

Topology changing Dirac-Nambu-Goto branes on higher dimensional black hole backgrounds

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V.G.Cz. and Antonino Flachi, Phys. Rev. D **80** 104017 (2009),

V.G.Cz. Phys. Rev. D **82** 024035 (2010),

V.G.Cz. Phys. Rev. D **83** 064026 (2011).

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Introduction

Higher dimensional black objects and branes are of importance and interest in several areas of present days physics.

- Classical black hole uniqueness theorems fail in higher dimensions;
- New type of black objects appear (strings, rings, cigars, etc.);
- The properties of possible transitions between the different types, or *phases*, are of special interest; [1]

In a recent paper [2], Frolov suggested a simple toy model for studying *merger* and *topology* changing transitions which also shows similarities in certain aspects with

- Other topology change transitions in classical and quantum gravity;
- Choptuik critical collapse phenomenon [3] (self similarity);

And it also turned out to be a very a useful model in the study of

Holographic phase transition of fundamental matter [4].

[1] B. Kol, Phys. Rep. **422**, 119 (2006); [2] V.P. Frolov, Phys. Rev. D **74**, 044006 (2006); [3] M.W. Choptuik, Phys. Rev. Lett. **70** 9 (1993); [4] D. Mateos, R.C. Myers and R.M. Thomson, Phys. Rev. Lett. **97** 091601 (2006); JHEP **05** 067 (2007).

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The brane - black hole toy model

BBH system

The toy model is a static test brane interacting with a bulk static, spherically symmetric black hole (BBH system). The metric of the bulk N -dimensional space-time is

$$ds^2 = g_{ab}dx^a dx^b = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_{N-2}^2 ,$$

where $f = f(r)$, and $d\Omega_{N-2}^2$ is the metric of the $(N - 2)$ -dimensional unit sphere S^{N-2} .

The explicit form of f is not important, it is assumed only that $f(r_0) = 0$ at the horizon and it grows monotonically from 0 to 1 at spatial infinity, where it has the asymptotic form

$$f = 1 - \left(\frac{r_0}{r}\right)^{N-3} .$$

Coordinates on the sphere

The coordinates $\theta_i (i = 1, \dots, N - 2)$ on the sphere are given by the relations

$$d\Omega_{i+1}^2 = d\theta_{i+1}^2 + \sin^2 \theta_{i+1} d\Omega_i^2 .$$

Bulk and brane coordinates

The bulk coordinates are $x^a \{t, r, \theta_1, \dots, \theta_{N-2}\}$, while the coordinates on the brane world sheet are $\zeta^\mu \{t, r, \theta_1, \dots, \theta_{D-2}\}$. We assume that $D \leq N - 1$ and that the brane is static and spherically symmetric, so that its world sheet geometry possesses the group of symmetry $O(D - 1)$. If $D < N - 1$ we choose the brane surface to obey the equation $\theta_D = \dots = \theta_{N-2} = \pi/2$.

With this parametrization and symmetry properties the brane world sheet is uniquely defined by the function $\theta_{D-1} = \theta(r)$ only, and the induced metric on the brane is

$$\gamma_{\mu\nu} d\zeta^\mu d\zeta^\nu = -f dt^2 + \left[f^{-1} + r^2 \dot{\theta}^2 \right] dr^2 + r^2 \sin^2 \theta d\Omega_n^2.$$

Dirac-Nambu-Goto action

Test brane configurations in an external gravitational field, g_{ab} , can be obtained by solving the Euler-Lagrange equation from the Dirac-Nambu-Goto action,

$$S = \int d^D \zeta \sqrt{-\det \gamma_{\mu\nu}}, \quad \gamma_{\mu\nu} = g_{ab} \frac{\partial x^a}{\partial \zeta^\mu} \frac{\partial x^b}{\partial \zeta^\nu}.$$

