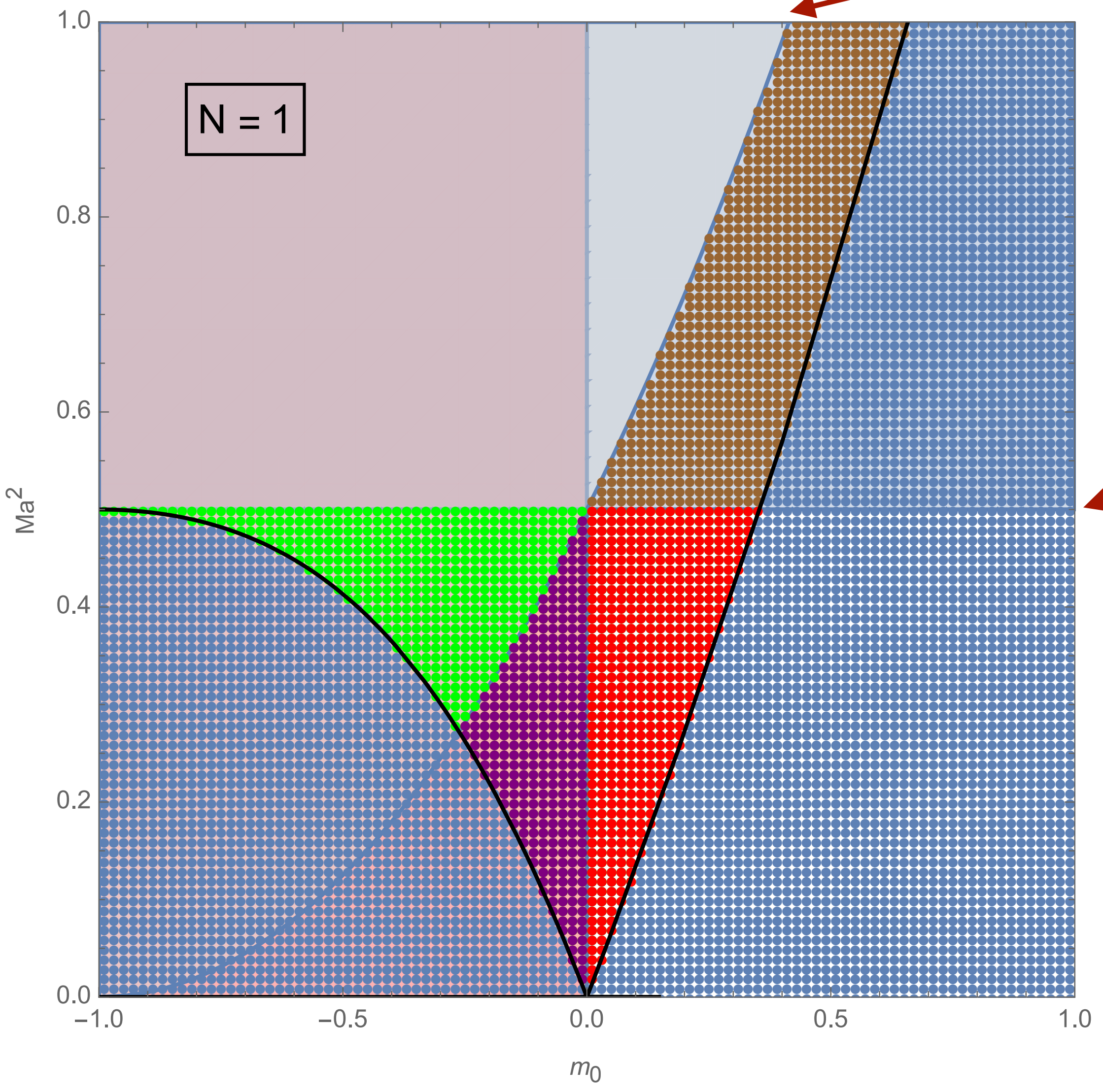

