What is Loop Quantum Cosmology

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LQC chronology

- 2000-2002: Early pioneer work by Martin Bojowald
- 2003: Rigorous foundations, by Ashtekar, Bojowald and Lewandowski
- 2006: Major conceptual, theoretical and technical developments, by Ashtekar, Pawlowski and Singh. First full quantum treatment. Derivation of the quantum bounce.
- Last years: lots of activity, both in the development of the theory and in applications - early universe, inflation.

Mena Marugán, Martín-Benito, Olmedo

also Barrau, Chiou, Corichi, Sloan, Wilson-Ewing, and many others ...

What it is

Loop Quantum Gravity (LQG) takes GR at face value. No modifications of GR at the classical level. New effects / modifications are supposed to emerge upon quantization. It aims at a Quantum Theory of Gravity and assumes that this is possible without extra ingredients. Just background independence and nonperturbative methods.

Loop Quantum Cosmology (LQC) is LQG's little cousin: it takes standard classical homogeneous cosmological models at face value and applies LQG-inspired quantization techniques.

Modifications to classical GR in cosmological models are therefore coming from quantum effects.

Not surprisingly, the most worked out and best understood model is k=0 FLRW, for which there is a complete quantum treatment.

Many developments and applications are based on modified semi-classic, or effective equations, such as modified Friedmann equation and modified Raychaudhuri equation, coming from typical LQC effects.

All isotropic models are well understood, and also anisotropic models (Bianchi I, II and IX, confirmed with numerical simulations). Also good indications for inhomogeneous (Gowdy) models

What it is not

It is not equivalent to the old Wheeler-De Witt approach to quantum cosmology (quantum geometrodynamics).

It is not based on the standard quantization procedure. It follows a non-equivalent quantization procedure, inspired by LQG - this is in fact the reason why different results are obtained.

However, LQC is also not deducible or derivable from LQG, at least in the present state of afairs. It is inspired by LQG and mimics as much as possible the main steps, but there are choices involved and room for creativity.

What does it achieve

- The LQC approach avoids the big bang (and big crunch) singularity altogether, replacing it with a bounce

The existence of singularities, such as the big bang, points to the incompleteness of classical GR, which reaches its limits when the spacetime curvature is extremely large.

Einstein 1945: "One may not assume the validity of field equations at very high density of field and matter and one may not conclude that the beginning of the expansion should be a singularity in the mathematical sense."

There was hope that those singularities would be understood and avoided by a quantum approach to gravitational physics.

However, that hope was not satisfactorily fulfilled for a long time.

What does it achieve

- The key modifications of GR by LQC are well-captured in effective equations. Modified effective Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\mathrm{crit}}}\right), \quad \rho_{\mathrm{crit}} = 0.41\,\rho_{\mathrm{Planck}}$$

- Singularity removal in LQC is generic: different types of singularities, other homogeneous models, Gowdy (inhomogeneous) models

How is it done: LQG

Hamiltonian formulation of GR (for globally hyperbolic spacetimes).

Inital data: induced spatial 3-metric q_{ab} and extrinsic curvature K_{ab}.

Replace:

1) the spatial 3-metric with the densitized triads

$$E_i^a = \sqrt{|q|} e_i^a, \quad E_i^a E_j^b \delta^{ij} = |q| q^{ab} \tag{1}$$

2) the extrinsic curvature with the Ashtekar-Barbero connection

$$A_a^i = \Gamma^i_a + \gamma K_a^i \tag{2}$$

γ is the Immirzi parameter, Γ^i_a is the su(2)-connection compatible with the co-triad and $K_a^i = K_{ab} e^{ib}$

Connnections and triads form a canonical set for classical GR

Over-complete new set of variables: holonomies and fluxes

SU(2)-holonomies along (piecewise analytic) curves e

$$h_e = P\ exp \int_e A^i_a \tau_i dx^a \tag{3}$$

Smeared fluxes of the vector densities E^a_i through surfaces S

$$E(S, f) = \int_S E^a_i f^i \epsilon_{abc} dx^a\, dx^b \tag{4}$$

- There is essentially a UNIQUE way of quantizing this variables.
- In LQG, the connection itself cannot be quantized, only the holonomies.
- Geometry is fundamentally discrete: there is an area gap Δ

How is LQC done: homogeneous and isotropic cosmology

- LQC: apply LQG quantization techniques to homogeneous and isotropic (FLRW) cosmologies.
- Fiducial triad $\tilde{e}_i^a$

$$A_a^i = c\,\tilde{e}_a^i, \quad E_i^a = p\sqrt{|\tilde{q}|}\,\tilde{e}_i^a \tag{5}$$

The canonical pair (c, p) describe the geometry degrees of freedom

The relation with the usual scale factor is $c = \gamma\dot{a}$, $|p| = a^2$

$$ds^2 = -dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2\right)$$

LQC quantization

Elementary variables: holonomies along straight edges

$$h_i(\mu) = cos(\mu c/2)\mathbf{1} + 2sin(\mu c/2)\tau_i$$

and fluxes accross squares.

