Scalar-tensor black holes, cosmological and isolated

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OVERVIEW

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2. Notions of horizon
3. Some cosmological black holes and their evolving horizons in scalar-tensor and $f(R)$ gravity
4. Isolated scalar-tensor black holes
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MOTIVATION

Horizon = "a frontier between things observable and things unobservable" (Rindler '56). The horizon, the product of strong gravity, is the most impressive feature of a BH spacetime.
"Textbook" kinds of horizons: Rindler horizons, BH horizons, cosmological horizons; also event, Killing, inner, outer, Cauchy, apparent, trapping, quasi-local, isolated, dynamical, and slowly evolving horizons (Poisson; Wald; Booth; Nielsen; Ashtekar & Krishnan '04; Gourghoulhon & Jaramillo '08). Some horizon notions coincide for stationary BHs.
The (now classic) black hole mechanics and thermodynamics (1970s) focus on *stationary* BHs and *event horizons* but highly dynamical situations are of even greater interest:

- Gravitational collapse.
- Merger BH/compact object.
- Hawking radiation and evaporation of a small BH.
- BHs interacting with non-trivial environments (accretion/emission, backreaction).

- Study the spatial variation of fundamental constants (scalar-tensor gravity embodies the variation of G).
- Old problem of whether/how the cosmic expansion affects local systems (Carrera & Giulini RMP '10) $\rightarrow$ McVittie solution '33. It has a complex structure, not yet completely understood (Kleban *et al.*; Lake & Abdelqader '11; Anderson '11; Nandra *et al.* '12 ; VF, Zambrano & Nandra '12; Silva, Fontanini & Guariento '12; ...).
- Cosmology again: no dark energy but backreaction of inhomogeneities *in GR*; exact inhomogeneous universes useful as toy models.
- Another idea: we live in a giant void which mimics an accelerated expansion. Analytical GR solutions considered are related to BHs in expanding universes (Boleiko *et al.* '11 review).

Here the concept of event horizon fails. If "background" is not Minkowski, internal energy (in 1st law) must be defined carefully (quasi-local energy, related to the notion of horizon).

Notion of BH event horizon is useless for practical purposes in dynamical situations because it requires knowledge of the entire causal structure of spacetime, including $\mathscr{I}^+$),
Is this relevant for astrophysics?

- Astronomy $\longrightarrow$ stellar/supermassive BHs play important roles in astrophysical systems.
- Now nearing the detection of GWs; large efforts to predict in detail GWs emitted by astrophysical BHs, build template banks for interferometers. In numerical calculations "BHs" are identified with outermost marginally trapped surfaces and apparent horizons (*e.g.*, Thornburg '07, Baumgarte & Shapiro '03, Chu, Pfeiffer, Cohen '11).
- If primordial BHs formed in the early universe, they would have had a scale $\sim$ Hubble scale and very dynamical horizons. How fast could these BHs accrete/grow?
- Spherical accretion (especially of dark/phantom energy) by BHs, much discussion, no definitive conclusion (Babichev *et al.* '04; Chen & Jing '05; Izquierdo & Pavon '06; Pacheco & Horvath '07;

Maeda, Harada & Carr '08; Gao, Chen, VF, Shen '08; Guariento *et al.* '08; Sun '08, '09; Gonzalez & Guzman '09; He *et al.* '09; Babichev *et al.* '11; Nouicer '11; Chadburn & Gregory '13), again relevant for primordial BHs.

Much theoretical effort into generalizing BH thermodynamics from event to moving horizons (see Collins '92, Hayward '93, Ashtekar & Krishnan '04; also for cosmological horizons).
"Slowly moving" horizons should be meaningful in some adiabatic approximation; will fast-evolving horizons be thermodynamical systems? Will they require non-equilibrium thermodynamics?
BH thermodynamics of event horizons does not postulate field equations (well, almost ...). Much interest in alternative gravity, motivated by

- Quantum gravity: low-energy effective actions contain ingredients foreign to GR (nonminimally coupled scalars, higher derivatives, non-local terms, ...).
- Do we see the first deviations from GR in the cosmic

acceleration? $f(R)$ gravity and other modifications. Theories designed to produce a time-varying effective Λ, BH spacetimes are dynamical and asymptotically FLRW.

- Even Newtonian gravity is doubted at galactic/cluster scales: MOND, TeVeS, non-minimally coupled matter.

Only a few exact solutions of GR (and even less of other theories of gravity) are known for which the horizons are explicitly time-dependent. Focus on some solutions describing BHs in cosmological "backgrounds".

