Dynamics of scalar fields around black holes.

Juan Carlos Degollado

Universidade de Aveiro

Motivation:

Scalar fields are a common theme in modern cosmology. They play a central role in inflation, and they have frequently been used to describe dark energy in place of the cosmological constant. Classical scalar fields have also been proposed as possible candidates for the dark matter component of the universe. One of the SFDM models proposes that galactic haloes are formed by a Bose-Einstein condensation of a scalar field (Matos 2000, Arbey 2001).

Given the fact that super-massive black holes seem to exist at the centre of most galaxies, a scalar field configuration should be stable in the presence of a central black hole, or at least be able to survive for cosmological time-scales.

SFDM

A simple proposal is to consider that dark matter is made of a massive scalar field, whose properties could explain the behaviour of dark matter at all scales. The scenario of galactic formation, for instance, is as follows: a sea of scalar field particles fills the Universe and forms localized primordial fluctuations that could collapse to form stable objects, which are interpreted as the dark matter halos of galaxies.

Scalar fields as cosmological background:
It is assumed that the scalar particles decouple after inflation in the early universe, after which the field has a simple quadratic potential with no interactions. Ultra-light particles form a pure ground state condensate with a high critical temperature behaving like cold dark matter today. The condition for the formation of BEC in an expanding Universe can be derived in a simple way if it is assumed that the scalar field particles reached thermal equilibrium with other particles at early times, then the thermal evolution of scalar field particles can be described by the behaviour of the scalar number density of particles n_ϕ during an adiabatic process.

The critical temperature below which condensation occurs is found in terms of the charge density of dark matter particles (the excess of particles over antiparticles).

$$q_\phi := n_\phi - \overline{n_\phi} \ , \quad n_\phi = \frac{1}{2\pi^2}\left[\int_0^\infty \frac{k^2\,dk}{\exp[\beta(E_k - \mu(T))] - 1}\right] \ ,$$

$$\overline{n_\phi} = n_\phi(-\mu(T))$$

For high temperatures, $m << T$, the charge density in excited states is $q_\phi = \frac{\mu(T_\phi)}{3}T_\phi^2$.

The maximum charge density allowed by the excited states at a given temperature occurs when $\mu = m$.

The critical temperature T_c and a critical charge density $q_{\phi,c}$ for the formation of a BEC in an ultra-relativistic Bose gas of SFDM particles are

$$T_c = \left(\frac{3q_\phi}{m}\right)^{1/2} \ , \quad q_{\phi,c} = \frac{m}{3}T_\phi^2 \ ,$$

The formation of a BEC will happen if the excited states are not able to accommodate all the bosonic particles $q_\phi > q_{\phi,c}$.

The entropy density of an ideal Bose gas is

$$s_\phi(T_\phi) = \frac{4\pi^2}{45} T_\phi^3 \ .$$

If the evolution of SFDM particles proceeds separately at constant entropy, $S_\phi = s_\phi a^3 =$const, then $aT_\phi =$const. Furthermore, $a^3 q_\phi =$const.

In an adiabatic cosmological expansion it is expected that the conservation of the charge density is $q_\phi = \eta_\phi T_\phi^3$ after decoupling.

The condition $q_{\phi,c} < q_\phi$ for the formation a BEC translates to $T_\phi > T_{\phi,c}$, where the critical temperature is mainly determined by the mass of the particle. Given,

$$T_c = \left(\frac{T_\phi}{T_{\phi,c}}\right)^{1/2} T_\phi \ , \quad T_{\phi,c} := \frac{m}{3\eta_\phi} \ ,$$

as long as $T_\phi > T_{\phi,c}$ is satisfied, $T < T_c$ is necessary for the formation of a BEC.

When $T << T_c$ the majority of the bosons will condensate to the ground state. Once the BEC is formed in the early Universe at high temperatures, its corresponding charge density q_{BEC} is

$$q_{BEC} = q_\phi - q_{\phi,c} = q_\phi \left(1 - \frac{T_{\phi,c}}{T_\phi}\right) \ ,$$

If the BEC occurs and most particles are in the ground state,

$$\rho_{DM} \approx (n_\phi) m \ , \quad \text{assuming} \quad \rho^0_{DM} \approx 23\% \rho_c$$

with

$$\rho_c \approx 4.19 \times 10^{-11} \text{eV}, \quad n \approx \frac{\rho_{DM}}{m} \approx 10^{12} \text{eV} \ ,$$

For a scalar field of mass of $m \sim 10^{-23}$ eV the critical temperature is $T_c \sim 1.7 \times 10^{17}eV\sim 10^{21}$K which corresponds to a very pure condensate today (Lundgren et al. 2010). If the scalar particles decouple from regular matter before Standard model particles annihilate, their temperature will be about 0.9K and thus Big Bang nucleosynthesis remains unaffected.

