Charged scalar solitons and black holes in a cavity

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Dolan, Ponglertsakul and EW, `arXiv:1507.02156`, *to appear*

CFP

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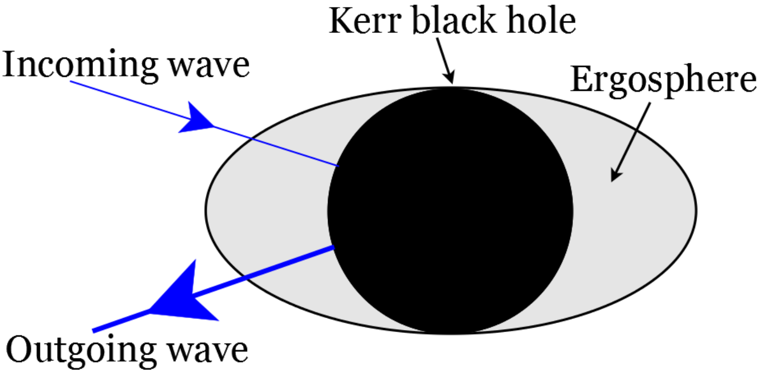
Outline

Superradiance



Bosonic wave amplification

$$\sigma < m\Omega_H$$

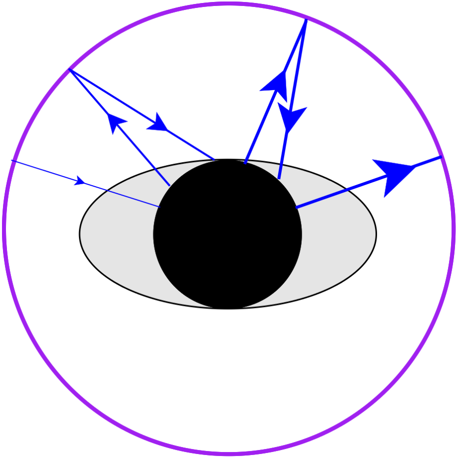
[Starobinski, Churilov *Sov Phys JETP* **38** 1 (1973)]

Black hole bomb

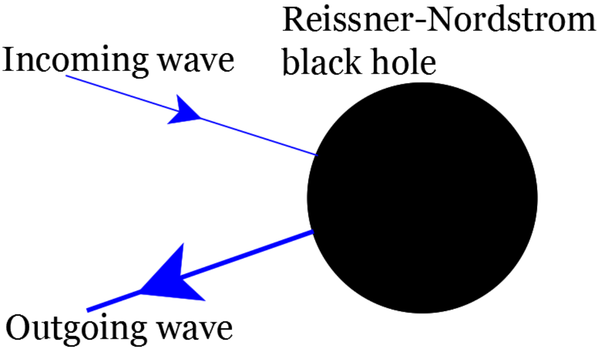
Black hole bomb

- Surround black hole by reflecting mirror
- Superradiant instability
- Wave grows exponentially with time

[Press, Teukolsky *Nature* **238** 211 (1972)]



Charge superradiance



Charged-scalar wave amplification

$$\sigma < q\Phi_H$$

[Bekenstein *PRD* **7** 949 (1973)]

Charge superradiance instability in a cavity

Massless charged scalar field Φ on RN black hole background

- Klein-Gordon equation

$$D_\mu D^\mu \Phi = 0 \qquad D_\mu = \nabla_\mu - iqA_\mu$$

- Time-periodic perturbations

$$\Phi \sim e^{-i\sigma t} R(r)$$

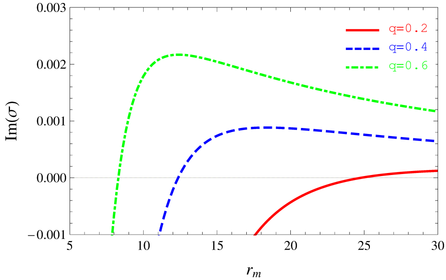
- Boundary conditions:
  - Horizon $r = r_h$: ingoing wave
  - Mirror $r = r_m$: $\Phi = 0$

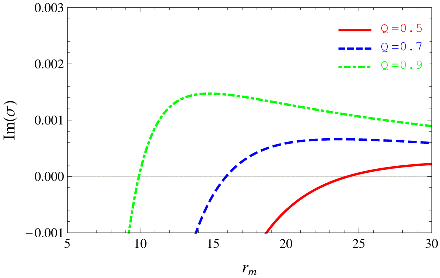
[Herdeiro, Degollado, Runarsson *PRD* **88** 063003 (2013)]