VARIOUS NOTIONS OF HORIZON

Basic notions: congruence of null geodesics (tangent $l^a = dx^a/d\lambda$, affine parameter λ): $l_a l^a = l^c \nabla_c l^a = 0$. Metric h_{ab} in the 2-space orthogonal to l^a: pick another null vector field n^a such that $l^c n_c = -1$, then

$$h_{ab} \equiv g_{ab} + l_a n_b + l_b n_a \,.$$

h_{ab} purely spatial, $h^a{}_b$ is a projection operator on the 2-space orthogonal to l^a. The choice of n^a is not unique but the geometric quantities of interest do not depend on it once l^a is fixed. Let $\eta^a =$ geodesic deviation, define $B_{ab} \equiv \nabla_b \, l_a$, orthogonal to the null geodesics. The transverse part of the deviation vector is

$$\tilde{\eta}^a \equiv h^a{}_b \, \eta^b = \eta^a + (n^c \eta_c) l^a$$

and the orthogonal component of $l^c \nabla_c \eta^a$ is

$$(\widetilde{l^c \nabla_c \eta^a}) = h^a{}_b h^c{}_d B^b{}_c \, \tilde{\eta}^d \equiv \tilde{B}^a{}_d \, \tilde{\eta}^d \,.$$

Decompose transverse tensor $\widetilde{B}_{ab}$ as

$$\widetilde{B}_{ab} = \widetilde{B}_{(ab)} + \widetilde{B}_{[ab]} = \left(\frac{\theta}{2} h_{ab} + \sigma_{ab} \right) + \omega_{ab} \,,$$

where expansion $\theta = \nabla_c l^c$ propagates according to the Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{2} - \sigma^2 + \omega^2 - R_{ab} l^a l^b \,;$$

If congruence is not affinely parametrized, the geodesic equation is

$$l^c \nabla_c l^a = \kappa \, l^a$$

(where κ used as a surface gravity) and

$$\theta_l = h^{ab} \nabla_a l_b = \left[g^{ab} + \frac{l^a n^b + n^a l^b}{(-n^c l^d g_{cd})} \right] \nabla_a l_b \,.$$

Raychaudhuri eq. becomes

$$\frac{d\theta}{d\lambda} = \kappa\,\theta - \frac{\theta^2}{2} - \sigma^2 + \omega^2 - R_{ab} l^a l^b \,.$$

A compact and orientable surface has two independent directions orthogonal to it: ingoing and outgoing null geodesics with tangents l^a and n^a. Basic definitions for closed 2-surfaces:

- A *normal surface* corresponds to $\theta_l > 0$ and $\theta_n < 0$.
- A *trapped surface* has $\theta_l < 0$ and $\theta_n < 0$. Both outgoing and ingoing null rays converge here, outward-propagating light is dragged back by strong gravity.
- A *marginally outer trapped* (or *marginal*) *surface (MOTS)* corresponds to $\theta_l = 0$ (where l^a is the outgoing null normal to the surface) and $\theta_n < 0$.
- An *untrapped surface* is one with $\theta_l \theta_n < 0$.
- An *antitrapped surface* corresponds to $\theta_l > 0$ and $\theta_n > 0$.
- A *marginally outer trapped tube (MOTT)* is a 3-D surface which can be foliated entirely by marginally outer trapped (2-D) surfaces.

Penrose ’65: if a spacetime contains a trapped surface, the null energy condition holds, and there is a non-compact Cauchy surface for the spacetime, then this contains a singularity.
Trapped surfaces are essential features in the concept of BH.
“Horizons” of practical utility are identified with boundaries of spacetime regions containing trapped surfaces. (Conditions for the existence/uniqueness of MOTSs not completely clear.)

Event horizons

An *event horizon* is a connected component of the boundary $\partial(J^-(\mathscr{I}^+))$ of the causal past $J^-(\mathscr{I}^+)$ of future null infinity $\mathscr{I}^+$. Causal boundary separating a region from which nothing can come out to reach a distant observer from a region in which signals can be sent out and eventually arrive to this observer.
Generated by the null geodesics which fail to reach infinity.
Provided that it is smooth, it is a null hypersurface.
To define and locate an event horizon one must know all the future history of spacetime: a globally defined concept has teleological nature. In a collapsing Vaidya spacetime, an event horizon forms and grows starting from the centre and an

observer can cross it and be unaware of it even though his or her causal past consists entirely of a portion of Minkowski space: the event horizon cannot be detected by this observer with a physical experiment. The event horizon "knows" about events belonging to a spacetime region very far away and in its future but not causally connected to it ("clarvoyance" Ashtekar & Krishnan '04, Ben Dov '07, BengtssonSenovilla '11, Bengtsson '11).

The event horizon $\mathcal{H}$ is a tube in spacetime. Language abuse: referring to the intersections of $\mathcal{H}$ with surfaces of constant time (which produce 2-surfaces) as "event horizons".

Killing horizons

When present, a Killing vector field k^a defines a *Killing horizon* $\mathcal{H}$, which is *a null hypersurface everywhere tangent to a Killing vector field k^a which becomes null on $\mathcal{H}$* This Killing vector field is timelike, $k^c k_c < 0$, in a spacetime region which has $\mathcal{H}$ as boundary. Stationary event horizons in GR are Killing horizons. If the spacetime is stationary and asymptotically flat (but not necessarily static), it must be axisymmetric and an event horizon is a Killing horizon for the Killing vector

$$k^a = (\partial/\partial t)^a + \Omega_H (\partial/\partial\varphi)^a \ ,$$

linear combination of the time and rotational symmetry vectors, $\Omega_H =$ angular velocity at the horizon (this statement requires the assumption that the Einstein-Maxwell equations hold and some assumption on the matter stress-energy tensor). Attempts to use conformal Killing horizons have not been fruitful.