In our case the action simplifies to

$$S = \Delta t \mathcal{A}_n \int \mathcal{L} dr \quad \text{with} \quad \mathcal{L} = r^n \sin^n \theta \sqrt{1 + fr^2 \dot{\theta}^2},$$

where Δt is an arbitrary interval of time and $\mathcal{A}_n = 2\pi^{n/2}/\Gamma(n/2)$ is the surface area of a unit n -dimensional sphere. The Euler-Lagrange equation

$$\frac{d}{dr} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$

then takes the form

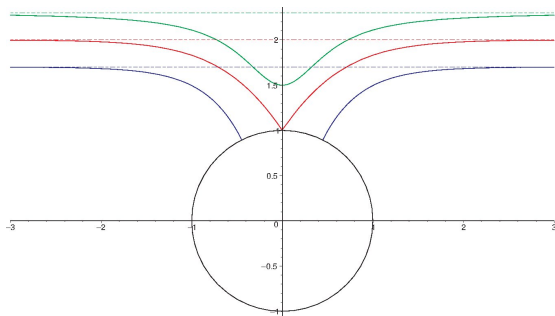
Euler-Lagrange equation

$$\ddot{\theta} + B_3 \dot{\theta}^3 + B_2 \dot{\theta}^2 + B_1 \dot{\theta} + B_0 = 0,$$

$$B_0 = -\frac{n \cot \theta}{fr^2}, \quad B_1 = \frac{n+2}{r} + \frac{\dot{f}}{f},$$

$$B_2 = -n \cot \theta, \quad B_3 = r \left[\frac{1}{2} r \dot{f} + (n+1)f \right].$$

Two topologically different solutions



Supercritical solution

For a brane crossing the horizon the EL equation has a **regular singular** point at $r = r_0$. A regular solution at this point has the following expansion near it

$$\theta = \theta_0 + \dot{\theta}_0(r - r_0) + \dots, \quad \text{with} \quad \dot{\theta}_0 = \left. \frac{n \cot \theta}{\dot{f} r^2} \right|_{r_0},$$

and it is uniquely determined by the initial value θ_0 .

Subcritical solution

In the subcritical case the brane does not cross the horizon, and its surface reaches the minimal distance from the black hole at $r_1 > r_0$ which, for symmetry reasons, occurs at $\theta = 0$. A regular solution near this point has the behavior

$$\theta = \eta\sqrt{r - r_1} + \sigma(r - r_1)^{3/2} + \dots, \quad \eta = \sqrt{\frac{2(n+1)}{B_3(r)}} \Big|_{r_1}$$

such a solution is also uniquely determined by the parameter r_1 .

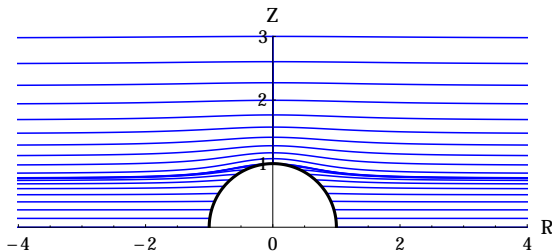


Figure: N=6, D=5

Near horizon behavior

Frolov studied the near critical solutions under the following conditions.

Rindler zone condition

Assuming that the radius R_0 of the surface of the intersection of the brane with the bulk horizon is much smaller than the size of the horizon r_0 , the space-time close to the bulk black hole horizon can be approximated by the Rindler space, where the horizon is an $(n+1)$ -dimensional plane.

And he found that

Properties

- The critical solution is an attractor, and both families are attracted to the critical solution asymptotically;
- The close to critical solutions have a self-similar behavior;
- There exist a critical dimension $D^* = 6$, such that for $D > D^*$ this symmetry is continuous, while for $D \leq D^*$ it is discrete;

These properties are very similar to the ones in the case of a caged black hole - black string transition and the Choptuik critical collapse phenomenon.

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Holographic phase transition of fundamental matter

AdS/CFT

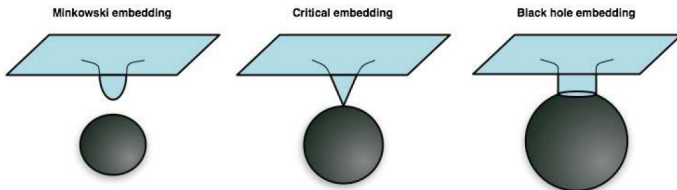
The gauge/gravity correspondence is a useful tool to study the nonperturbative physics of gauge theories in diverse dimensions. The classical supergravity regime corresponds to the large N_c (strong 't Hooft coupling) limit of the gauge theory. This allows the study of a large class of theories that share some of the important features of 4-dimensional QCD, such as **confinement/deconfinement**, **thermal phase transitions**, etc.