The evolution of the density of the field follows the standard ΛCDM model at times later than nucleosyntesis $a \sim 10^{-10}$. During radiation domination, the scalar field is a subdominant contribution to the density of the universe, and during matter domination it might be a replacement for dark matter. The macroscopically-occupied ground state has $\rho \sim a^{-6}$ at early times.

The idea is to solve the Klein-Gordon equation for the field Φ in the FRW universe.

$$ds^2 = a(\tau)^2(-d\tau^2 + dx^2 + dy^2 + dz^2)\ , \quad dt = a d\tau\ .$$

The density and the pressure are defined as

$$\rho = \frac{1}{2}(\partial_t\Phi^\dagger\partial_t\Phi + \partial_j\Phi^\dagger\partial^j\Phi + m^2|\Phi|)\ , \quad p = \frac{1}{2}(\partial_t\Phi^\dagger\partial_t\Phi + \partial_j\Phi^\dagger\partial^j\Phi - m^2|\Phi|)\ .$$

Decomposing the field into modes

$$\Psi = a\Phi = \int d^3k A_k\psi_k \mathrm{e}^{ikx} + \text{Hermitian conjugate}\ ,$$

the KG equation becomes

$$\frac{d^2\psi_k}{d\tau^2} + \left[k^2 - \frac{a''}{a} + a^2m^2\right]\psi_k = 0\ ,$$

The Hubble parameter is approximated in different epochs by power laws of the form $H = a'/a^2 = H_0 a^{-n}$. Where $n = 2$ during the radiation domination era and $n = 3/2$ during the matter domination era. In the Friedman equation, $H = \frac{8\pi G}{3}(\rho_{rad} + \rho_\Lambda + \rho_m)$, $\rho_{rad} \sim a^{-4}$ and $\rho_m \sim a^{-3}$.

In the radiation domination regime ($a'' = 0$) the density for the excited states is (Lundgren 2010)

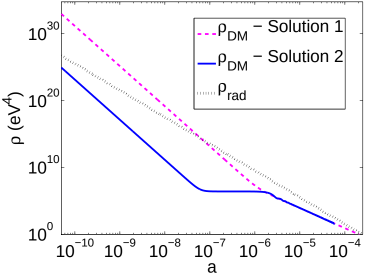
$$\rho_{ex} = \frac{T^4\pi^4}{15} + \frac{T^2H^2}{12} - \frac{m^2}{12}\frac{T^2}{a^2} + \ldots .$$

Whereas at early times the pressure and the density of the ground states

$$\rho_0 = \frac{H_{0r}^{3/2}}{4a^6 m^{1/6} C_0} + \frac{m^{5/2} C_0}{2H_{0r}^{3/2}} \ ,$$

$$H_{0r} = 1.4 \times 10^{-35} eV,$$

$$p_0 = \frac{H_{0r}^{3/2}}{4a^6 m^{1/6} C_0} - \frac{m^{5/2} C_0}{2H_{0r}^{3/2}} \ .$$



In the matter domination regime $n = 3/2$

$$\frac{d^2\psi_k}{d\tau^2} + \left[k^2 - \frac{H_{0m}^2}{2a} + a^2m^2\right]\psi_k = 0 \ ,$$

$H = H_{0m}a^{-3/2}$ with $H_{0m} \sim 7.8 \times 10^{-34}$eV. For the ground state $k = 0$

$$\omega_0 = \frac{am}{C_1 \sin(2mt + \alpha) + C_2} \ ,$$

The pressure averages to zero on cosmological scales (the period is $\sim$ few years for $m = 10^{-23}$eV) causing the ground state scalar field particles to behave like pressureless matter.

Scalar field fluctuations:
The Compton wave-length associated to this boson (m$\sim 10^{-23}$eV) is about kpc that corresponds to the dark-halo size of typical galaxies.