Apparent horizons (AHs)

A *future apparent horizon* is *the closure of a 3-surface which is foliated by marginal surfaces*; defined by the conditions on the time slicings (Hayward '93)

$$\theta_l = 0 \, ,$$
$$\theta_n < 0 \, ,$$

where θ_l and θ_n are the expansions of the future-directed outgoing and ingoing null geodesic congruences, respectively (outgoing null rays momentarily stop expanding and turn around at the horizon). Inequality distinguishes between BHs and white holes.

AHs defined quasi-locally but depend on the choice of the foliation of the 3-surface with marginal surfaces (non-symmetric slicings of the Schwarzschild spacetime exist for which there is no AH (Wald & Iyer '91; Schnetter & Krishnan '06). Quite distinct from event horizons in non-stationary situations.

In GR, a BH AH lies inside the event horizon provided that the null curvature condition $R_{ab} l^a l^b \geq 0 \; \forall$ null vector l^a is satisfied. But Hawking radiation itself violates the weak and the null energy conditions, as do quantum matter and non-minimally coupled scalars.

Trapping horizons

A *future outer trapping horizon (FOTH)* is *the closure of a surface* (usually a 3-surface) *foliated by marginal surfaces such* *that on its 2-D "time slicings"* (Hayward '93)

$$\theta_l = 0 \,,$$
$$\theta_n < 0 \,,$$
$$\mathcal{L}_n \theta_l = n^a \nabla_a \theta_l < 0 \,,$$

Last condition distinguishes between inner and outer Hs and between AHs and trapping Hs (sign distinguishes between future and past horizons).
Past inner trapping horizon (PITH): exchange l^a with n^a and reverse signs in the inequalities,

$$\theta_n = 0 \,,$$
$$\theta_l > 0 \,,$$
$$\mathcal{L}_l \theta_n = l^a \nabla_a \theta_n > 0 \,.$$

The PITH identifies a white hole or a cosmological horizon. As one moves just inside an outer trapping horizon, one encounters trapped surfaces, while trapped surfaces are encountered as as one moves just outside an inner trapping horizon.

Example: Reissner-Nordström BH with natural foliation. The event horizon $r = r_+$ is a future outer trapping horizon, the inner (Cauchy) horizon $r = r_-$ is a future inner trapping horizon, while the white hole horizons are past trapping horizons.

BH trapping horizons have been associated with thermodynamics; claims that entropy is associated with trapping horizon, not event horizon, area (Haijcek '87; Hiscock '89; Collins '92; Nielsen) – controversial (Sorkin '97; Corichi & Sudarsky '02; Nielsen & Firouzjaee '12). The Parikh-Wilczek ('00) "tunneling" approach is in principle applicable also to AHs and trapping horizons.

In general, trapping horizons do not coincide with event horizons. Dramatic examples are spacetimes possessing trapping but not event horizons (Roman & Bergmann '83, Hayward '06).

Isolated, dynamical, and slowly evolving horizons

Isolated horizons correspond to isolated systems in thermal equilibrium. Introduced in relation with loop quantum gravity

(Ashtekar, Beetle & Fairhurst '99; Ashtekar *et al.* 00). Too restrictive when one wants to allow mass-energy to cross the "horizon". A *weakly isolated horizon* is *a null surface* $\mathcal{H}$ *with null normal* l^a *such that* $\theta_l = 0$, $-T_{ab} l^a$ *is a future-oriented and causal vector, and* $\mathcal{L}_l \left(n_b \nabla_a l^b \right) = 0$. l^a is a Killing vector for the intrinsic geometry on $\mathcal{H}$, without reference to the surroundings: a "completely local Killing horizon" when there are no energy flows across $\mathcal{H}$. The vector field l^a generates a congruence of null geodesics on $\mathcal{H}$, which can be used to define a surface gravity κ via the (non-affinely parametrized) geodesic equation

$$l^a \nabla_a l^b = \kappa \, l^b \rightarrow \kappa = -n_b l^a \nabla_a l^b \, .$$

Since n^a is not unique also this surface gravity is not unique. A Hamiltonian analysis of the phase space of isolated horizons, identifying boundary terms with the energies of these boundaries, leads to a 1st law for isolated horizons with rotational symmetry,

$$\delta H_{\mathcal{H}} = \frac{\kappa}{8\pi} \, \delta A + \Omega_{\mathcal{H}} \delta J \, ,$$

where J = angular momentum, $H_{\mathcal{H}}$ = Hamiltonian, A = area of 2-D cross-sections of $\mathcal{H}$, $\Omega_{\mathcal{H}}$ = angular velocity of horizon.