Holographic dual picture

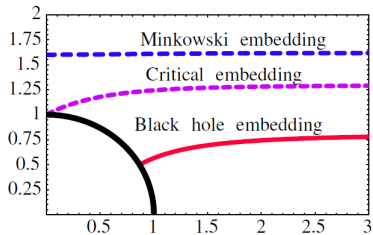
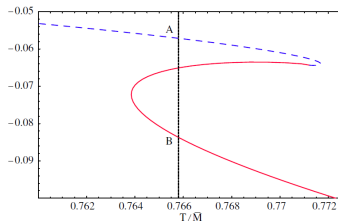
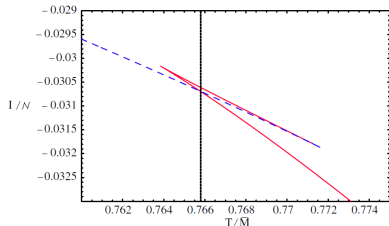
In the dual picture, a small number of flavors of fundamental matter,

$$N_f \ll N_c$$

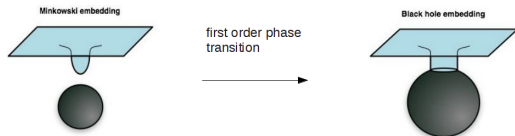
may be described by N_f probe Dq -branes in the gravitational background of N_c probe Dp -branes. At a sufficiently high temperature T , the background geometry contains a black hole. It was recently shown that these systems generally undergo a universal **first order phase transition** characterized by a change in the behavior of the fundamental matter. [D. Mateos, R.C. Myers and R.M. Thomson, Phys. Rev. Lett. **97** 091601 (2006); JHEP **05** 067 (2007)]



- Increasing the temperature increases both the radial position and the energy density of the event horizon in the Dp -brane throat. For sufficiently small temperature the probe branes are gravitationally attracted towards the horizon but their tension balance this attractive force. The probe branes then lie entirely outside the black hole. This case is called “Minkowski” embedding.
- Above a critical temperature T_{fun} , the gravitational force overcomes the tension and the branes are pulled into the horizon. Such configurations are referred as “black hole” embedding.
- In between the two phases, there exists a critical solution which just “touches” the horizon.



Frolov has shown that in the vicinity of the critical solution the embeddings show a self similar behavior. As a result multiple solutions exist for a given temperature close to T_{fun} . From thermodynamic considerations to select the true ground state, a **first order phase transition** reveals at T_{fun} , where the probe branes jump discontinuously from a Minkowski to a black hole embedding.



In the dual field theory this transition is exemplified by discontinuities in the quark condensate. The most striking feature of this phase transition is found in the mass spectrum of the mesons (i.e. the quark-antiquark bound states).

Minkowski phase (low-temperature)

The mesons are stable (to leading order within the approximation of large N_c and strong coupling) and the spectrum is discrete with a finite mass gap.

Black hole phase (high-temperature)

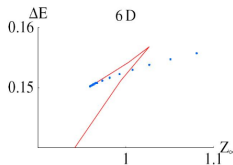
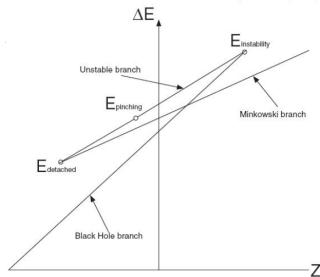
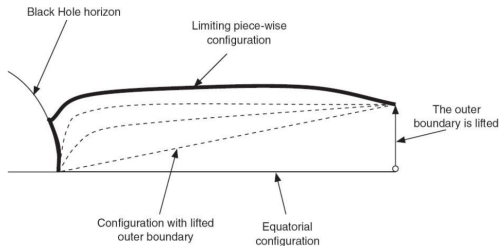
Stable mesons cease to exist, rather one finds a continuous and gapless spectrum.

Hence the first order phase transition is characterized by the dissociation or "melting" of mesons. This physics is particularly interesting in theories that exhibit a confinement/deconfinement phase transitions such as QCD, where the question of quark deconfinement has been a long standing problem.

