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Black holes in expanding spacetimes

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Outline

- Introduction - some historical remarks

– The paper by McVittie

– Charged objects in expanding spacetimes

– Rotating objects in expanding spacetimes

Our work (with M. Rodrigues, CQG 2015):

- Charged McVittie metric
- Charged Vaidya metric
- Charged Sultana-Dyer metric
- Charged Thakurta metric
- Summary/Conclusion

THE MASS-PARTICLE IN AN EXPANDING UNIVERSE.

G. C. McVittie, Ph.D.

I. *Introduction and Summary*

In the astronomical applications of General Relativity two types of metric for the universe are used. For discussing the motion of planets round the Sun, the statical Schwarzschild metric is employed, which may be written in isotropic co-ordinates * as

$$ds^2 = \left(\frac{1 - m/2r_1}{1 + m/2r_1}\right)^2 dt^2 - \frac{1}{c^2}\left(1 + \frac{m}{2r_1}\right)^4 \{dr_1{}^2 + r_1{}^2(d\theta^2 + \sin^2\theta d\phi^2)\}; \qquad (1)$$

on the other hand, for dealing with the phenomenon of the recession of the spiral nebulæ non-statical metrics are used, which can be subdivided into two classes : the Lemaître class, in which

$$ds^2 = dt^2 - \frac{e^{\beta(t)}}{c^2}\left\{\frac{dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)}{(1 + r^2/4R^2)^2}\right\}, \qquad (2)$$

and the de Sitter class, in which

$$ds^2 = dt^2 - \frac{e^{\beta(t)}}{c^2}\{dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\}. \qquad (3)$$

In (2) the constant $1/R^2$, which may be positive or negative, gives the curvature of space as a whole, local irregularities being disregarded. In (3) the curvature of space is zero. A particular case of (3) is the original de Sitter universe, for which $\beta(t) = t/a$ where a is a constant.†

One important respect in which the metric (1) differs from (2) and (3) is

Introduction: McVittie paper - citations

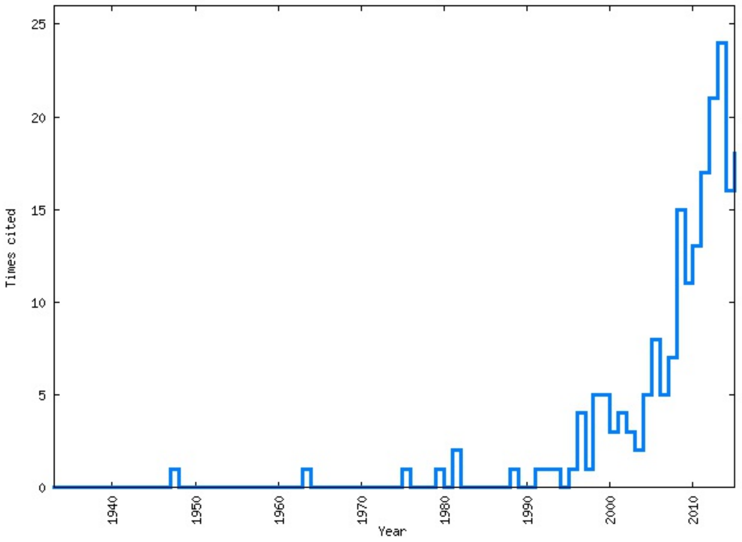

Figure: Citations from inspirehep.net, 82 years

Introduction: McVittie spacetime

Some references on the structure of McVittie spaceitme

- B. C. Nolan (PRD 58, 1998): "A point mass in an isotropic universe: Existence, uniqueness, and basic properties".
  – "A point mass in an isotropic universe: II. Global properties" (CQG 16, 1227, 1999).
  – "A point mass in an isotropic universe: III. The region $R \leq 2m$" (CQG 16, 3183 ,1999).
- N. Kaloper, M. Kleban, D. Martin (PRD 81, 2010): "McVittie's legacy: Black holes in an expanding universe".
  – Progress on understanding the global structure.
- K. Lake, M. Abdelgader (PRD 84, 2011): "More on McVittie's legacy: A Schwarzschild–de Sitter black and white hole embedded in an asymptotically ΛCDM cosmology".
  – Maximal analytic extension.