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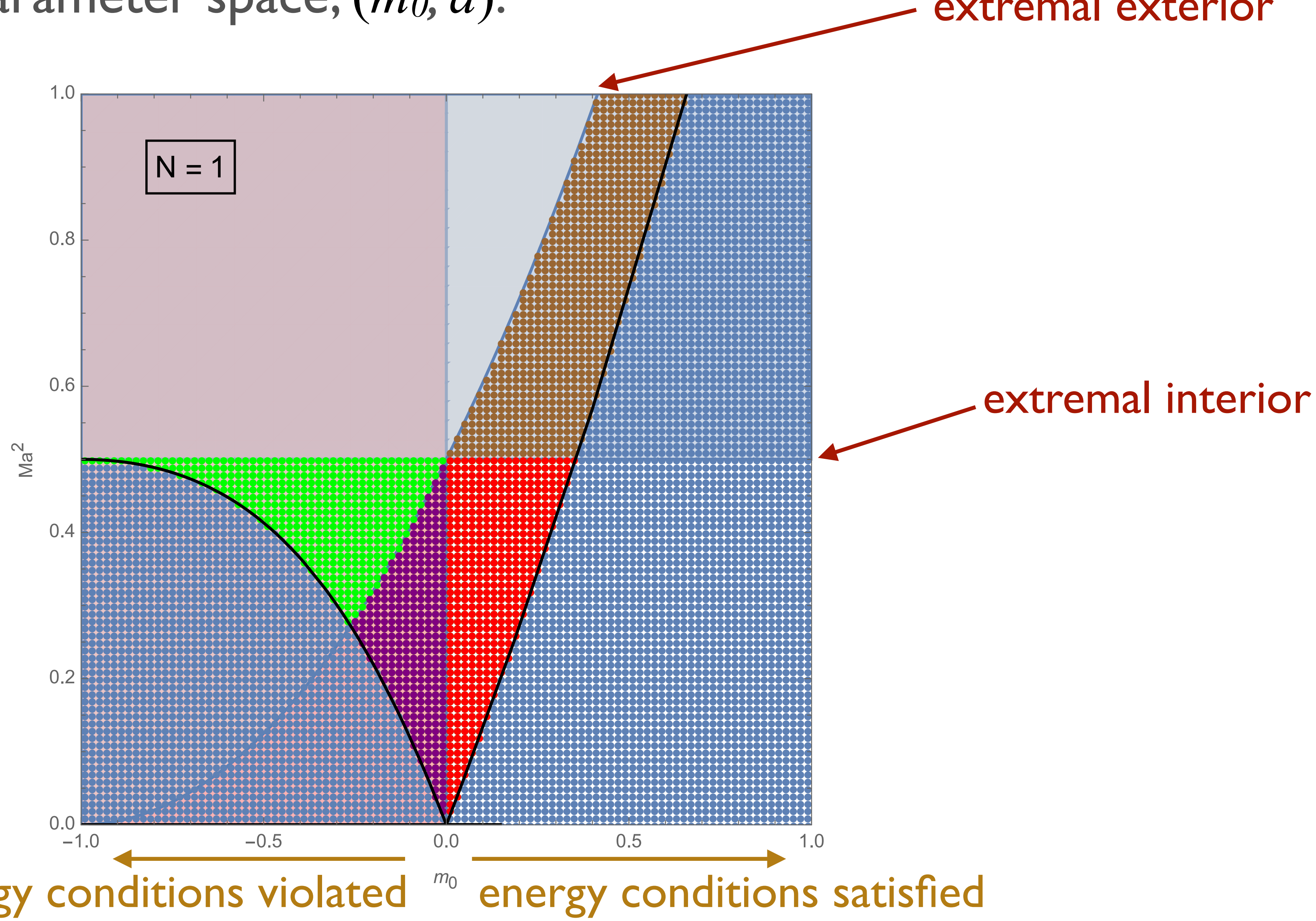