Gravity side

When the outer boundary is slightly lifted the initial configuration is relaxed to the one with the least action which is the piecewise limiting configuration drawn below.

A. Flachi, O. Pujolás, M. Sasaki and T. Tanaka, Phys. Rev. D **74**, 045013 (2006)



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Thickness corrections

MOTIVATION

Finite 't Hooft coupling corrections (not too large N) corresponds to higher-derivative corrections to the D-brane action. These corrections are likely to spoil the scaling symmetry of the solution close to the critical embedding and hence the self similar behavior. This **may change the** structure of the **phase transition** discussed earlier and thus would give a more realistic picture of the meson spectrum or the deconfinement.

Curvature corrections to the DNG brane action [B. Carter and R. Gregory, Phys. Rev. D 51 5839 (1995)]

Carter and Gregory showed that the corrections to the Dirac-Nambu-Goto action is in the quadratic order in the thickness and can be expressed in terms of the intrinsic Ricci scalar R and the extrinsic curvature scalar K as

$$S = \int d^D \zeta \sqrt{-\det \gamma_{\mu\nu}} \left[-\frac{8\mu^2}{3\ell} (1 + C_1 R + C_2 K^2) \right], \quad C_1 = \frac{\pi^2 - 6}{24} \ell^2, \quad C_2 = -\frac{1}{3} \ell^2.$$

The thickness $\ell = 1/\mu\sqrt{2\lambda}$ originates from a ϕ^4 field theoretical domain-wall model where μ is the mass parameter and λ is the coupling constant of the scalar field.

The Ricci scalar of the intrinsic metric of the brane can be obtained by the Gauss formula

Gauss formula

$$R = K^2 - K_b^a K_a^b \equiv K^2 - Q ,$$

and for K and Q we get

Curvature functions

$$K = \frac{1}{F} \left[\frac{r\dot{\theta}f}{2} + \frac{B}{2F^2} + (n+1)f\dot{\theta} - \frac{n \cot \theta}{r} \right] ,$$

$$Q = \frac{1}{F^2} \left[\frac{r^2\dot{\theta}^2 f^2}{4} + \frac{B^2}{4F^4} + \frac{f\dot{\theta}B}{F^2} + f^2\dot{\theta}^2 + n \left[f\dot{\theta} - \frac{\cot \theta}{r} \right]^2 \right] ,$$

where

$$F = \sqrt{1 + fr^2\dot{\theta}^2} , \quad B = [rf + 2f] \dot{\theta} + 4rf\ddot{\theta} .$$

New brane action

The new effective action can be re-expressed as

$$S = \Delta t \mathcal{A}_n \int \mathcal{L} dr, \quad \mathcal{L} = -\frac{8\mu^2}{3\ell} \mathcal{L}_0 [1 + \varepsilon\delta],$$

with

$$\varepsilon = \frac{\ell^2}{L^2}, \quad \delta = \left[\frac{C_1 L^2}{\ell^2} R + \frac{C_2 L^2}{\ell^2} K^2 \right], \quad \frac{1}{L} \sim \max\{K, \sqrt{|R|}\}.$$

Euler-Lagrange equation

Since we have now **second derivative terms** in the Lagrangian, the Euler-Lagrange equation becomes *4th* order

$$\frac{d^2}{dr^2} \left(\frac{\partial \mathcal{L}}{\partial \ddot{\theta}} \right) - \frac{d}{dr} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) + \frac{\partial \mathcal{L}}{\partial \theta} = 0,$$

and can be separated in the following way

$$\frac{d}{dr} \left(\frac{\partial \mathcal{L}_0}{\partial \dot{\theta}} \right) - \frac{\partial \mathcal{L}_0}{\partial \theta} - \varepsilon \left[\frac{d^2}{dr^2} \left(\frac{\partial (\mathcal{L}_0 \delta)}{\partial \ddot{\theta}} \right) - \frac{d}{dr} \left(\frac{\partial (\mathcal{L}_0 \delta)}{\partial \dot{\theta}} \right) + \frac{\partial (\mathcal{L}_0 \delta)}{\partial \theta} \right].$$