Introduction: McVittie spacetime

Some references on the structure of McVittie spaceitme

- V. Faraoni, A. F. Zambrano-Moreno, R. Nandra (PRD 85, 2012): "Making sense of the bizarre behavior of horizons in the McVittie spacetime".
- A. Maciel da Silva, M. Fontanini, D. C. Guariento (PRD 87, 2013): "How the expansion of the Universe determines the causal structure of McVittie spacetimes".

Charged objects in expanding universes

Some references

- P. C. Vaidya, Y. P. Shah (Current Science **66**, 1966): "The gravitational field of a charged particle embedded in an expanding universe"
  – Modifications of the **extremal** Reissner-Nordström metric.
- Y. P. Shah, P. C. Vaidya (Tensor **9**, 1968): "The gravitational field of a charged particle embedded in a homogeneous universe"
  – It is a generalization of the McVittie metric including electric charge.
- D. Kastor, J. Traschen (Phys. Rev. D 47, 1993): "Cosmological multi-black-hole solutions".
  – Extremely charged black holes.
  – Exact solution for coalescing black holes.
- C. J. Gao, S. N. Zhang (Phys. Lett. B 28, 2004):
  – Similar to the solution by Shah & Vaidya, made further analysis.

Charged objects in expanding universes

Some references

- M. L. McClure, C. C. Dyer (CQG 23, 2006): "Asymptotically Einstein–de Sitter cosmological black holes and the problem of energy conditions".
  – Analyzed some charged static solutions.
- G. W. Gibbons, K.-I. Maeda (PRL 104, 2010), "Black Holes in an Expanding Universe"
  – Generalized Kastor & Traschen work to $p = \omega\rho$ cosmic fluid.

Rotating objects in expanding universes

Some references

- P. C. Vaidya (Pramāna **8**, 1977): "Kerr metric in cosmological background".
  – Obtained the apparent horizon (He called event horizon). Asymptotic to flat FLRW metric
- S. N. G. Thakurta, (Indian J. Phys **55B**, 1981): "Kerr metric in an expanding universe".
  – Conformal to Kerr in Boyer-Lindquist coordinates. Asymptotic to general FLRW.
- P. C. Vaidya (Pramāna **22** , 1984): "Kerr metric in the de Sitter background".

Rotating charged objects in expanding universes

- L. K. Patel and H. B. Trivedi (J. Astrophys. Astron. **3**, 1982).
  "Kerr-Newman metric in cosmological background".
  – Followed Vaidya (Pramāna **8**, 1977).

The McVittie metric - flat FLRW background

A recipe?

- Write the Schwarzschild black hole solution in isotropic coordinates
- Change the “radial” coordinate r to $a(t)r$, $a(t)$ to be interpreted as the cosmological scale factor.

The McVittie metric (MNRAS, 1933) is usually written in the form

$$ds^2 = a^2(t)\left(1+\frac{m}{2ra(t)}\right)^4\left(dr^2+r^2d\Omega^2\right)-\left(1-\frac{m}{2ra(t)}\right)^2\left(1+\frac{m}{2ra(t)}\right)^{-2}dt^2,$$

where m is a constant.

Or, using the areal radial coordinate $R = a(t)\,r\left(1+\frac{m}{2a(t)\,r}\right)^2$,

$$ds^2 = -\left(F^2 - H^2R^2\right)dt^2 - \frac{2HR}{F}dRdt + \frac{dR^2}{F^2} + R^2d\Omega^2\,, \qquad (1)$$

$F(R) = \sqrt{1-\frac{2m}{R}}$, and $H(t) = \frac{\dot{a}}{a}$ is the Hubble factor.