From the equation of scalar fluctuations it is possible to determine a Jeans length for scalar dark matter and show that modes larger than this length are growing modes. This fact implies a cut off in the mass power spectrum that is used to fix the parameters of the model (Matos & Urena 2008).

The study of structure formation was done in terms of a linear perturbation theory (Review: Magana 2012) considering a Newtonian gauge,

$$\Phi(\mathbf{x},t) = \Phi_0(t) + \delta\Phi(\mathbf{x},t)\ , \quad g_{00} = -a^2(1+2\phi)\ ,$$
$$g_{0i} = 0\ , \quad g_{ij} = a^2(1-2\psi)\delta_{ij}\ .$$

After a decomposition of the form

$$\delta\Phi(x^i,t) = \int d^3k\,\delta\Phi_k \exp(ik_i x^i)\ ,$$

The perturbed Klein-Gordon equation takes the form

$$\delta\ddot{\Phi}_k + \left(\frac{k^2}{a^2} + V_{,\Phi\Phi}\right)\delta\Phi_k = -3H\delta\dot{\Phi}_k + 4\dot{\phi}\dot{\Phi}_0 - 2\phi V_{,\Phi}\ ,$$

that may have growing solutions depending of the sign of the second factor.

Scalar field in galaxies:

Scalar field particles could form gravitationally stable structures made of particles in quantum coherent states, like boson stars for a complex scalar field (Colpi et al. 1986, Seidel & Suen 1990, LRR Liebling & Palenzuela 2012) or oscillatons for a real scalar field (Seidel & Suen 1991, Alcubierre et al. 2002).

It was shown that the scalar field can collapse to form structures of the size of galaxies.

The stability and gravitational wave signatures of compact scalar stars have been studied numerically in (Khlopov et at, 1985, Seidel & Suen 1990, Balakrishna et al, 2006).

In addition, it has been shown that a oscillaton-like solution can be stable, non-singular and asymptotically flat.
The critical mass of an oscillaton, the maximum mass for which a stable configuration exists, will depend on the mass of the boson. If one takes $m = 1.1 \times 10^{-23}$eV then $M_{crit} \sim 0.6\frac{m_p^2}{m_\phi} \sim 10^{12} M_\odot$, which is of the order of magnitude of the dark matter content of a standard galactic halo. The oscillaton is a spherically symmetric solution to the Einstein's equations

$$ds^2 = -\alpha^2(r,t)dt^2 + a^2(r,t)dr^2 + r^2 d\Omega^2 \ ,$$

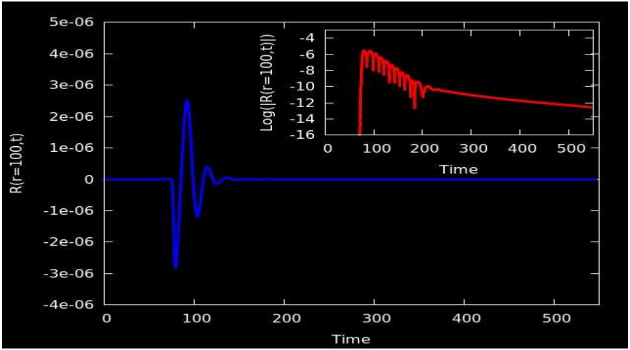
coupled to the KG equation. Halos formed from ultra-light scalar with Compton wavelength of galactic scales do not lead to over-abundance of dwarf galaxies unlike cold dark matter simulations with heavier bosons (Navarro et al, 1996, Alcubierre et al. 2002, Salucci et al. 2003).

The comparison with the observed rotation curves in galaxies seems promising. Scalar field halos have been fit to rotation curves in spiral galaxies (Schunk & Liddle 1997, Arbey et al. 2001, Guzman & Urena 2003, Bochmer and Harko 2007). By using the mass and scattering length as a free parameters to fit rotation curves, it has been obtained a mass of $m = 0.4 - 1.6 \times 10^{-23}$eV for non-interacting ultra-light bosons.

Several improvements are needed before concluding that a scalar field is the best galactic dark matter candidate on the market. It is necessary to extend the comparison to various types of individual galaxy rotation curves, with the drawback that more degrees of freedom must be included in realistic models of the baryonic components gas, bulge, etc.

Scalar fields around black holes

On the other hand, the dynamics of massless scalar fields around black holes is a bit different. It can be split into three stages. After a first burst of radiation depending on the initial configuration, the field undergoes damped oscillations (quasinormal ringing). Finally, the field follows a power law decay. caused by backscattering of the background potential.