[Degollado, Herdeiro *PRD* **89** 063005 (2014)]

Charge superradiance instability in a cavity

- $\text{Im}(\sigma) > 0$ for r_m sufficiently large
- Charge superradiance instability





[Herdeiro, Degollado, Runarsson *PRD* **88** 063003 (2013)]

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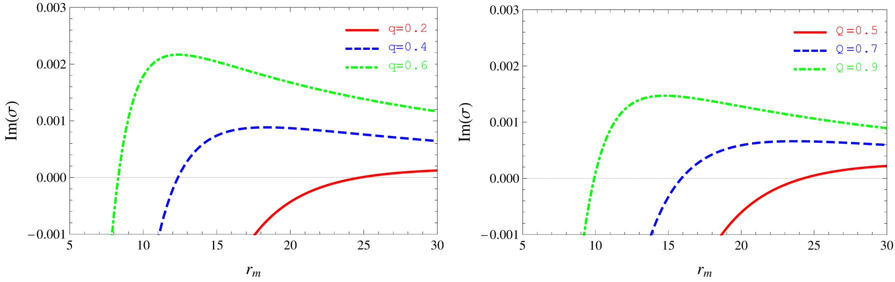

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What is the end-point of this instability?

Einstein-charged-scalar field theory

$$S = \int \sqrt{-g} \left[\frac{R}{2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} g^{\mu\nu} D^{*}_{(\mu} \Phi^{*} D_{\nu)} \Phi \right] d^4 x$$

Field equations

$$G_{\mu\nu} = T^{F}_{\mu\nu} + T^{\Phi}_{\mu\nu} \qquad \nabla_{\mu} F^{\mu\nu} = J^{\nu} \qquad D_{\mu} D^{\mu} \Phi = 0$$

Stress-energy tensor

$$T^{F}_{\mu\nu} = F_{\mu\rho} F_{\nu}{}^{\rho} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}$$

$$T^{\Phi}_{\mu\nu} = D^{*}_{(\mu} \Phi^{*} D_{\nu)} \Phi - \frac{1}{2} g_{\mu\nu} \left[g^{\rho\sigma} D^{*}_{(\rho} \Phi^{*} D_{\sigma)} \Phi \right]$$

Field current

$$J^{\mu} = \frac{iq}{2} \left[\Phi^{*} D^{\mu} \Phi - \Phi \left(D^{\mu} \Phi \right)^{*} \right] \qquad D_{\mu} = \nabla_{\mu} - iq A_{\mu}$$

Spherically symmetric configurations

$$ds^2 = -f(r)h(r)\,dt^2 + f^{-1}(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\varphi^2\right)$$

$$A_\mu = [A_0(r), 0, 0, 0] \qquad \Phi = \phi(r)$$

Static field equations

$$h' = r\left[\left(\frac{qA_0\phi}{f}\right)^2 + h(\phi')^2\right],$$

$$(A_0')^2 = -\frac{2}{r}\left[f'h + \frac{1}{2}fh' + \frac{h}{r}(f-1)\right],$$

$$0 = fA_0'' + \left(\frac{2f}{r} - \frac{fh'}{2h}\right)A_0' - q^2\phi^2 A_0,$$

$$0 = f\phi'' + \left(\frac{2f}{r} + f' + \frac{fh'}{2h}\right)\phi' + \frac{(qA_0)^2}{fh}\phi.$$

Boundary conditions

Event horizon $r = r_h$

- $A_0(r_h) = 0$
- Solutions parametrized by $q, r_h, \phi_h = \phi(r_h), E_h = A_0'(r_h)$
- $|E_h| < \sqrt{2}/r_h$

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Origin $r = 0$

- Regular metric and curvature
- Solutions parametrized by q, $\phi_0 = \phi(0)$, $a_0 = A_0(0)$
- Scaling symmetry $\Rightarrow$ fix $q = 0.1$

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Mirror $r = r_m$

$$\phi(r_m) = 0$$

Perturbation equations

Perturbations

$$f = \bar{f}(r) + \delta f(t,r) \qquad h = \bar{h}(r) + \delta h(t,r)$$

$$A_0 = \bar{A}_0(r) + \delta A_0(t,r) \qquad \Psi = r\Phi = \bar{\psi}(r) + \delta\psi(t,r)$$