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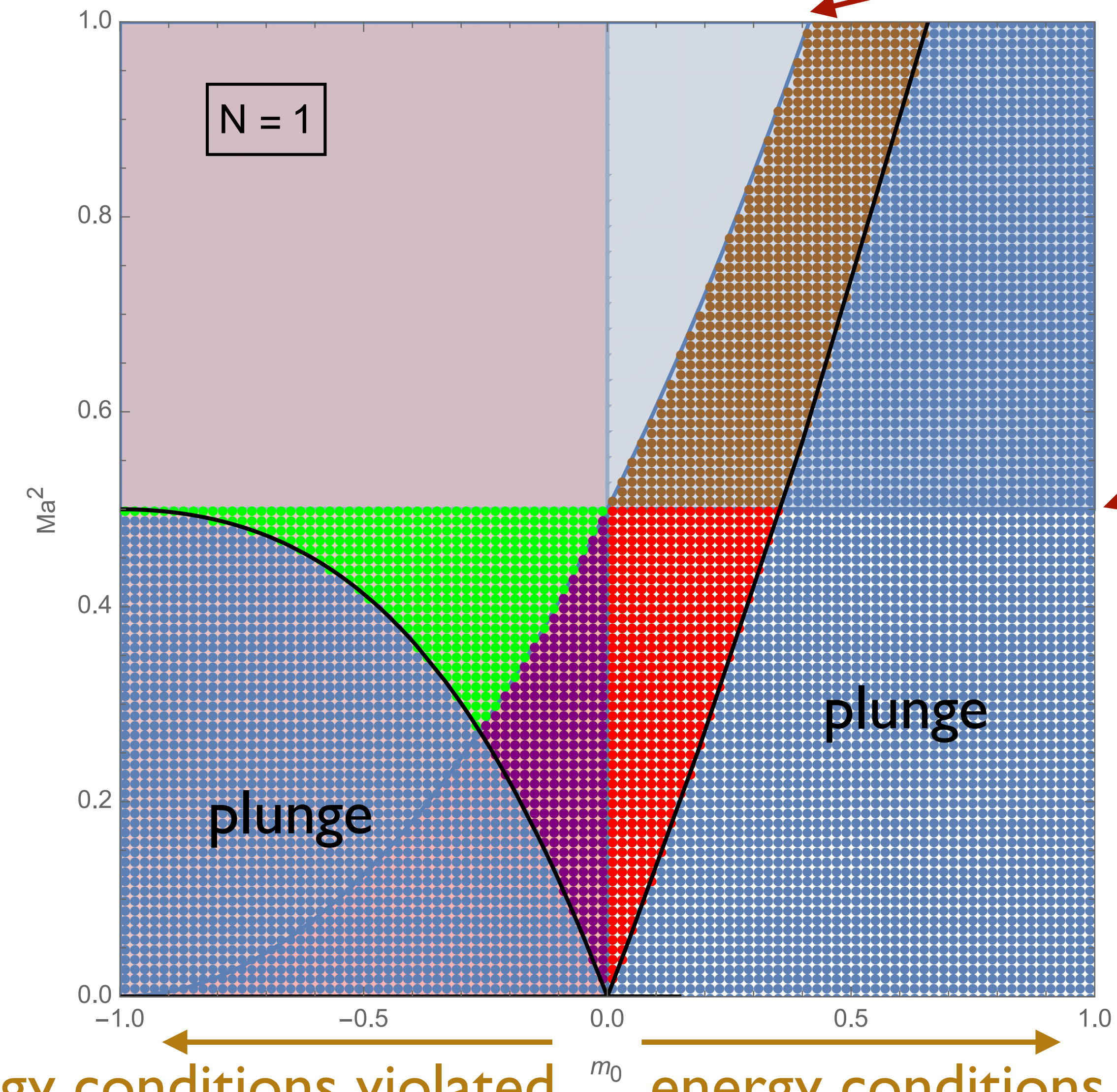