The McVittie metric - Singularity and horizons

Curvature singularity at $F(R) = 0 \Rightarrow R = 2m$.

Apparent horizons (zeros of the expansion of outgoing null geodesics):

$$-H^2(t)R^2 + \left(1 - \frac{2m}{R}\right) = 0.$$

The McVittie metric - global structure

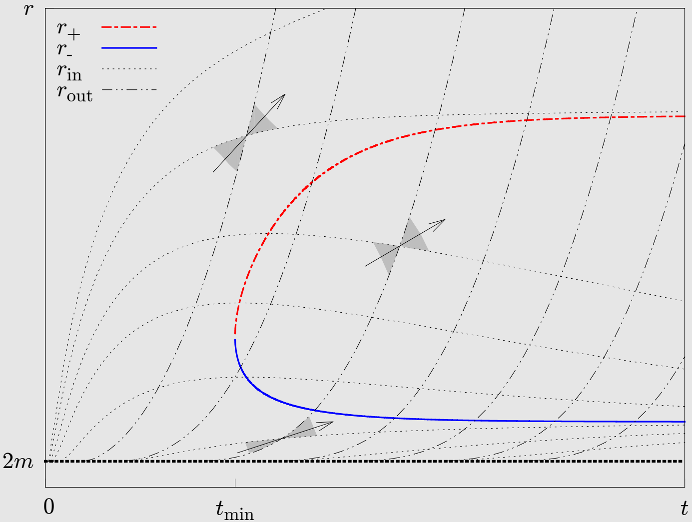

Figure: Apparent horizons - McVittie spacetime: $a(t)$ asymptotes a de Sitter expansion (from Silva, Fontanini, Guariento - PRD, 2013)

The McVittie metric - global structure

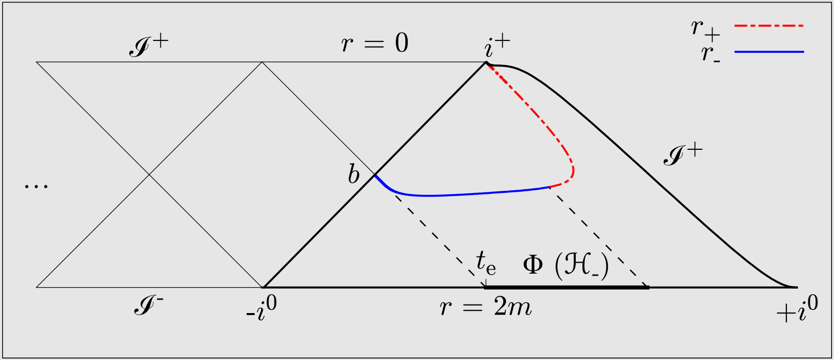

Figure: A possible maximal analytical extension of McVittie spacetime (from Silva, Fontanini, Guariento - PRD, 2013)

Our work

- Motivated by the work of M. L. McClure & C. C. Dyer (CQG 23, 2006).
- Investigate the possible electromagnetic sources, singularities and apparent horizons.
- The charged McVittie solution is revisited
- Charged versions of solutions originally put forward by Vaidya (Vd), Sultana and Dyer (SD), and Thakurta are also analyzed.

Charged objects in expanding spacetimes: general metric

The generic metric

A sufficiently general metric for all cases we are interested here is of the form

$$ds^2 = -f_0(r,t)\,dt^2 + a^2(t) f_1(r,t)\,dr^2 + 2\,a(t) f_2(r,t)dt\,dr + a^2(t)\,r^2 f_3(r,t)\,d\Omega^2,$$

t= cosmological time, r= spherical coordinate, $a(t)$= the expansion factor, $d\Omega^2$ is the metric on the unit sphere.