Time-periodic perturbations

$$\delta\psi(t,r) = e^{-i\sigma t}\delta\psi(r)$$

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Time-periodic perturbations

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Perturbation equations

- Metric perturbations written in terms of matter perturbations
- Three perturbation equations for δA_0, $\mathrm{Re}(\delta\psi)$, $\mathrm{Im}(\delta\psi)$
- Two of these are dynamical and involve $\delta\ddot{\psi}$
- Third is a constraint equation involving $\delta A_0''$ and no time derivatives

Perturbation boundary conditions

Event horizon $r = r_h$

Ingoing boundary conditions

$$\delta\psi \sim e^{-i\sigma(t+r_*)}\delta\tilde{\psi}(r) \qquad \frac{dr_*}{dr} = \frac{1}{\bar{f}\sqrt{\bar{h}}}$$

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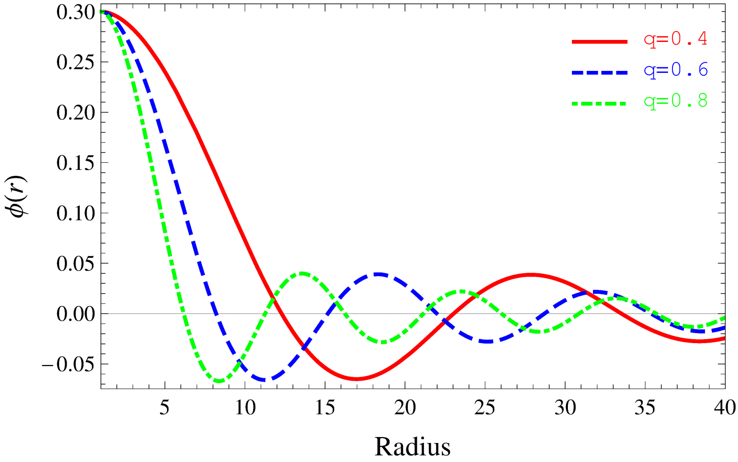
Mirror $r = r_m$

$$\delta\tilde{\psi} = 0$$

Black holes

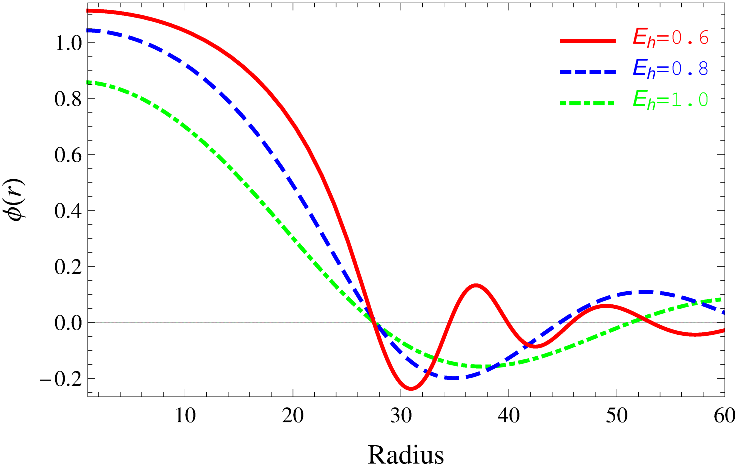
Scalar field profiles



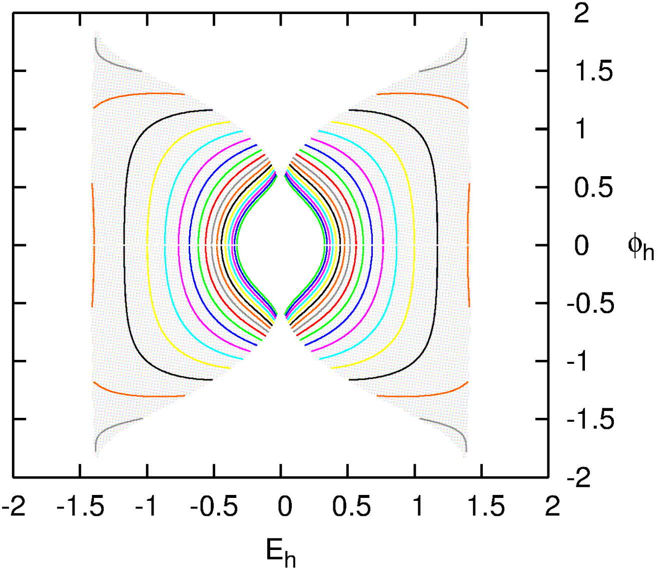
- $\phi = 0$ at mirror $r = r_m$
- Mirror at first zero of ϕ