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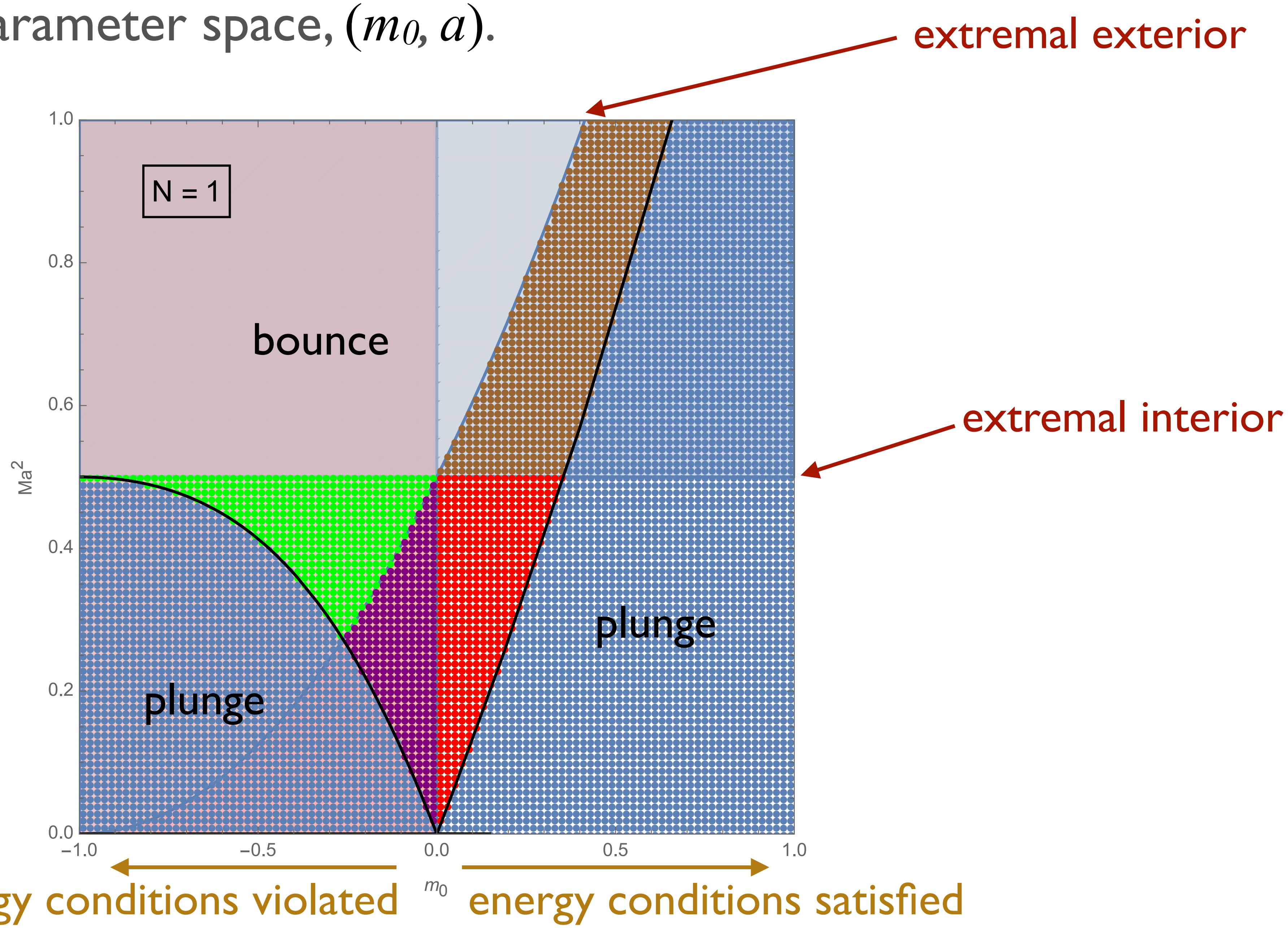