A *dynamical horizon* (Ashtekar & Krishnan '04) is *a spacelike marginally trapped tube foliated by marginally trapped 2-surfaces* (MTT). Definition allows energy fluxes across the dynamical horizon. A set of flux laws describing the related changes in the area of the dynamical horizons have been formulated. An AH which is everywhere spacelike coincides with a dynamical horizon.

Slowly evolving horizons (Booth & Fairhurst '04; Kavanagh & Booth '06) are "almost isolated" FOTHs intended to represent BH horizons which evolve slowly in time.

Kodama vector

In dynamical situations there is no timelike Killing vector and these surface gravities defined in various ways are inequivalent. In spherical symmetry, the Kodama vector mimics the properties of a Killing vector and originates a (miracolously) conserved current and a surface gravity.

Defined only for spherically symmetric spacetimes. Let the metric be

$$ds^2 = h_{ab}dx^a dx^b + R^2 d\Omega_{(2)}^2 \,,$$

where $a, b = 0, 1$ and R is the areal radius and $d\Omega_{(2)}^2 = d\theta^2 + \sin^2\theta \, d\varphi^2$. Let ϵ_{ab} =volume form of h_{ab}; the *Kodama vector* is

$$K^a \equiv -\epsilon^{ab}\nabla_b R \,.$$

In a static spacetime the Kodama vector is parallel (not equal) to the timelike Killing vector. When timelike, the Kodama vector defines a class of preferred observers (it is timelike in asymptotically flat regions).

Divergence-free, $\nabla_a K^a = 0$, so the Kodama energy current $J^a \equiv G^{ab}K_b$ is covariantly conserved, $\nabla^a J_a = 0$ even if there is no timelike Killing vector ("Kodama miracle"). The Noether charge associated with the Kodama conserved current is the Misner-Sharp-Hernandez energy.

Spherical symmetry

Misner-Sharp-Hernandez mass defined in GR and for spherical symmetry, coincides with the Hawking-Hayward quasi-local mass (Hawking '68; Hayward '94). Use areal radius R, write

$$ds^2 = h_{ab} dx^a dx^b + R^2 d\Omega_{(2)}^2 \qquad (a, b = 1, 2). \tag{1}$$

then

$$1 - \frac{2M}{R} \equiv \nabla^c R \, \nabla_c R$$

Formalism of Nielsen and Visser '06, general spherical metric is

$$ds^2 = -e^{-2\phi(t,R)} \left[1 - \frac{2M(t,R)}{R} \right] dt^2 + \frac{dR^2}{1 - \frac{2M(t,R)}{R}} + R^2 d\Omega_{(2)}^2$$

where $M(t, R)$ *a posteriori* is the Misner-Sharp-Hernandez mass. Recast in Painlevé-Gullstrand coordinates as

$$ds^2 = -\frac{e^{-2\phi}}{(\partial\tau/\partial t)^2} \left(1 - \frac{2M}{R} \right) d\tau^2 + \frac{2e^{-\phi}}{\partial\tau/\partial t} \sqrt{\frac{2M}{R}} \, d\tau dR + dR^2 + R^2 d\Omega_{(2)}^2$$

with $\phi(\tau, R)$ and $M(\tau, R)$ implicit functions. Use

$$c(\tau, R) \equiv \frac{e^{-\phi(t,R)}}{(\partial\tau/\partial t)}, \qquad v(\tau, R) \equiv c \sqrt{\frac{2M}{R}},$$

then line element becomes

$$ds^2 = -\left[c^2(\tau, R) - v^2(\tau, R)\right] d\tau^2 + 2v(\tau, R)\, d\tau dR + dR^2 + R^2 d\Omega_{(2)}^2 \,. \tag{2}$$

Outgoing radial null geodesics have tangent

$$l^\mu = \frac{1}{c(\tau, R)} \Big(1, c(\tau, R) - v(\tau, R), 0, 0 \Big) \,, \tag{3}$$

ingoing radial null geodesics have tangents

$$n^\mu = \frac{1}{c(\tau, R)} \Big(1, -c(\tau, R) - v(\tau, R), 0, 0 \Big) \,, \tag{4}$$

with $g_{ab} l^a n^b = -2$. Expansions are

$$\theta_{l,n} = \pm \frac{2}{R} \left(1 \mp \sqrt{\frac{2M}{R}} \right)$$

A sphere of radius R is *trapped* if $R < 2M$, *marginal* if $R = 2M$, *untrapped* if $R > 2M$. AHs located by

$$\frac{2M\left(\tau, R_{AH}\right)}{R_{AH}(\tau)} = 1 \iff \nabla^c R \nabla_c R \,|_{AH} = 0 \iff g^{RR} \,|_{AH} = 0 \,,$$

Inverse metric is

$$\left(g^{\mu\nu}\right) = \frac{1}{c^2} \left(\begin{array}{cccc} 1 & -v & 0 & 0 \\ -v & -(c^2 - v^2) & 0 & 0 \\ 0 & 0 & \frac{1}{R^2} & 0 \\ 0 & 0 & 0 & \frac{1}{R^2 \sin^2\theta} \end{array} \right) .$$

Condition $g^{RR} = 0$ is a very convenient recipe to locate the AHs in spherical symmetry.