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Perturbative results

Linear perturbation method

$$\tilde{\theta}(r) = \theta(r) + \varepsilon\varphi(r) + o(\varepsilon^2),$$

Perturbation equation

$$\ddot{\varphi} + q_1\dot{\varphi} + q_0\varphi + q = 0,$$

with

$$q_1 = B_1 + 2\dot{\theta}B_2 + 3\dot{\theta}^2B_3,$$

$$q_0 = \frac{n}{\sin^2\theta} \left[\frac{1}{r^2 f} + \dot{\theta}^2 \right],$$

$$q = q\left(\theta^{(4)}, \theta^{(3)}, \ddot{\theta}, \dot{\theta}, \theta, f^{(3)}, \ddot{f}, \dot{f}, f^4, f^3, f^2, f, r\right).$$

Far distance solution

$$n = 1 \quad \varphi = \frac{P + P' \ln r}{r} - \frac{E_1 + E_2(1 + \ln r)}{4r^3}$$

$$n > 1 \quad \varphi = \frac{P}{r} + \frac{P'}{r^n} + \frac{E}{2(n-3)r^3}$$

$$n = 3 \quad \varphi = \frac{P}{r} + \frac{P'}{r^3} + \frac{E[1 + 2 \ln r]}{4r^3}$$

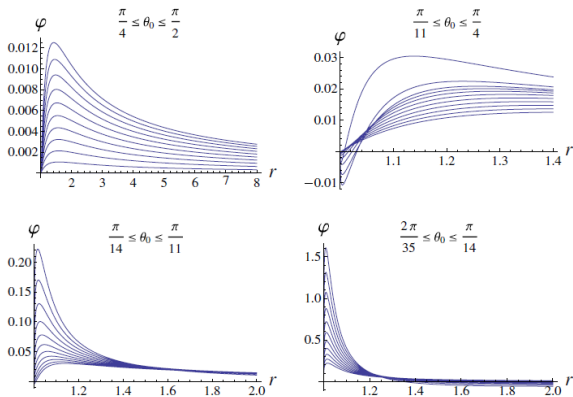
Black hole embedding

For the brane crossing the horizon, the source term q has a $1/f$ singular behavior as $r \rightarrow r_0$, and the differential equation has a **regular singular point**. Requiring regularity implies the condition:

$$\dot{\varphi}_0 = -\frac{1}{m} \left[\frac{n\varphi_0}{\sin^2 \theta_0} + \alpha_0 \right]$$

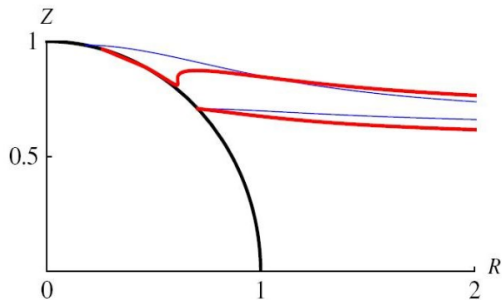
where $m = N - 3$ and α is a constant depends on the initial conditions. Thus the black hole embedding perturbations are uniquely determined by the initial value φ_0 .

Perturbative solutions - supercritical case



The picture shows the numerical solution of the perturbation Eq. (42) with varying the parameter θ_0 of the brane initial inclination in the case of $N = 5$, $n = 2$, black hole embedding.

Perturbative solutions - supercritical case



The picture shows the thick (red) brane configurations together with their thin (blue) counterparts in a cylindrical coordinate system, in the case of an $N = 5$, $n = 2$ black hole embedding. The initial conditions are $\theta_0 = \frac{\pi}{4}$ (bottom curves) and $\frac{\pi}{17.5}$ (top curves), and the thickness parameter ℓ is chosen to be large for the purpose of making the effects visible. The black curve represents the black hole's event horizon.