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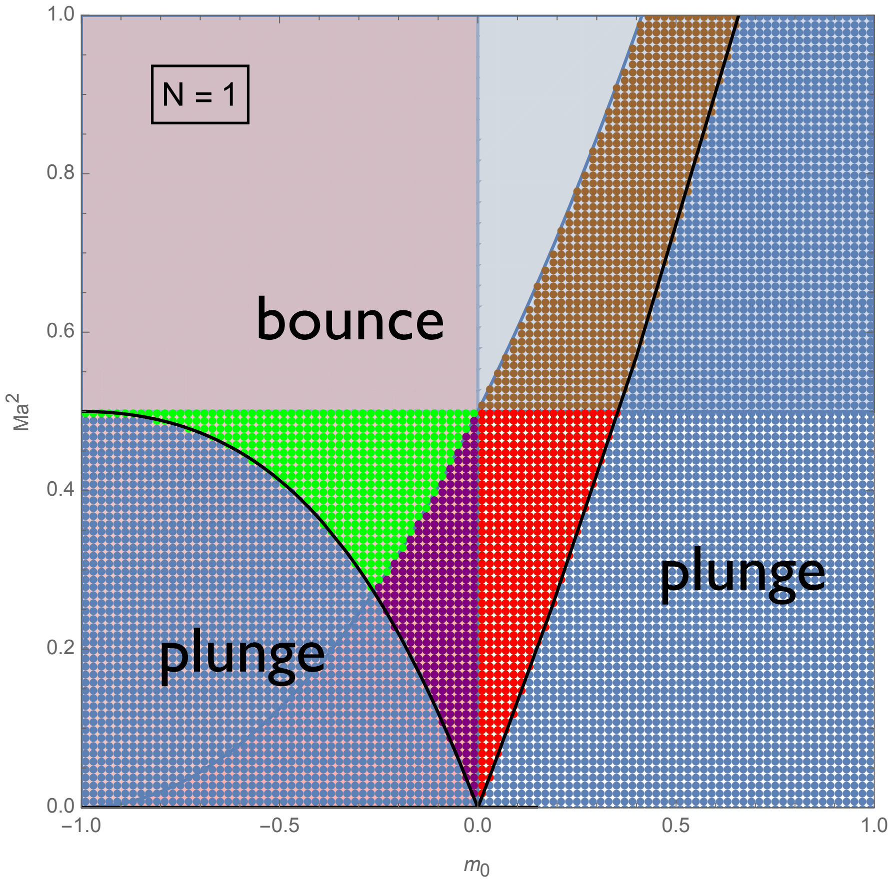