For a massive field new properties arise.

We restrict to the case of a scalar field as a test field in the background of a Schwarzschild black hole (Barranco et al. 2012a, 1212b).

We assume that the energy associated with the scalar field configuration is very small compared to the mass of the black hole, so that the gravitational back-reaction associated to the scalar field distribution can be disregarded (to be consistent with the test field approximation).

The equation of motion for that field is given by the Klein-Gordon equation

$$(\Box - \mu^2)\phi = 0 \ ,$$

Considering a decomposition into spherical harmonics we obtain a family of reduced equations

$$\left[\frac{1}{N(r)}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial r}N(r)\frac{\partial}{\partial r} + \mathcal{U}_\ell(\mu, M; r)\right]\psi_{\ell m} = 0 \ ,$$

where we have defined

$$\mathcal{U}_\ell(\mu, M; r) := \frac{\ell(\ell+1)}{r^2} + \frac{2M}{r^3} + \mu^2 \ .$$

Initial data

What do we know about stationary solutions?

In order to look for the stationary solutions we make a further decomposition of the functions $\psi_{\ell m}(t,r)$ into oscillating modes of the form:

$$\psi_{\ell m}(t,r) = e^{i\omega t} u(r) \ ,$$

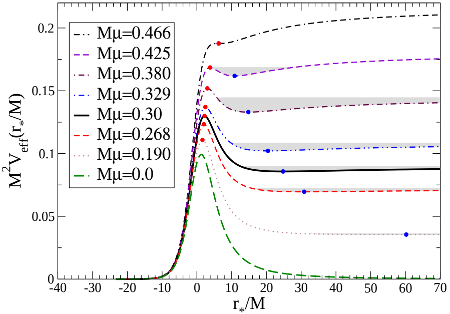
with ω a real frequency, and $u(r)$ a complex function of r in the interval $(2M,\infty)$.

This equation can be rewritten as a time-independent Schrödinger-like equation:

$$\left[-\frac{\partial^2}{\partial r^{*2}} + V_{\rm eff}(r^*)\right] u(r^*) = \omega^2 u(r^*) \ , \qquad -\infty < r^* < \infty,$$

with the effective potential $V_{\rm eff}(r^*)$ defined as

$$V_{\rm eff}(r^*) := N(r)\,\mathcal{U}_\ell(\mu, M; r).$$



The existence of the potential well. is reflected in a bound for the values of $(M\mu)^2$

$$
\begin{aligned}
(M\mu)^2 &< -\frac{1}{32}(\ell^2+\ell-1)(\ell^2+\ell+1)^2 \\
&+ \frac{1}{288}\sqrt{3(3\ell^4+6\ell^3+5\ell^2+2\ell+3)^3}\,,
\end{aligned}
$$

The condition $0 < \omega^2 < \mu^2$ guarantee that the solution for the scalar field decays exponentially at spatial infinity and is “localized” close to the black hole, but the scalar field can still escape towards the black hole horizon.

All purely stationary solutions, *i.e.* those with real ω, require waves to move outward from the horizon to compensate for the waves that tunnel out through the barrier and move toward the horizon (otherwise the situation would not be stationary). Imposing the condition of no waves coming out from the horizon clearly improves the situation at the cost of introducing complex valued frequencies ω. When ω is complex the solution corresponds to quasi-bound states (Detweiler 1981, Ohashi 2004, Cardoso & Yoshida 2005, Dolan 2007) in the usual Boyer Lindquist foliation.

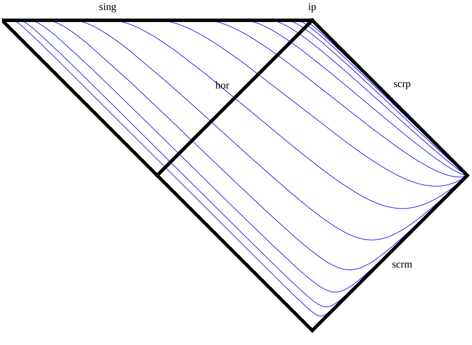
Time evolution

The numerical evolution was performed in penetrating coordinates. We use ingoing Eddington-Finkelstein coordinates

$$ds^2 = -\left(1 - \frac{2M}{r}\right)\, d\bar{t}^2 + \frac{4M}{r}\, d\bar{t}dr + \left(1 + \frac{2M}{r}\right)\, dr^2 + r^2\, d\Omega^2 \ .$$

This coordinate system covers a region that includes the black hole interior $r \in (0, \infty)$, and is regular at the horizon.