Scalar field profiles

Different values of the parameters, same mirror radius r_m



Phase space of black hole solutions $r_h = 1, q = 0.1$

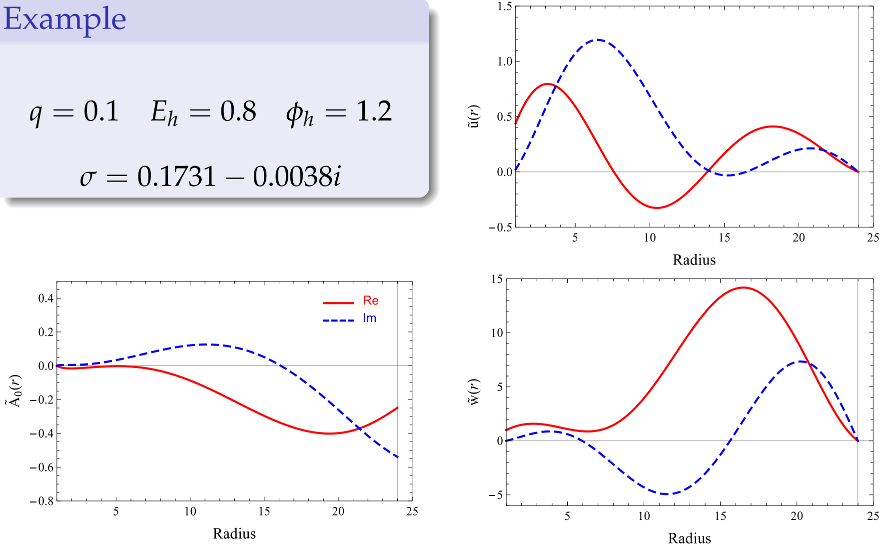


Solving the perturbation equations

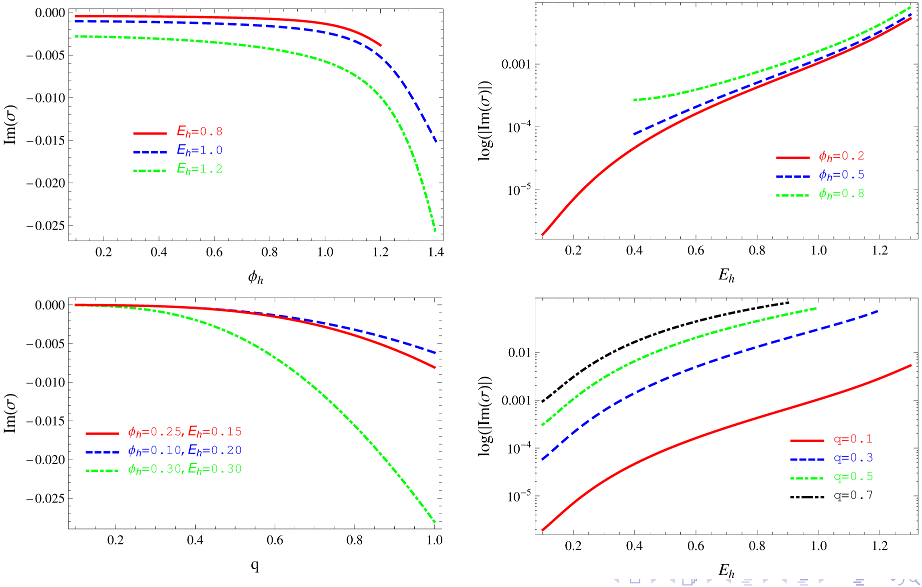
Example

$$q = 0.1 \quad E_h = 0.8 \quad \phi_h = 1.2$$

$$\sigma = 0.1731 - 0.0038i$$

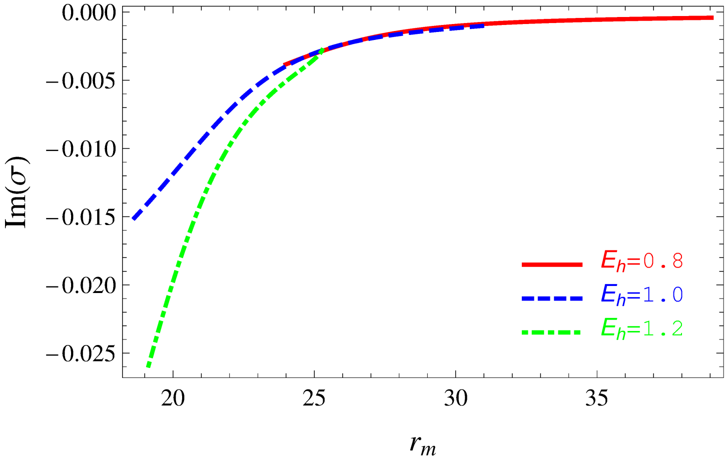