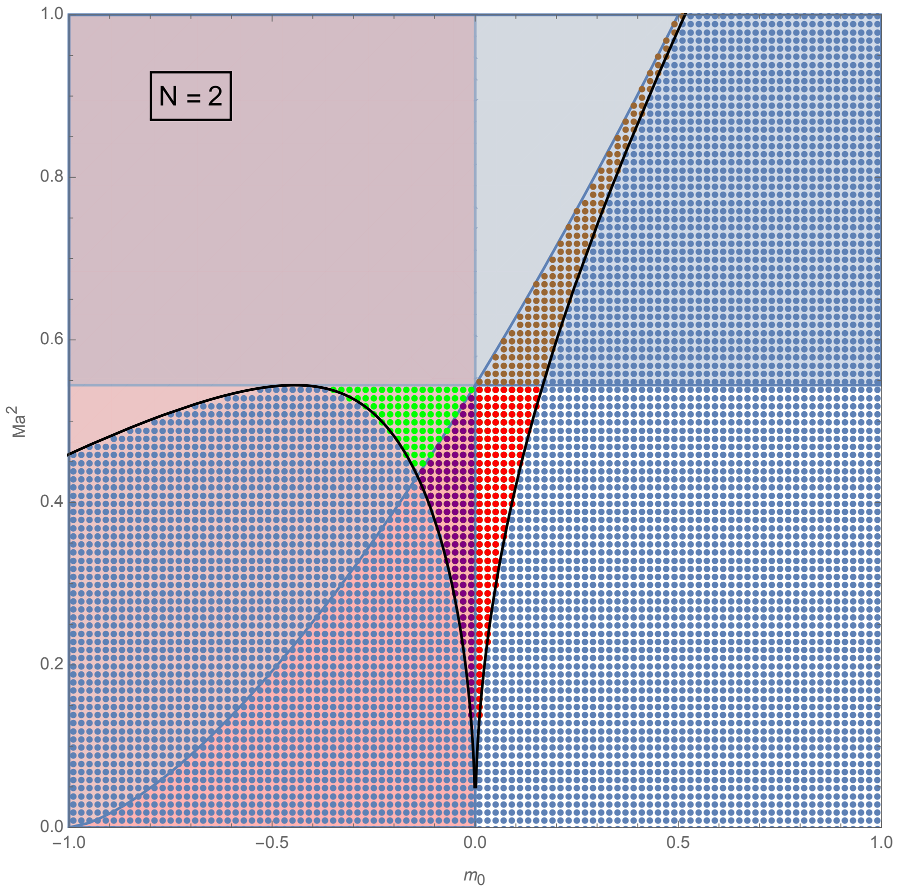

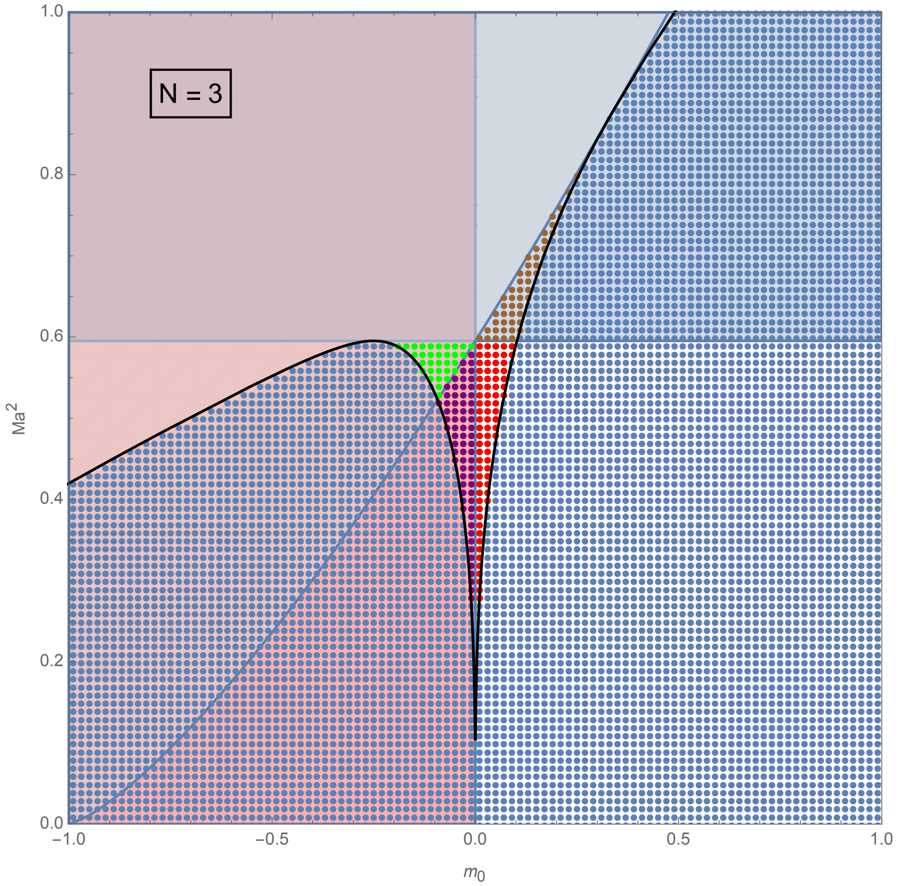

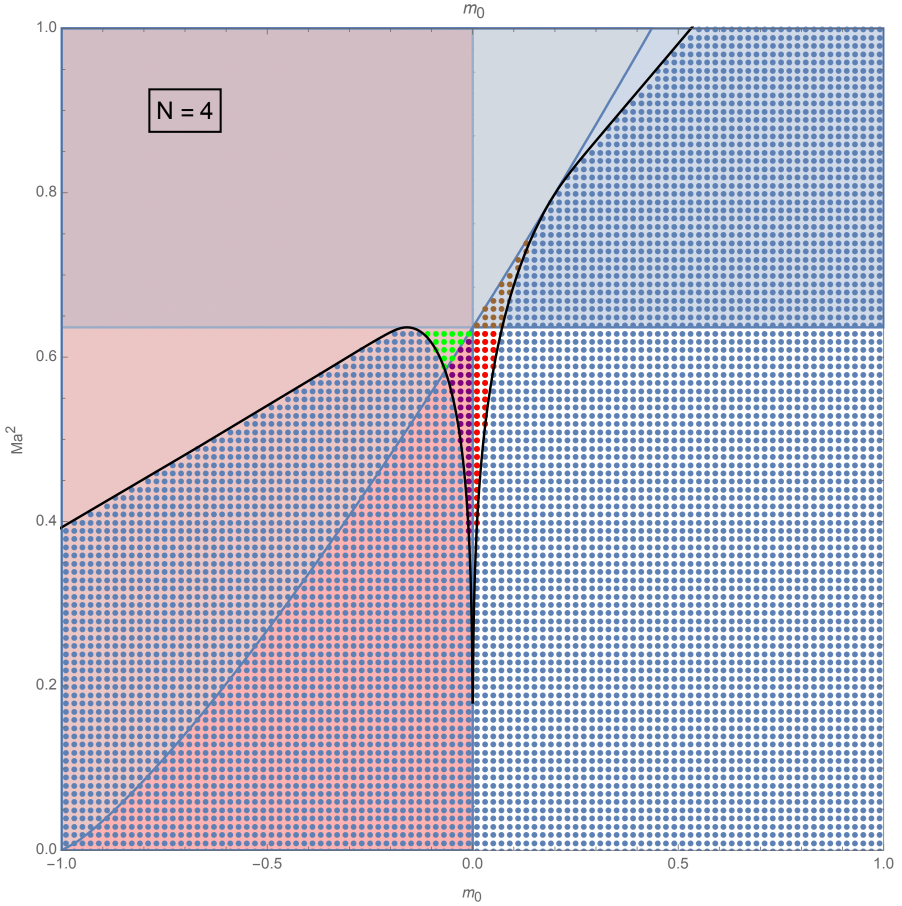

Outline

✓ Introduction

✓ Background: cohomogeneity-1 black holes

✓ Rotating thin shells & cosmic censorship

✦ Conclusion & outlook

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  - Full collapse onto rotating, asymptotically flat BH starting from rest at infinity (and satisfying energy conditions) respects the CCC.

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Thank you.

Introduction: (Weak) Cosmic censorship conjecture

- Some arguments supporting the conjecture:

1. Collapse of a homogeneous (spherically symmetric) ball of dust yields a BH.

[Oppenheimer, Snyder (1939)]