We define a set of functions given by first order derivatives, and obtain a system of the form $\partial_t \vec{\Psi} + B \partial_r \vec{\Psi} = S$.

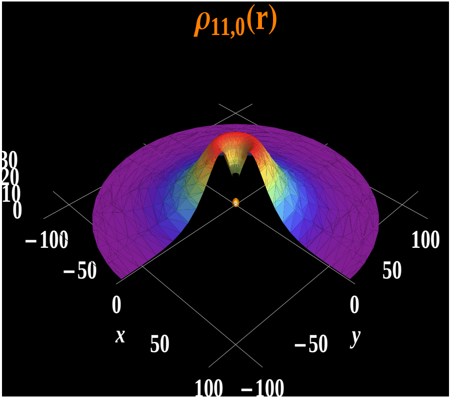


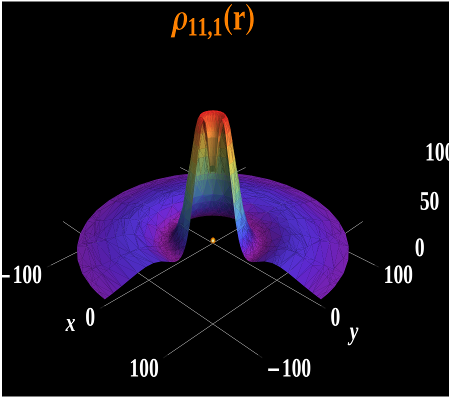
We start constructing configurations that are arbitrarily close to the quasi-stationary solutions in the following way:

Choose any stationary solution that decays at spatial infinity and set it to zero by hand in the interval $r \in (2M, 2M + \epsilon)$, for some small parameter $\epsilon > 0$ with dimensions of length.

The resulting configuration can be seen as a combination of the stationary solution plus a perturbation around the horizon. The frequency was taken as the real part of the frequency for quasi-bound state.

We construct pseudo-resonant initial data configurations for different values of ℓ and $M\mu$, and for each pair of these parameters we study the states up to the fifth mode.





In the test field limit all configurations have the conserved energy $E = \sum_{\ell,m} E_{\ell m}$, with

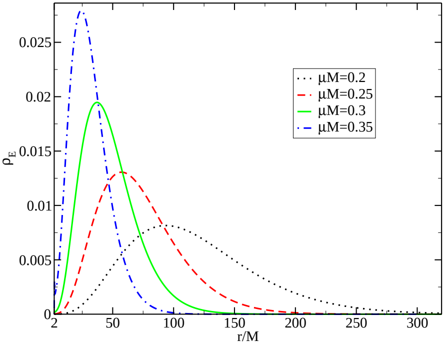
Energy associated to the time killing vector field

$$E_{\ell m} = \int_{2M}^{\infty} \rho_E(r) dr \ ,$$

and where

$$\rho_E(r) = \frac{1}{2}\left(\frac{1}{N(r)}\left|\frac{\partial \psi_{\ell m}}{\partial t}\right|^2 + N(r)\left|\frac{\partial \psi_{\ell m}}{\partial r}\right|^2 + \mathcal{U}_\ell(\mu, M; r)|\psi_{\ell m}|^2\right)$$

This quantity is given by the conserved charge $Q_a = \int_\Sigma {k_a}^\mu T_{\mu\nu} n^\nu \sqrt{\gamma}\, d\Sigma$.
We used this quantity to monitor the state of the configuration.



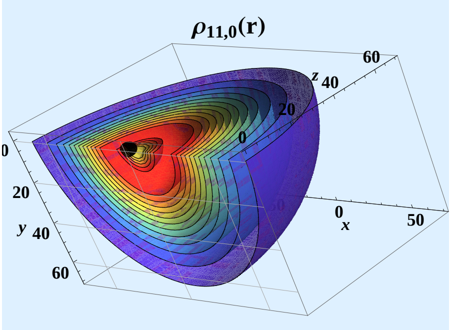


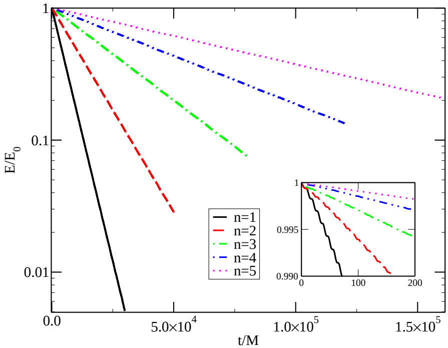
Fig: Radial energy density ρ_E for $\ell = 1$ and the first pseudo-resonant mode $n = 1$, for configurations with different values of $M\mu$. The solution has been normalized so that $E_{lm} = 1$.