In practice, one needs to quantize p and functions of c of the type $e^{i\mu c}$, $\mu \in \mathbb{R}$

Guided by LQG, we look for quantum representations of the holonomies, or $e^{i\mu c}$, which do not lead to a quantization of the connection c.

Nonregular representations, avoids Stone-von Neumann uniqueness

Effectively, p is discrete and c is compactified - manifestation in LQC of the existence of an area gap in LQG / discrete geometry

Simple model: FLRW with minimally coupled massless scalar field

To have an interesting and fully treatable quantum model, consider a massless scalar field ϕ (minimally coupled).

Classical Hamiltonian constraint

$$C = -\frac{3}{8\pi G}\frac{\pi_a^2}{a} + \frac{\pi_\phi^2}{2a^3} = -\frac{3}{4\gamma^2}b^2|v| + \frac{\pi_\phi^2}{4\pi G|v|}, \tag{6}$$

where

$$b = c/|p|^{1/2}, \quad v = sgn(p)|p|^{3/2} \tag{7}$$

$$\left(\text{Friedmann equation} \quad \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho, \quad \rho = \frac{\pi_\phi^2}{2a^6}\right)$$

Classical singular trajectories

Equation of state: $w = P/\rho = 1$. Integration yields $\rho \propto a^{-6}$

ϕ is a monotonic function, can play the role of internal time

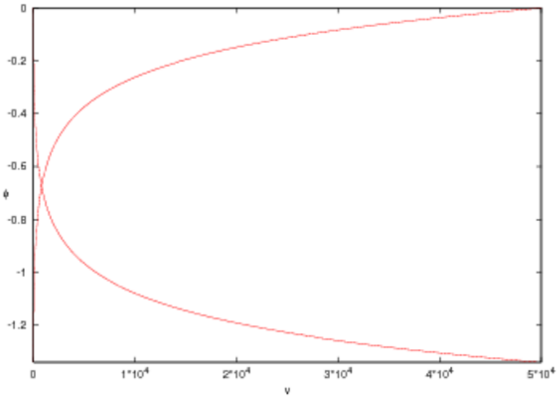
The classical trajectories can be obtained:

$$\phi = \pm \frac{1}{\sqrt{12\pi G}} \ln\left(\frac{v}{v_0}\right) + \phi_0$$

where v_0 and ϕ_0 are constants.

One trajectory corresponds to the expanding universe with big bang singularity in the past evolution. The other trajectory corresponds to the contracting universe with big crunch singularity in the future evolution.

Classical singular trajectories



Singularity resolution: what does it mean?

Do to the discreteness of p, and therefore of v, the Hamiltonian constraint (7) becomes a difference equation (ϕ, π_ϕ are quantized in the usual way).

$$\widehat{C}\,\Psi(\phi, v) = 0 \tag{8}$$

With a full quantum theory available, one can construct self-adjoint

observables and extract physical predictions.

Complete set of observables: field momentum π_ϕ and the volume v

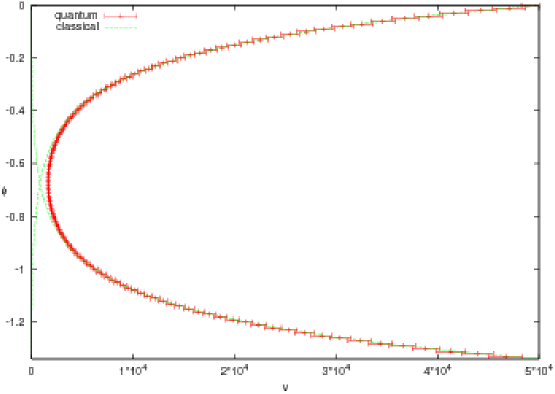
One can choose a semiclassical state, sharply peaked on a classical trajectory at late times, and evolve the state numerically using the quantum hamiltonian constraint. The expectation values of the observables can then be computed and compared with the classical trajectory.

It turns out that the state does not encounter the big bang: it bounces at a certain volume (determined by the field momentum π_ϕ).