2. Numerical studies of axially symmetric star collapses do not show evidence of naked singularity formation.

[Nakamura (1981)]
[Stark, Piran (1985)]

3. High-energy BH collisions in 4D with arbitrary impact parameter invariably yield a Kerr BH.

[Sperhake et al. (2009)]

4. Once formed BHs are hard to kill. E.g., stability of Kerr(-Newman).

[Whiting (1989)]
[Zilhão et al. (2014)]
[Dias, Godazgar, Santos (2015)]

5. CCC survives Wald's spin-up process with test particles for a wide variety of BHs (spinning, charged, higher dimensions, non-spherical horizon topology, AdS).

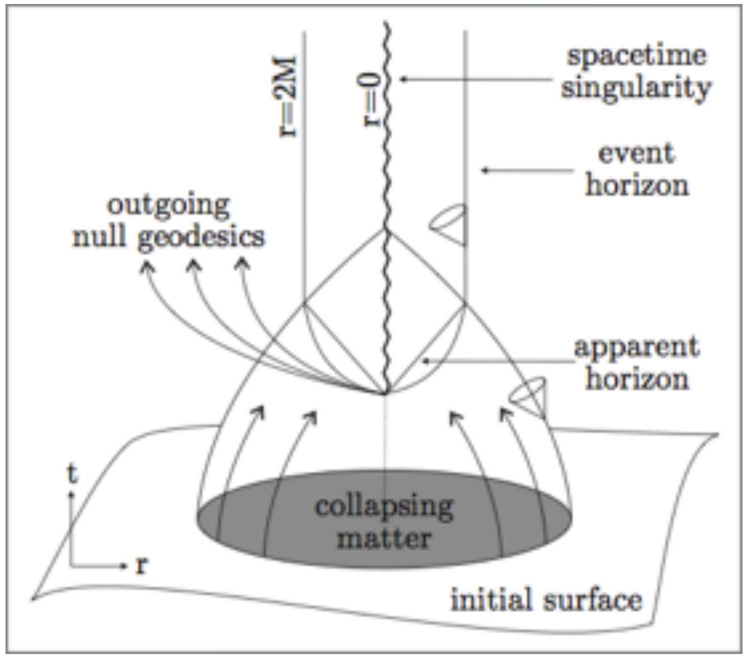
[Wald (1974)]
[Bouhmadi-López, Cardoso, Nerozzi, JVR (2010)]
[JVR, Santarelli (2014)]