Black holes are stable: $\mathrm{Im}(\sigma) < 0$



Dependence on mirror radius r_m

Charged-scalar hairy black holes exist for $r_m \gtrsim 18$ when $r_h = 1$, $q = 0.1$



Solitons

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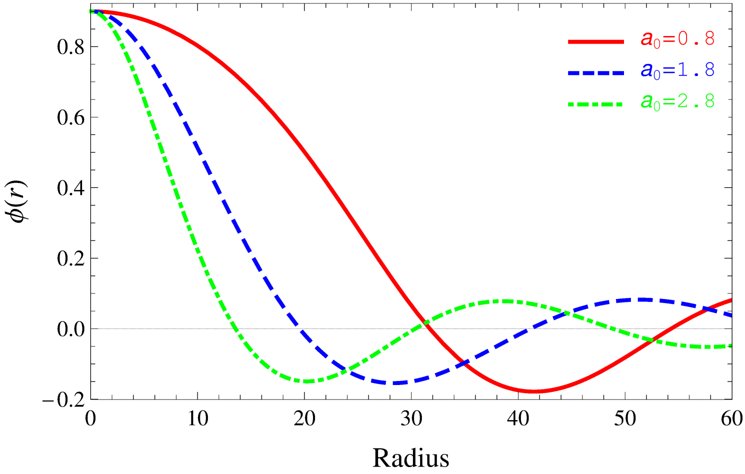

- $\phi = 0$ at mirror $r = r_m$
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Scalar field profiles