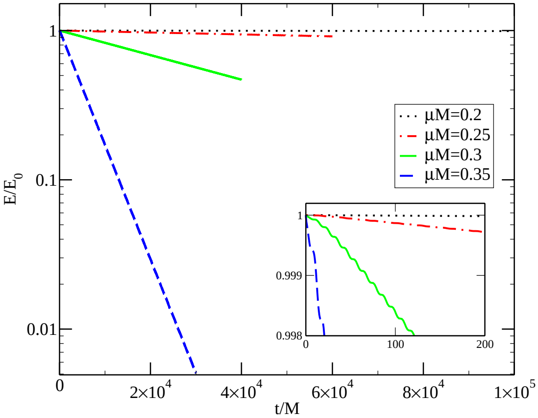
Our attention was mainly focused on the evolution of the pseudo-resonant states.
We start by evaluating the total energy loss and studying some spectral characteristics of the different configurations by means of a time Fourier analysis, and finish with more explicit considerations about how long such configurations can last.

The energy shows a exponential decay of the form

$$E(t) = E_0 \exp(-s\, t/M)\ ,$$

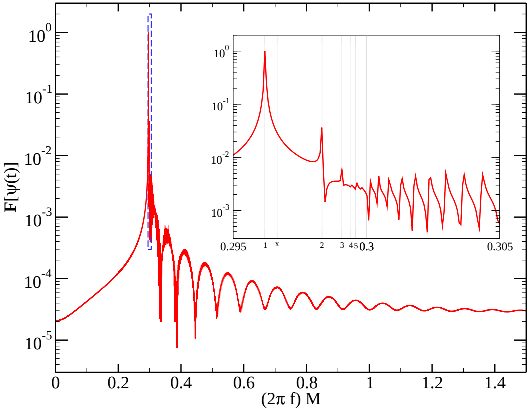
with s constant, except for some very small oscillations that remain during the whole evolution. Given the exponential decay that dominates the overall behaviour, we can perform a linear fit of $\ln(E/E_0)$ as a function of t/M to calculate the parameter s.





Energy of the scalar field vs. time for the evolution of the initial data corresponding to the first $(n = 1)$ pseudo-resonant mode with $\ell = 1$, and $M\mu = 0.2$, 0.25, 0.3 and 0.35.

Discrete Fourier transform in time for the evolution of non-resonant data with $M\mu = 0.3$, $\ell = 1$, and frequency $\omega_x = 0.29664794$.



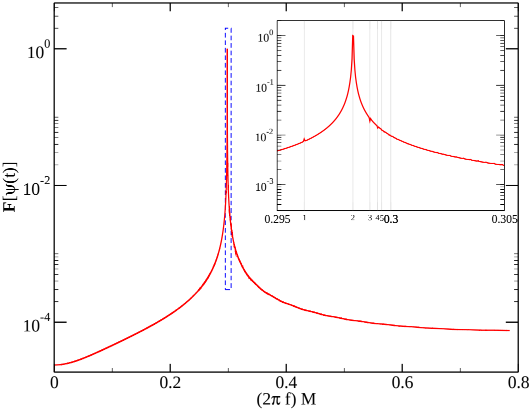


Fig: The Fourier transform in time vs. frequency for the evolution of the resonant data with $M\mu = 0.3$, $\ell = 1$, and frequency corresponding to the second pseudo-resonant mode.

We also construct a two-parameter family of initial data, of the form

$$u_0(r) = \begin{cases} N(r - R_1)^4 (r - R_2)^4 & \text{for } R_1 \le r \le R_2 \\ 0 & \text{otherwise} \end{cases},$$

During the initial stages of the evolution some SF falls into the BH, while some is radiated away, both with rates that depend greatly on the initial data chosen. However, at late times, all evolutions show a similar steady behaviour with slow accretion into the BH. Similar results are obtained when studying the long-term evolution of a variety of configurations, some of them very different in size and spatial distribution.