Introduction: (Weak) Cosmic censorship conjecture

- However, there are indications of naked singularity formation:

non-homogeneous spherical collapse

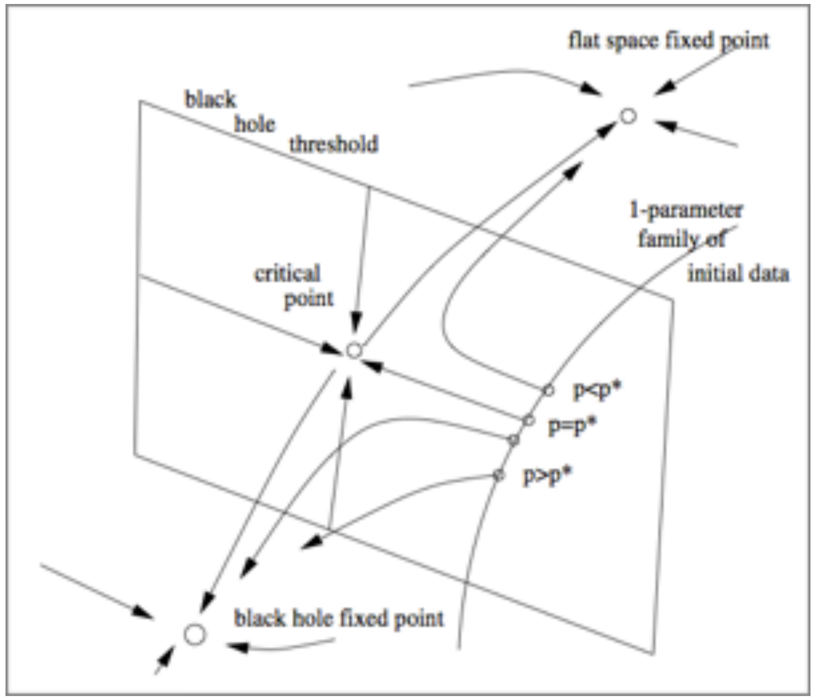
[Eardley, Smarr (1979)]



from [Joshi, Malafarina (2012)]

critical collapse

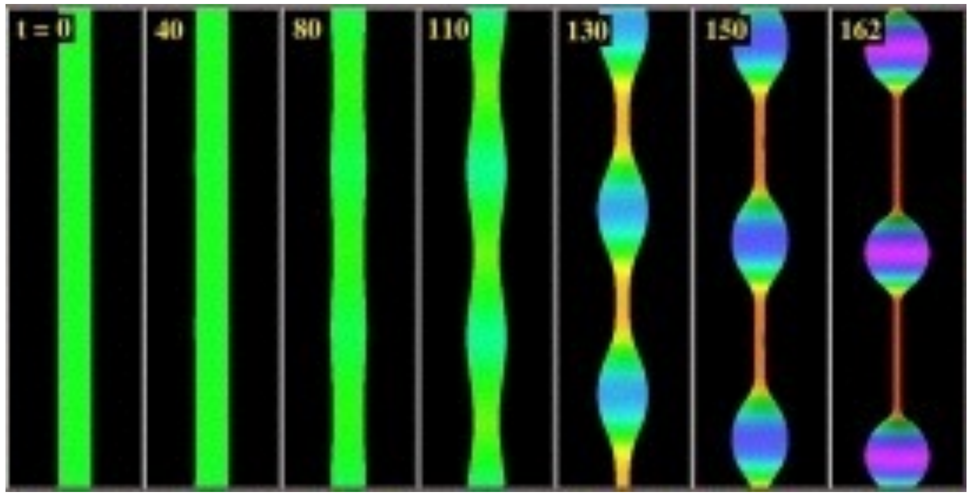
[Choptuik (1983)]



from [Gundlach, Martín-García (2007)]

endpoint of Gregory-Laflamme instability in 5D

[Lehner, Pretorius (2010)]



endpoint of superradiant instability in AdS

[Dias, Horowitz, Santos (2011)]
[Niehoff, Santos, Way (2015)]
[Green, Hollands, Ishibashi,, Wald (2015)]

- *Under what conditions (genericity, dimensionality, matter content, …) can the CCC hold?*