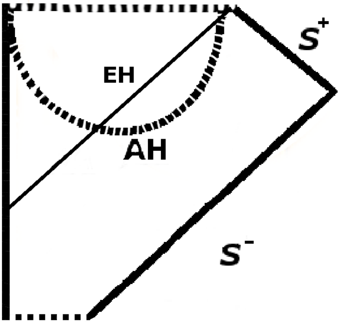


Figure: The conformal diagram of an hypothetical cosmological BH. The top horizontal line is a spacelike BH singularity. An AH can change from timelike, to null, to spacelike, and it can be located inside or outside the EH, according to whether the energy conditions are satisfied or not.

COSMOLOGICAL BHs IN SCALAR-TENSOR AND $f(R)$ GRAVITY

or A COSMOLOGICAL BESTIARY

Few solutions known. Simplest theory: Brans-Dicke gravity, with action

$$S_{BD} = \int d^4x \sqrt{-g} \left[\phi R^c{}_c - \frac{\omega}{\phi} g^{ab} \nabla_a \phi \nabla_b \phi + 2\kappa \mathcal{L}^{(m)} \right]$$

$\phi \approx G_{eff}^{-1}$

The Husain-Martinez-Nuñez solution of GR

HMN '94 new phenomenology of AHs. This spacetime describes an inhomogeneity in a spatially flat FLRW "background" sourced by a free, minimally coupled, scalar field.

$$ds^2 = (A_0\eta + B_0)\left[-\left(1-\frac{2C}{r}\right)^{\alpha} d\eta^2 + \frac{dr^2}{\left(1-\frac{2C}{r}\right)^{\alpha}} + r^2\left(1-\frac{2C}{r}\right)^{1-\alpha} d\Omega^2_{(2)}\right],$$

$$\phi(\eta, r) = \pm\frac{1}{4\sqrt{\pi}} \ln\left[D\left(1-\frac{2C}{r}\right)^{\alpha/\sqrt{3}} (A_0\eta + B_0)^{\sqrt{3}}\right],$$

where $A_0, B_0, C, D \geq 0$ constants, $\alpha = \pm\sqrt{3}/2$, $\eta > 0$. The additive constant B_0 becomes irrelevant and can be dropped whenever $A_0 \neq 0$. When $A_0 = 0$, the HMN metric degenerates into the static Fisher spacetime (Fisher '48)

$$ds^2 = -V^\nu(r)\, d\eta^2 + \frac{dr^2}{V^\nu(r)} + r^2 V^{1-\nu}(r) d\Omega^2_{(2)} \,,$$

where $V(r) = 1 - 2\mu/r$, μ and ν are parameters, and the Fisher scalar field is

$$\psi(r) = \psi_0 \ln V(r) \,.$$

(a.k.a. Janis-Newman-Winicour-Wyman solution, rediscovered many times, naked singularity at $r = 2C$, asympt. flat). The general HMN metric is conformal to the Fisher metric with conf. factor $\Omega = \sqrt{A_0 \eta + B_0}$ equal to the scale factor of the "background" FLRW space and with only two possible values of the parameter ν. Set $B_0 = 0$. Metric is asympt. FLRW for $r \to +\infty$ and is FLRW if $C = 0$ (in which case the constant A_0 can be eliminated by rescaling η).

Ricci scalar is

$$R^a{}_a = 8\pi \nabla^c \phi \nabla_c \phi = \frac{2\alpha^2 C^2 \left(1 - \frac{2C}{r}\right)^{\alpha-2}}{3r^4 A_0 \eta} - \frac{3A_0^2}{2\left(A_0\eta\right)^3 \left(1 - \frac{2C}{r}\right)^{\alpha}} \,,$$

spacetime singularity at $r = 2C$ (for both values of α). ϕ also diverges there, Big Bang singularity at $\eta = 0$. $2C < r < +\infty$ and $r = 2C$ corresponds to zero areal radius

$$R(\eta, r) = \sqrt{A_0 \eta}\, r \left(1 - \frac{2C}{r}\right)^{\frac{1-\alpha}{2}} \,.$$

Use comoving time t, then

$$t = \int d\eta\, a(\eta) = \frac{2\sqrt{A_0}}{3}\, \eta^{3/2} \,, \qquad \eta = \left(\frac{3}{2\sqrt{A_0}}\, t\right)^{2/3} \,, \quad a(t) = a_0\, t^{1/3} \,,$$

HMN solution in comoving time reads

$$ds^2 = -\left(1 - \frac{2C}{r}\right)^{\alpha} dt^2 + a^2 \left[\frac{dr^2}{\left(1 - \frac{2C}{r}\right)^{\alpha}} + r^2 \left(1 - \frac{2C}{r}\right)^{1-\alpha} d\Omega^2_{(2)}\right]$$

with

$$\phi(t,r) = \pm\frac{1}{4\sqrt{\pi}} \ln\left[D\left(1-\frac{2C}{r}\right)^{\alpha/\sqrt{3}} a^{2\sqrt{3}}(t)\right] .$$