The scale factor $a(t)$:

- Case (a): used by Lake & Abdelgader (PRD84,2011)
  $a(t) = \left[\sinh\left(\dfrac{3k\,t}{2}\right)\right]^{2/3}$, k being a constant to be chosen appropriately.
  – Initial power-law expansion with $a(t) \sim t^{2/3}$ and a final de Sitter accelerated phase $a(t) \sim \mathrm{e}^{kt}$.
- Case (b): $a(t) = t^{\alpha}$, with constant α.
  – Perfect fluid whose equation of state is of the form $p = \omega\,\rho$, with $\omega = (2-3\alpha)/2$. Our choice is $\alpha = 2/3$, and so $\omega = 0$.

Charged objects in expanding spacetimes: general metric

The Maxwell equations

The Maxwell equations ($F^{tr} = -F^{rt} \equiv E(r,t)$) give the two relations

$$\frac{\partial Q(r,t)}{\partial r} = 4\pi a^3(t)\, r^2 f_3(r,t)\, g_1(r,t)\, J^t(r,t), \tag{2}$$

$$\frac{\partial Q(r,t)}{\partial t} = -4\pi\, a^3(t)\, r^2 f_3(r,t)\, g_1(r,t)\, J^r(r,t), \tag{3}$$

$$Q(r,t) \equiv a^3(t)\, r^2 f_3(r,t)\, g_1(r,t)\, E(r,t), \tag{4}$$

with $J^t(r,t)$ and $J^r(r,t)$ being the only nonzero components of the electromagnetic current-density. ${g_1}^2(r,t) \equiv f_0(r,t) f_1(r,t) + {f_2}^2(r,t)$.
$J^t(r,t) = 0 \Rightarrow Q = q_0 h(t)$, q_0 constant, $h(t)$ arbitrary.

$$E(r,t) = \frac{q_0 h(t)}{a^3(t)\, r^2 f_3(r,t)\, g_1(r,t)}, \tag{5}$$

$$J^r(r,t) = -\frac{q_0 \dot{h}(t)}{4\pi a^3(t)\, r^2 f_3(r,t)\, g_1(r,t)}. \tag{6}$$

Charged McVittie

$$ds^2 = -\frac{f^2(r,t)}{g^2(r,t)}dt^2 + a^2(t)g^2(r,t)\left(dr^2 + r^2 d\Omega^2\right), \tag{7}$$

$$f(r,t) = 1 - \frac{m^2}{4a^2(t)\,r^2} + \frac{q^2}{4a^2(t)\,r^2}, \quad g(r,t) = \left(1 + \frac{m}{2a(t)\,r}\right)^2 - \frac{q^2}{4a^2(t)\,r^2}. \tag{8}$$

$$Q(r,t) = q_0 h(t); \quad E(r,t) = \frac{q_0 h(t)}{a^3(t)\,r^2 f(r,t)\,g^2(r,t)}.$$

–The solution presented by Shah & Vaidya (Tensor 19, 1968, also Gao & Zhang, PLB 595, 2004) assumes $h(t) = 1$.

–Leading to $Q(r,t) = q_0 =$ constant, so that $q_0 = q$, q is the charge parameter of the metric.

$J^\mu(r,t) = 0$ for $a^3(t)\,r^2 f(r,t)\,g^2(r,t) \neq 0$.

The charged McVittie metric - Singularity and horizons

The areal radius: $R = a(t)\, r\, g(r,t) = a(t)\, r \left(1 + \frac{M}{2a(t)\, r}\right)^2 - \frac{Q^2}{4a(t)\, r}$.

Curvature singularity at
$F(R) = 1 - 2M/R + Q^2/R^2 = 0 \Longrightarrow R_\pm = 2M \pm \sqrt{M^2 - Q^2}$.