Minkowski embedding

If the brane does not cross the horizon the brane surface reaches its minimal distance to the black hole at $r = r_1 > r_0$. Near this point θ has the asymptotic form

$$\theta = \eta\sqrt{r - r_1} + \sigma(r - r_1)^{3/2} + \dots ,$$

and one can find that q has a singular behavior

$$q \sim \frac{c_1}{\sqrt{r - r_1}} + \frac{c_3}{(r - r_1)^{3/2}} + \frac{c_5}{(r - r_1)^{5/2}},$$

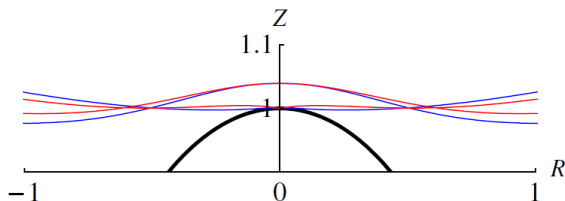
as $r \rightarrow r_1$, where

$$c_5 = \frac{n(n-1)\eta}{8(n+1)r_1} \left[2(a + 2b)(n + 1)f + (a + 3b)r\dot{f} \right]_{r_1} ,$$

The perturbation equation near the axis ($\theta = 0$) has the asymptotic form

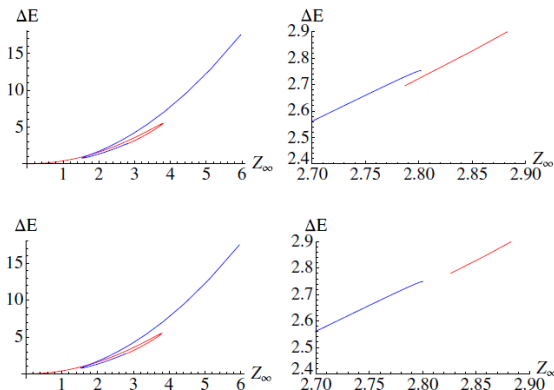
$$\ddot{\varphi} + \frac{n + 3}{2(r - r_1)}\dot{\varphi} + \left[\frac{n}{4(r - r_1)^2} + \frac{\xi}{r - r_1} \right] \varphi + \frac{c_1}{\sqrt{r - r_1}} + \frac{c_3}{(r - r_1)^{3/2}} + \frac{c_5}{(r - r_1)^{5/2}} = 0.$$

Perturbative solutions - subcritical case



The picture shows the thick (red) brane configurations together with their thin (blue) counterparts in a cylindrical coordinate system, in the case of an $N = 5$, $n = 1$ Minkowski embedding. The initial parameters are $r_1 = 1.001$ (bottom curves) and $r_1 = 1.04$ (top curves), and the thickness parameter ℓ is chosen to be large for the purpose of making the effects visual. The black thick curve represents the black hole's event horizon.

Phase transition



The figure shows the energy difference of a brane configuration (that quasistatically evolves from the equatorial configuration) with respect to the equatorial configuration, as a function of the parameter Z_∞ in the case of $N = 5$ bulk dimensions. The top pictures belong to the thin, while the bottom pictures to the thick system. The red (blue) curves represent the black hole (Minkowskian) embedding branch. The pictures on the right hand side are the zooms into the near region of θ_{\min} , where the effects of the perturbations are the largest, and the difference become apparent between the thin (top) and thick (bottom) cases.

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Nonperturbative solution

4th order Euler-Lagrange equation

$$\theta^{(4)} + T_1(\ddot{\theta}, \dot{\theta}, \theta, \dot{f}, f, r)\theta^{(3)} + T_2(\ddot{\theta}, \dot{\theta}, \theta, f^{(3)}, \ddot{f}, \dot{f}, f, r) = 0 ,$$

Black hole embedding asymptotics

$$\frac{y_2}{f^2} + \frac{y_1}{f} + y_0 + \dots = 0 ,$$

Minkowski embedding asymptotics

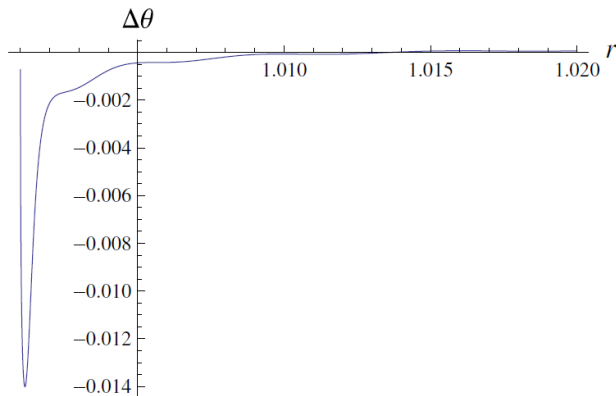
$$\frac{s_3}{\theta^3} + \frac{s_2}{\theta^2} + \frac{s_1}{\theta} + s_0 + \dots = 0 ,$$