Areal radius $R(t,r)$ increases for $r > 2C$. In terms of R, setting

$$A(r) \equiv 1-\frac{2C}{r}, \qquad B(r) \equiv 1-\frac{(\alpha+1)C}{r},$$

we have $R(t,r) = a(t)rA^{\frac{1-\alpha}{2}}(r)$ and a time-radius cross-term is eliminated by introducing a new T with $dT = \frac{1}{F}\left(dt + \beta dR\right)$,

$$\beta(t,R) = \frac{HRA^{\frac{3(1-\alpha)}{2}}}{B^2(r) - H^2R^2A^{2(1-\alpha)}};$$

then

$$ds^2 = -A^{\alpha}(r)\left[1-\frac{H^2R^2A^{2(1-\alpha)}(r)}{B^2(r)}\right]F^2dt^2$$

$$+\frac{H^2R^2A^{2-\alpha}}{B^2(r)}\left[1+\frac{A^{1-\alpha}(r)}{B^2(r)-H^2R^2A^{2(1-\alpha)}(r)}\right]dR^2+R^2d\Omega^2_{(2)}$$

AHs located by $g^{RR}=0$, or

$$\frac{1}{\eta}=\frac{2}{r^2}\Big[r-(\alpha+1)C\Big]\left(1-\frac{2C}{r}\right)^{\alpha-1} .$$

For $R\rightarrow+\infty$, reduces to $R\simeq H^{-1}$, cosmological AH in FLRW. Let $x\equiv C/r$, then the AH eq. is

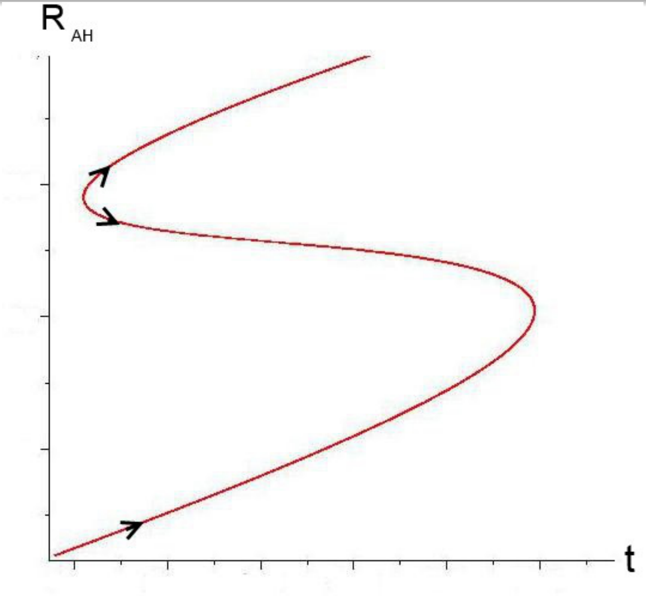
$$HR=\left[1-\frac{(\alpha+1)C}{r}\right]\left(1-\frac{2C}{r}\right)^{\alpha-1} .$$

lhs is

$$HR=\frac{a_0}{3\,t^{2/3}}\,\frac{2C}{x}\,(1-2x)^{\frac{1-\alpha}{2}},$$

rhs is $[1-(\alpha+1)x]\,(1-2x)^{\alpha-1}$ and

$$
\begin{aligned}
t(x) &= \left\{ \frac{2Ca_0}{3}\,\frac{(1-2x)^{3(1-\alpha)}}{x\,[1-(\alpha+1)x]} \right\}^{3/2} , \\
R(x) &= a_0\, t^{1/3}(x)\,\frac{2C}{x}\,(1-2x)^{\frac{1-\alpha}{2}} .
\end{aligned}
$$

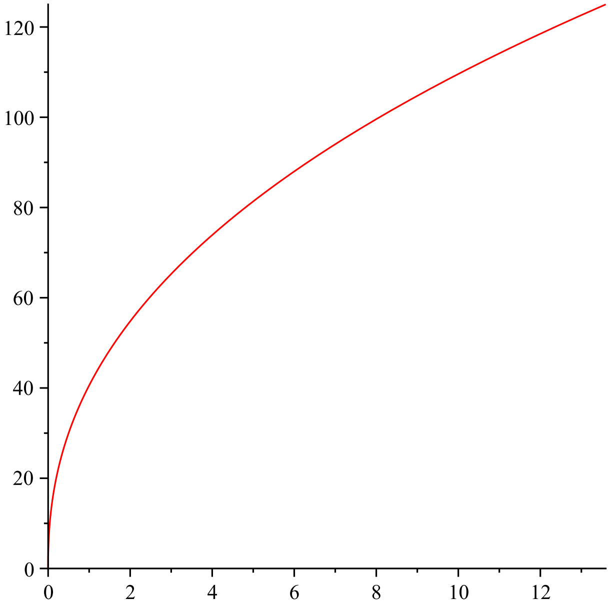


If $\alpha = \sqrt{3}/2$, between the Big Bang and a critical time t_* there is only one expanding AH, then two other AHs are created at t_*. One is a cosmological AH which expands forever and the other is a BH horizon which contracts until it meets the first (expanding) BH AH. When they meet, these two annihilate and a naked singularity appears at $R = 0$ in a FLRW universe.