Apparent horizons (zeros of the expansion of outgoing null geodesics):

$$-H^2(t)\, R^2 + \left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right) = 0.$$

The charged McVittie metric - global structure

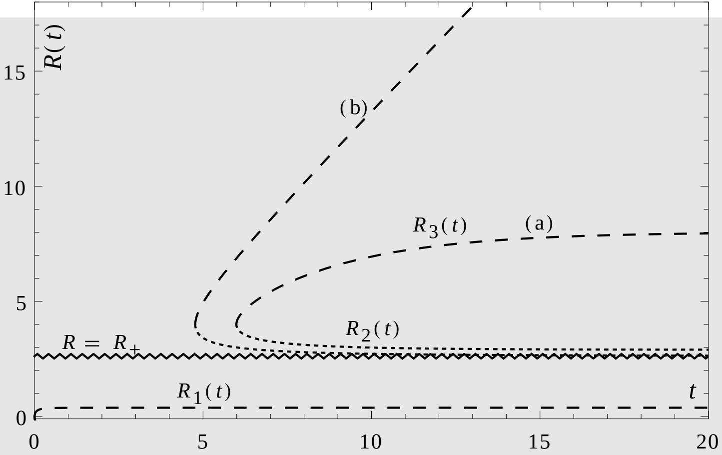

Figure: The evolution of the apparent horizons as a function of the time t of the charged McVittie spacetime, for two different expansion factors $a(t)$ of case (a) and case (b), as indicated. The plots are for $M = 2.0$, $Q = 1.0$. The singularity at $R = R_+$ is indicated. For late times, there are two apparent horizons (similar to the uncharged case). See also V. Faraoni, A. F. Zambrano-Moreno, A. Prain, PRD 89, 2014.

Charged Vaidya type metric

The metric

$$ds^2 = a^2(t)\left[dr^2 + r^2 d\Omega^2\right] + \left[\frac{2m(t)}{r} - \frac{q^2(t)}{r^2}\right]\left[\frac{dt}{a(t)} + dr\right]^2 - dt^2, \tag{9}$$

- Original Vaidya (cosmological) metric: $m(t)$ = constant, $q(t) = 0$.
- Faraday-Maxwell tensor field is $F^{tr} \equiv E(r,t) = q_0 h(t)/a^3(t)r^2$, q_0 being a constant.
- $Q(r,t) = q_0 h(t)$, $q_0 =$ constant, $h(t)$ arbitrary.
- Choosing $h(t) = 1$, i.e., $E(r,t) = q_0/a^3(t)r^2$, with constant q_0, the Maxwell equations are satisfied with zero current-density, $J^\mu(r,t) = 0$ for $a^3(t)r^2 \neq 0$.

Charged Vaidya type metric - apparent horizons

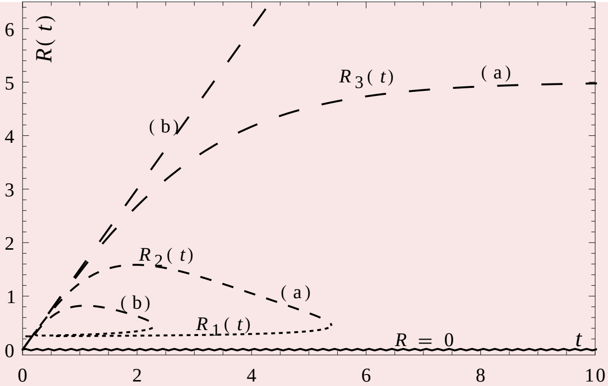

Figure: The evolution of the apparent horizons as a function of the time t of the charged Vaidya spacetime, for two different expansion factors $a(t)$, of case (a) and case (b), as indicated. The plots are for $M = m = 2.0$, $Q = q = 1.0$. The singularity at $R = 0$ is indicated.