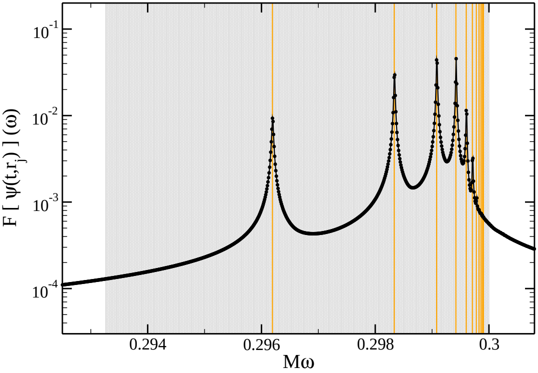


Fig: The Fourier transform in time vs. frequency for the evolution of a generic initial data with $M\mu = 0.3$, $\ell = 1$.

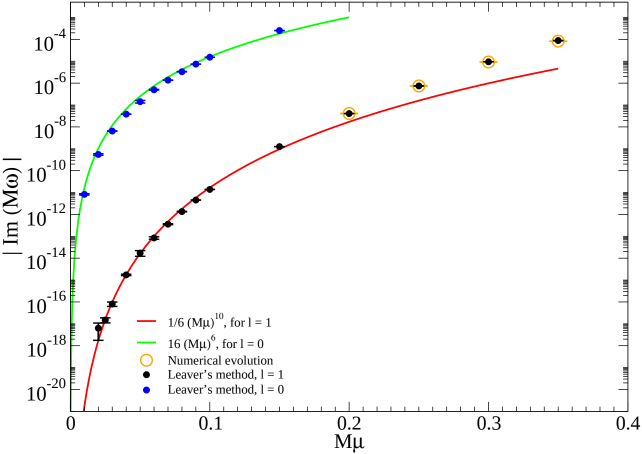
We found that the real part of the frequency of the quasi-resonant modes are very close with the frequency of the stationary and dynamical resonances, and that the imaginary part coincides with the decay rate of the dynamical resonances.

However numerical roundoff errors make it still prohibitive to obtain accurately very small values for the imaginary part of the quasi-resonant frequencies.

For the KG Potential, the quasi-resonant modes can be obtained semi-analytically in several ways, the most common are the continued fraction method introduced by (Leaver 1985), and the WKB approach (Iyer 1986)

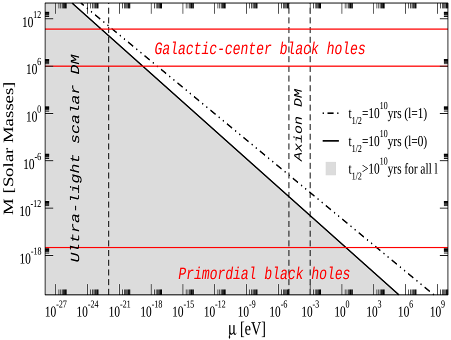
An analytic expression valid in the limit $M\mu \ll 1$ is known (Detweiler 1980).

For $\ell = 1$, the imaginary part of the frequency for the first quasi-resonant mode in this limit is given by $\mathrm{Im}(M\omega) = (M\mu)^{10}/6$.



There are two distinct regions of the parameter space of physical interest for which the configurations live longer than the age of the Universe:

- A scalar field mass smaller than 1 eV and black hole mass smaller than $10^{-17} M_{\odot}$, consistent with primordial black holes with an axion distribution (Sikivie 2009);
- An ultra-ligh scalar field with mass smaller than 10^{-23} eV (Hu 2000, Matos 2000) and a supermassive black holes with mass smaller than $5 \times 10^{10} M_{\odot}$, as could be the case for a dark matter halo surrounding a black hole at a galactic center (Arbey 2001).



The interaction of a Klein-Gordon field with a Schwarzschild black hole has been considered in the test field approximation. We have found dynamically long-lived dynamical scalar field configurations that, for values of $M\mu \lesssim 10^{-3}$, can survive in the vicinity of the BH for cosmological time scales.

Although we were unable to reach very small values for the combination $M\mu$ by means of numerical simulations we match our results with other methods to show that such configurations are possible.

Also our results seem to indicate that, at late times, even quite generic distributions evolve as a combination of the dynamical resonant modes, which can last for cosmological time-scales.

Thanks!