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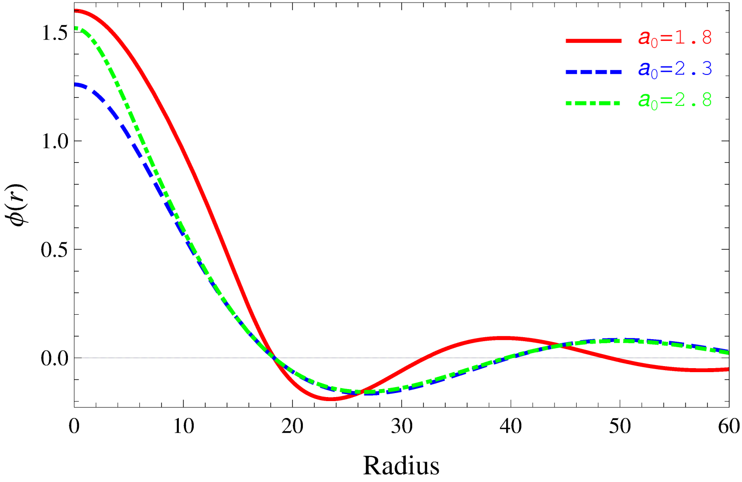
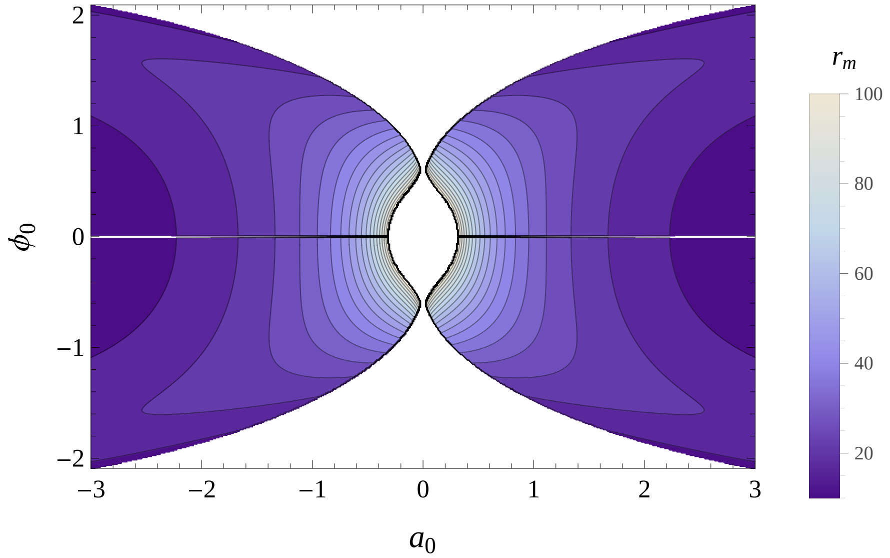
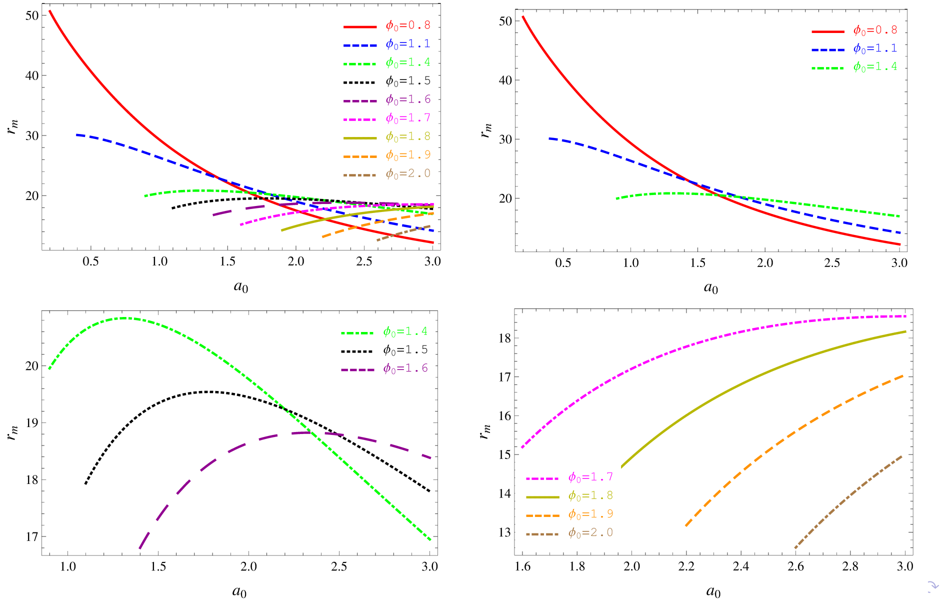