2-dim problem

$$\dot{\theta}|_{r_1} = \pm \sqrt{\frac{-(b+an)}{2(2a+b-an)r^2 f}} \Big|_{r_1} \quad \frac{-(b+an)}{2(2a+b-an)} \geq 0 \quad n = 1???$$

Nonperturbative solutions - subcritical case

$$\Delta\theta(r) = \theta(r) - \theta_{DNG}(r),$$



The picture shows a near horizon $\Delta\theta$ curve with minimum horizon distance $r_1 = 1.001$.

Nonperturbative solutions - supercritical case

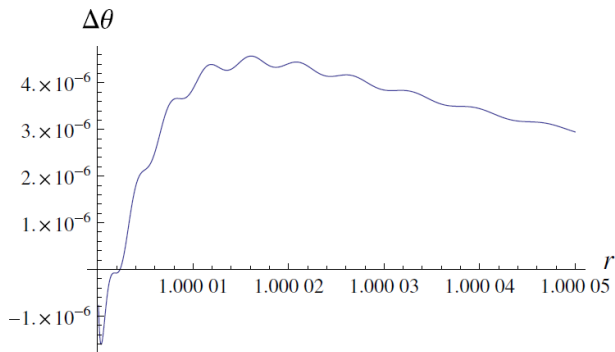
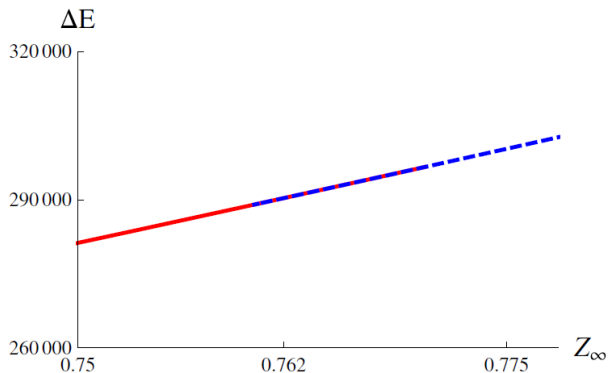


FIG. 5 (color online). The $\Delta\theta$ function in the black hole embedding case with $\theta_0 = \frac{\pi}{900}$ at the very near horizon region.

Nonperturbative solution - Phase transition



The enlargement of the overlap region

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2-dim case

Regularity

A regular solution of the problem must exist although analytic solution could not be found at the axis of the system. Thus the point r_1 on the axis must be a **regular singular point** of the differential equation. Even though the brane equation is **highly nonlinear**, general results from the theory of *local analysis* of **linear differential equations** can be applied, because we know from physical considerations that the brane equation should not develop any nontrivial singular points in its domain.

Theorem (Fuchs)

If a solution is not analytic at a regular singular point, its singularity must be either a pole or an algebraic or logarithmic branch point, and there is always at least one solution of the form

$$\theta(r) = (r - r_1)^\alpha A(r) ,$$

where α is called the *indical* exponent and $A(r)$ is a function which is analytic in r_1 and has a convergent Taylor series.

$$\alpha = 1/2 \quad \theta(r) = A_1 \sqrt{r - r_1} + A_2 (r - r_1)^{\frac{3}{2}} + A_3 (r - r_1)^{\frac{5}{2}} + A_4 (r - r_1)^{\frac{7}{2}} + \dots$$

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Rotating background, work in progress ...