"S-curve" phenomenology appears also in Lemaître-Tolman-Bondi spacetimes (dust fluid) (Booth *et al.*)

Multiple "S"s are possible in other GR solutions, for example 5 AHs may appear. Also in Brans-Dicke and $f\left(R^c{}_c\right)$ gravity.

Scalar field is regular on AHs.



120
100
80
60
40
20
0
0
2
4
6
8
10
12

For $\alpha = -\sqrt{3}/2$ there is only one cosmological AH and the universe contains a naked singularity at $R = 0$.
AHs are *spacelike*: normal vector always lies inside the light cone in an (η, r) diagram. In agreement with a general result of (Booth *et al.* '06) that a trapping horizon created by a massless scalar field must be spacelike (but a $V(\phi)$ can make the trapping horizon non-spacelike).
Singularity at $R = 0$ is timelike for both values of parameter α. Created with the universe in the Big Bang, does not result from a collapse process.


Aliud animal est in nilo flumine quod
dicitur hydrus. phisiologus dicit
de eo quod satis est hoc animal inimicum
cocodrillo. et hanc habet naturam et con
suetudinem. Cum videt cocodrillum
in litoribus fluminis dormientem aperto
ore vadit et involvit se in limum luti quo

The conformal cousin of the HMN solution

Solution of BD gravity found by Clifton, Mota, and Barrow '05 by conformally transforming the HMN solution,

$$ds^2 = -A^{\alpha\left(1-\frac{1}{\sqrt{3}\,\beta}\right)}(r)\,dt^2 + A^{-\alpha\left(1+\frac{1}{\sqrt{3}\,\beta}\right)}(r)\,t^{\frac{2\left(\beta-\sqrt{3}\right)}{3\beta-\sqrt{3}}}\left[dr^2 + r^2 A(r) d\Omega_{(2)}^2\right],$$

$$\phi(t,r) = A^{\frac{\pm 1}{2\beta}}(r)\,t^{\frac{2}{\sqrt{3}\,\beta-1}},$$

where

$$A(r) = 1 - \frac{2C}{r}, \quad \beta = \sqrt{2\omega+3}, \quad \omega > -3/2.$$

Singularities at $r = 2C$ and $t = 0$ ($2C < r < +\infty$ and $t > 0$). The scale factor is

$$a(t) = t^{\frac{\beta-\sqrt{3}}{3\beta-\sqrt{3}}} \equiv t^{\gamma}.$$ (5)

Interpreted in VF & A. Zambrano Moreno '12. Rewrite as

$$ds^2 = -A^{\sigma}(r)\, dt^2 + A^{\Theta}(r)\, a^2(t) dr^2 + R^2(t,r) d\Omega^2_{(2)}\,,$$

where

$$\sigma = \alpha \left(1 - \frac{1}{\sqrt{3}\,\beta} \right)\,, \quad \Theta = -\alpha \left(1 + \frac{1}{\sqrt{3}\,\beta} \right)\,,$$

and

$$R(t,r) = A^{\frac{\Theta+1}{2}}(r)\, a(t)\, r$$

Study the area of the 2-spheres of symmetry:
$\partial R/\partial r = a(t) A^{\frac{\Theta-1}{2}}(r)\,(1 - r_0/r)$ where

$$r_0 = (1-\Theta) C\,, \quad R_0(t) = \left(\frac{\Theta+1}{\Theta-1} \right)^{\frac{\Theta+1}{2}} (1-\Theta) a(t)\, C\,.$$

Critical value r_0 lies in the physical region $r_0 > 2C$ if $\Theta < -1$. R has the limit

$$R(t,r) = \frac{r\, a(t)}{\left(1 - \frac{2C}{r} \right)^{\left| \frac{\Theta+1}{2} \right|}} \rightarrow +\infty \quad \text{as } r \rightarrow 2C^+ \qquad (6)$$

For $\Theta < -1$, $R(r)$ has a minimum at r_0, the area $4\pi R^2$ of the 2-spheres of symmetry is minimum there, and there is a *wormhole throat* joining two spacetime regions. Since

$$\Theta = \mp \frac{\sqrt{3}}{2} \left(1 + \frac{1}{\sqrt{3}\sqrt{2\omega + 3}} \right)$$

for $\alpha = \pm\sqrt{3}/2$, condition $\Theta < -1$ requires $\alpha = +\sqrt{3}/2$ (necessary but not sufficient condition for the throat). Sufficient condition $\Theta < -1$ constrains

$$\omega < \frac{1}{2} \left[\frac{1}{\left(2 - \sqrt{3}\right)^2} - 3 \right] \simeq 5.46 \equiv \omega_0 \, .$$

For $-3/2 < \omega < \omega_0$, wormhole throat is exactly comoving with the cosmic substratum.