Charged Sultana-Dyer type metric

The metric

$$ds^2 = a^2(t)\left[dr^2 + r^2 d\Omega^2\right] + \left[\frac{2m(t)}{r} - \frac{q^2(t)}{r^2}\right]\left[dt + a(t)\,dr\right]^2 - dt^2, \qquad (10)$$

- Original SD (GRG 37, 2005) metric: $m(t) = \text{constant}$, $q(t) = 0$.
- Equals the Vaidya metric if $ma^2(t) \to m$ and $qa(t) \to q$.
- Faraday-Maxwell tensor field is $F^{tr} \equiv E(r,t) = q_0 h(t)/a^3(t)r^2$, q_0 being a constant.
- $Q(r,t) = q_0 h(t)$, $q_0 = \text{constant}$, $h(t)$ arbitrary.
- Solution of McClure-Dyer (CQG, 2006): $h(t) = a(t)$, the Maxwell equations are satisfied with nonzero current-density, $J^r(r,t) = -q_0 H(t)/(4\pi a^2(t) r^2)$.
- Choosing $h(t) = 1$ (our choice), i.e., $E(r,t) = q_0/a^3(t)r^2$, with constant $q_0 = q \Rightarrow$ constant electric charge of the source.

Charged static Thakurta type metric

The metric

$$ds^2 = -\left(1 - \frac{2m(t)}{r} + \frac{q^2(t)}{r^2}\right) dt^2 + \frac{a^2(t)\, dr^2}{1 - \dfrac{2m(t)}{r} + \dfrac{q^2(t)}{r^2}} + a(t)^2 r^2 d\Omega^2, \quad (11)$$

- Original Thakurta (cosmological) metric: $m(t)$ = constant, $q(t) = 0$.
- Faraday-Maxwell tensor field as $F^{tr} \equiv E(r,t) = q_0 h(t)/a^3(t) r^2$, q_0= constant.
- $Q(r,t) = q_0 h(t)$, q_0 = constant, $h(t)$ arbitrary.
- Solution of McClure-Dyer (CQG, 2006): $h(t) = a(t)$, the Maxwell equations are satisfied with nonzero current-density, $J^r(r,t) = -q_0 H(t)/(4\pi a^2(t) r^2)$.
- Choosing $h(t) = 1$, i.e., $E(r,t) = q_0/a^3(t) r^2$, with constant q_0 = constant = electric charge of the source.
- Last choice implies modifications also in energy-momentum, but the singularity at spacetime region where $f(r,t) = 0$ is not removed.

Charged Thakurta type metric - apparent horizons

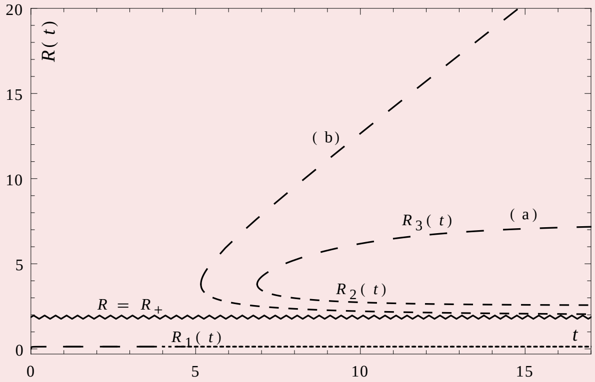

Figure: The evolution of the apparent horizons as a function of the time t in the undercharged case of the charged Thakurta spacetime, for two different expansion factors. The plots are for $M = m(t)a(t) = 1.0$, $Q = q(t)a(t) = 0.5$, $a(t)$ of case (a), and $a(t)$ of case (b), as indicated. The singularity at $R = R_+$ is indicated. For late times, there are three apparent horizons.

Summary/Conclusion

– A brief historical review of black hole solutions in expanding backgrounds was given

– Investigated the electromagnetic sources of Vaidia, Sultana-Dyer and Thakurta metrics

– The singularities and apparent horizons were found for each case

– Some progress on understanding the global structure of the mentioned spacetimes has been made.

Thank You!