Myers-Perry black hole with a single angular momentum

$$ds^2 = - \left(1 - \frac{F}{\Sigma}\right) dt^2 + \sin^2 \theta \left[r^2 + a^2 \left(1 + \frac{F}{\Sigma} \sin^2 \theta\right)\right] d\varphi^2 \\ + 2a \frac{F}{\Sigma} \sin^2 \theta dt d\varphi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + r^2 \cos^2 \theta d\Omega_{N-4}^2, \\ \Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - F, \quad F = \mu r^{5-N},$$

Coordinates

$$x^a = \{t, r, \varphi, \theta, \vartheta_1, \dots, \vartheta_{N-4}\}, \quad \zeta^\mu = \{t, r, \varphi, \vartheta_1, \dots, \vartheta_n\}, \quad \vartheta_{n+1} = \dots = \vartheta_{N-4} = \pi/2.$$

Lagrangian

$$\mathcal{L} = r^n \cos^n \theta \sin \theta \sqrt{\Sigma \left(1 + \Delta \dot{\theta}^2\right)}$$

Asymptotic and regularity analysis

Euler-Lagrange equation

$$\ddot{\theta} + \left(\alpha \Delta + \frac{\dot{\Delta}}{2} \right) \dot{\theta}^3 + \beta \dot{\theta}^2 + \left(\alpha + \frac{\dot{\Delta}}{\Delta} \right) \dot{\theta} + \frac{\beta}{\Delta} = 0,$$

$$\alpha = \frac{n}{r} + \frac{r}{\Sigma},$$

$$\beta = n \tan \theta - \cot \theta + \frac{a^2 \sin \theta \cos \theta}{\Sigma}.$$

Near horizon region

Essentially the same as the Schwarzschild case.

Far distance

Relevant differences.

Far distance solution

Asymptotic form and equation

$$\theta(r) = \theta_\infty + \nu(r), \quad \lim_{r \rightarrow \infty} \nu(r) = 0.$$
$$\theta_\infty = \arctan \left[\frac{1}{\sqrt{n}} \right], \quad \ddot{\nu} + \frac{n+3}{r} \dot{\nu} + \frac{2(n+1)}{r^2} \nu + \frac{a^2 \sqrt{n}}{(n+1)r^4} = 0.$$

Asymptotic solution

$$\nu(r) = \begin{cases} \frac{p \sin[\delta(r)] + p' \cos[\delta(r)]}{r^{1+\frac{n}{2}}} - \frac{a^2 \sqrt{n}}{2(n+1)r^2}, & \text{if } n \leq 4, \\ \frac{p + p' r^{\sqrt{-\gamma}}}{r^{1+\frac{n}{2} + \frac{\sqrt{-\gamma}}{2}}} - \frac{a^2 \sqrt{n}}{2(n+1)r^2}, & \text{if } n \geq 5, \end{cases}$$

with

$$\delta(r) = \frac{\sqrt{\gamma}}{2} \ln(r), \quad \gamma = -n^2 + 4n + 4.$$

Far distance solution

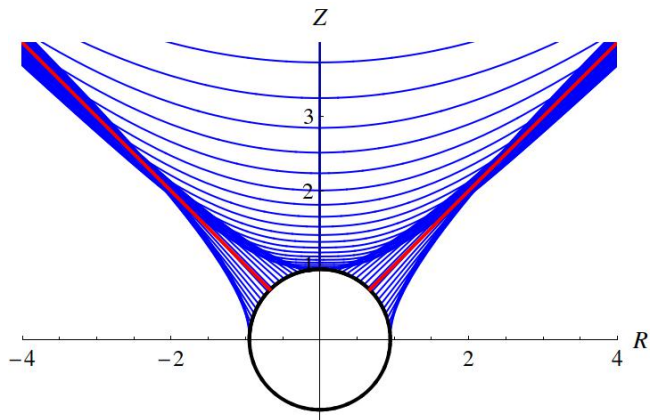


Figure: $N=6$, $n=1$, $a=0.4$

- 1 Introduction
- 2 The brane - black hole toy model
- 3 Holographic phase transition of fundamental matter
- 4 Thickness corrections
 - Perturbative results
 - Nonperturbative solutions
 - 2-dim special case
- 5 Rotating background
- 6 Summary

Summary

Introduction

- Toy model for merger transitions;
- Brane - black hole setup;
- Holographic dual picture;
- Instability zone, first order phase transition;

My results

- Higher derivative corrections to the action from the brane thickness;
- 4th order equation of motion;
- Linear perturbation method;
- Phase transition;
- Nonperturbative solution;
- 2-dim special case;
- Rotating background.

Thank you for your attention.