Phase space of soliton solutions $q = 0.1$



Mirror radius r_m

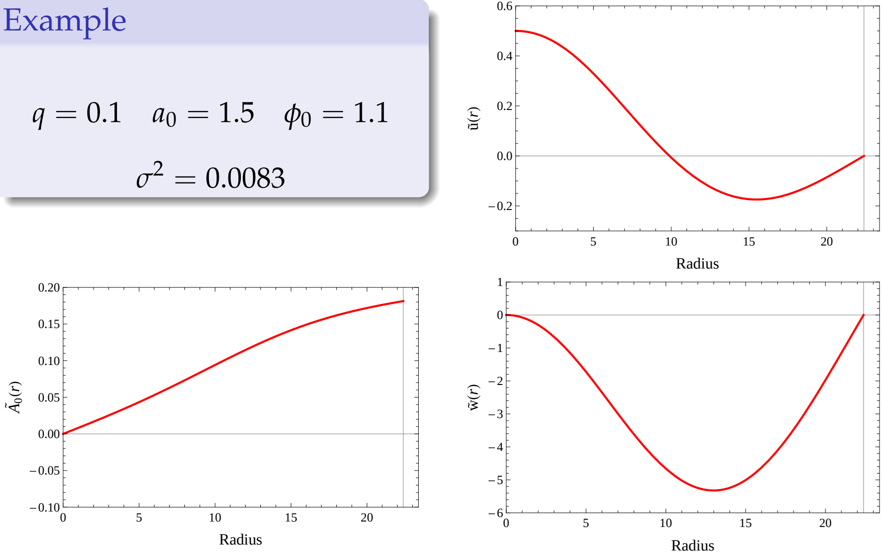


Solving the perturbation equations

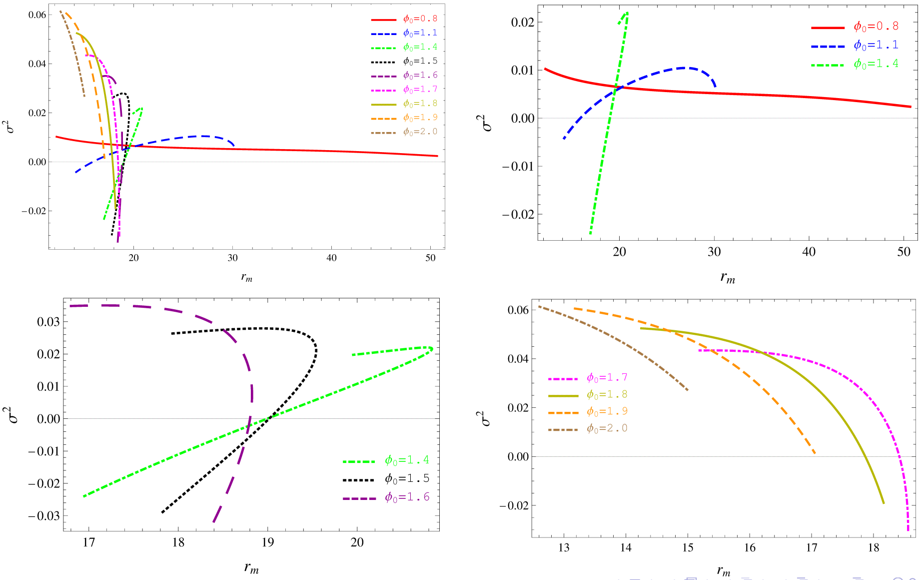
Example

$$q = 0.1 \quad a_0 = 1.5 \quad \phi_0 = 1.1$$

$$\sigma^2 = 0.0083$$

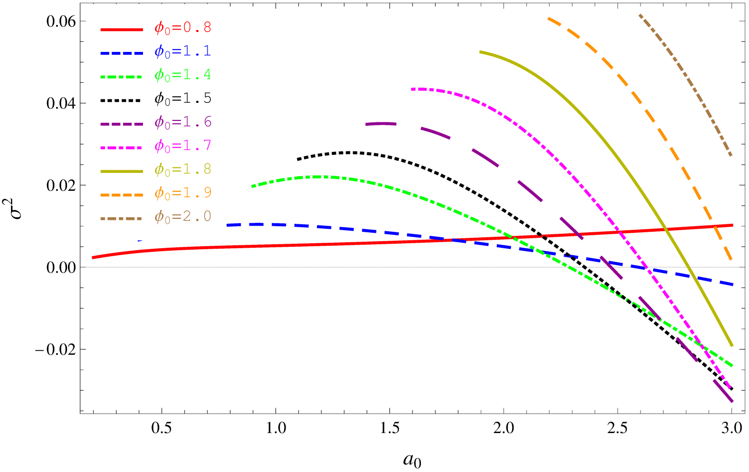


σ^2 as a function of mirror radius r_m



Stability of charged-scalar solitons

- All solitons stable for r_m sufficiently large
- Some unstable solitons for r_m sufficiently small



Conclusions

Charge superradiance instability

Reissner-Nordström black holes in a cavity

- Unstable to perturbations of charged scalar field
- Provided cavity is sufficiently large
- What is the end-point of this instability?

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Einstein-charged-scalar field theory

- Static, spherically symmetric solitons and black holes in a cavity
- Linear, spherically symmetric perturbations

Conclusions

Black holes

- Charged-scalar hairy black holes exist if the cavity is sufficiently large
- Stable if the mirror is at the first zero of the scalar field
- Possible end-point of the charge superradiance instability?

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- Charged-scalar hairy black holes exist if the cavity is sufficiently large
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- Possible end-point of the charge superradiance instability?

Solitons

- Charged-scalar solitons exist for all cavity sizes
- Stable if the cavity is sufficiently large
- Some solitons are unstable if the cavity is small