For sharply peaked states, the bounce always occur when the energy density of the field reaches a value

$$\rho_{crit} \approx 0.41\,\rho_{\rm Planck}$$

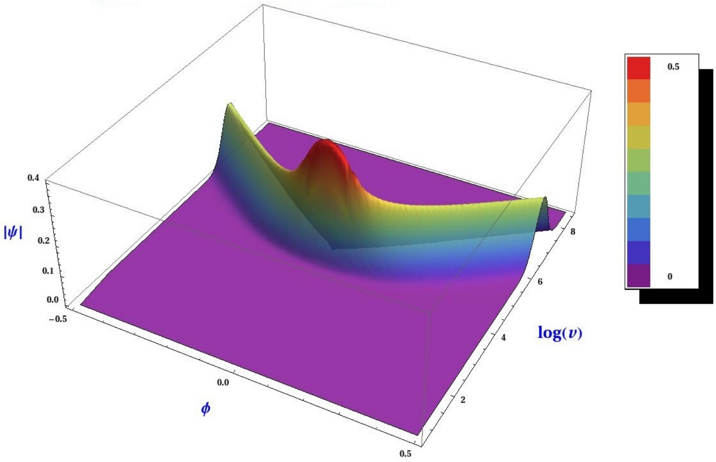
Big Bounce



Expectation values of the volume plotted against internal time ϕ

(Ashtekar, Pawlowski and Singh)

Wave function at the Bounce



(Martín-Benito, Mena Marugán and Olmedo)

Characteristics of the Big Bounce

Semiclassical states remain peaked. The trajectory deviates from GR only for $\rho \approx 0.01\rho_{\rm crit}$.

The physical scale for the emergence of quantum corrections is universal: $\rho_{\rm crit} \approx 0.41\rho_{Pl}$.

The matter density is BOUNDED by $\rho_{\rm crit}$.

Hubble rate also has an upper bound.

The trajectory of the wave function peak matches quite well an EFFECTIVE classical dynamics trajectory, obtained from semiclassical analysis techniques.

Effective dynamics

Obtained from effective Hamiltonian (improved dynamics)

$$C = -\frac{3}{4\gamma^2}\frac{\sin^2(\mu_0 b)}{\mu_0}|v| + C_{matter}, \tag{9}$$

where

$$b = c/|p|^{1/2}, \;\; v = sgn(p)|p|^{3/2} \tag{10}$$

Modified Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\rho\Big(1 - \frac{\rho}{\rho_{\rm crit}}\Big), \;\; \rho_{\rm crit} = 0.41\rho_{\rm Pl} \tag{11}$$

Also modified Raychaudhuri equation

Numerical simulations for various models show that GR is a good approximation already when $\rho \approx 0.01\rho_{\rm crit}$

Singularity resolution fairly generic in LQC

Using effective equations, P. Singh as shown that all **strong** singularities are RESOLVED in flat FLRW (for any kind of matter content).

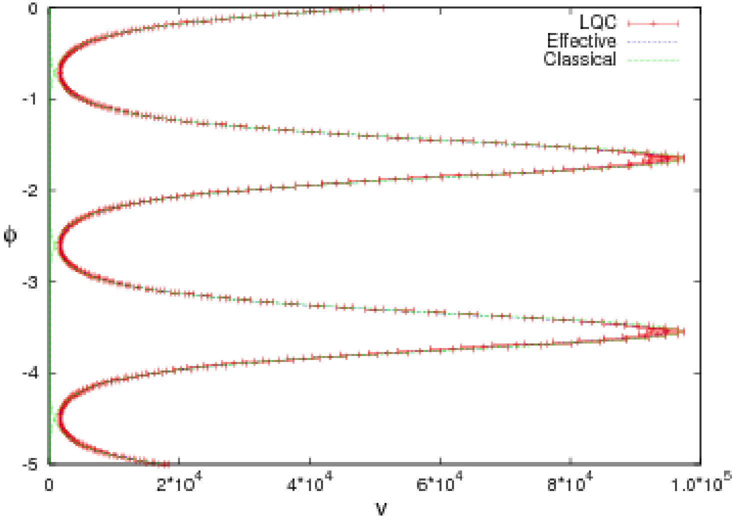
[Essentially, singularities are avoided in all cases for which $\rho \to \infty$, $H \to \infty$, classically]

Similar results were obtained for other homogeneous models, e.g. $k = \pm 1$ FRW and also anisotropic models (Bianchi I, II and IX)

Effective dynamics are used, but numerical simulations confirm the existence of the bounce.

Also very good indications for inhomogeneous models. Gowdy models with hybrid approach.

k=1 model



LQC
Effective
Classical
0
-1
-2
-3
-4
-5
φ
0
2*10^4
4*10^4
6*10^4
8*10^4
1.0*10^5
v