AHs: use $dr = \frac{dR - A^{\frac{\Theta+1}{2}}(r)\,\dot{a}(t)\,r dt}{A^{\frac{\Theta-1}{2}}\,a(t)\,\frac{C(\Theta+1)}{r} + A^{\frac{\Theta+1}{2}}(r)\,a(t)}$, turn line element into

$$ds^2 = -\frac{\left(D_1 A^\sigma - H^2 R^2\right)}{D_1}\,dt^2 - \frac{2HR}{D_1}\,dt dR + \frac{dR^2}{D_1} + R^2 d\Omega_{(2)}^2$$

Inverse metric is

$$\left(g^{\mu\nu}\right) = \begin{pmatrix} -\frac{1}{A^\sigma} & -\frac{HR}{A^\sigma} & 0 & 0 \\ -\frac{HR}{A^\sigma} & \frac{\left(D_1 A^\sigma - H^2 R^2\right)}{A^\sigma} & 0 & 0 \\ 0 & 0 & R^{-2} & 0 \\ 0 & 0 & 0 & R^{-2}\sin^{-2}\theta \end{pmatrix} .$$

AHs located by $g^{RR} = 0$, or $D_1(r)A(r) = H^2(t)R^2(t,r)$. There are solutions which describe apparent horizons with the "S-curve" phenomenology of the HMN solution of GR. AH eq. satisfied also if the rhs is time-independent, $H = \alpha/t = 0$,

$\gamma = 0$, $\beta = \sqrt{3}$, $\omega = 0$, which produces a static BD solution (inhomogeneity in Minkowski).
Other cases include a) $\omega \leq \omega_0$ and b) $\alpha = -3/2$: no wormhole throats, no AHs, naked singularity.
$\alpha = -\sqrt{3}/2$: it is $\Theta = \frac{\sqrt{3}}{2}\left(1 + \frac{1}{\sqrt{3}\,\beta}\right) > 0$, $R(t)$ is monotonic, naked singularity at $R = 0$.
Special case $\omega = 0$ ($\beta = \sqrt{3}$, $\gamma = 0$) produces the static solution

$$ds^2 = -A^{\frac{2\alpha}{3}}(r)dt^2 + \frac{dr^2}{A^{\frac{4\alpha}{3}}(r)} + \frac{r^2}{A^{\frac{4\alpha}{3}-1}(r)}\, d\Omega^2_{(2)}$$

$$\phi(t, r) = A^{\frac{\pm 1}{2\sqrt{3}}}(r)t\,,$$

(ϕ time-dependent even though metric is static) this is a Campanelli-Lousto metric. General CL solution of BD theory has form

$$
\begin{aligned}
ds^2 &= -A^{b+1}(r)dt^2 + \frac{dr^2}{A^{a+1}(r)} + \frac{r^2 d\Omega_{(2)}^2}{A^a(r)}\,, \\
\phi(r) &= \phi_0 A^{\frac{a-b}{2}}(r)\,,
\end{aligned}
$$

with $\phi_0 > 0, a, b$ constants,

$$\omega(a,b) = -2\left(a^2+b^2-ab+a+b\right)(a-b)^{-2}\ .$$

Reporduced by setting $(a,b) = \left(\frac{4\alpha}{3}-1, \frac{2\alpha}{3}-1\right)$, then $\omega\left(\frac{4\alpha}{3}-1, \frac{2\alpha}{3}-1\right) = 0$ for $\alpha = \pm\sqrt{3}/2$. The nature of the CL spacetime depends on sign(a) (choice $\alpha = \pm\sqrt{3}/2$ (Vanzo, Zerbini, VF '12). For $a \geq 0 \leftrightarrow \alpha = +\sqrt{3}/2$, $a \simeq 0.1547$, and $\Theta = -\frac{4\alpha}{3} \simeq -1.1547 < -1$ CL contains a wormhole throat which coincides with an AH at $r_0 = 2C\left(\frac{1-\Theta}{2}\right) > 2C$.
For $a < 0 \leftrightarrow \alpha = -\sqrt{3}/2\,, a \simeq -2.1547$, and $\Theta \simeq 1.1547 > 0$, there are no AHs and CL contains a naked singularity.


Mandra
gore.

The BD solutions of Clifton, Mota, and Barrow

Clifton, Mota, Barrow '05

$$ds^2 = -e^{\nu(\varrho)}dt^2 + a^2(t)e^{\mu(\varrho)}(d\varrho^2 + \varrho^2 d\Omega^2)\,,$$

where

$$e^{\nu(\varrho)} = \left(\frac{1-\frac{m}{2\alpha\varrho}}{1+\frac{m}{2\alpha\varrho}}\right)^{2\alpha} \equiv A^{2\alpha}$$

$$e^{\mu(\varrho)} = \left(1+\frac{m}{2\alpha\varrho}\right)^4 A^{\frac{2}{\alpha}(\alpha-1)(\alpha+2)}$$

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2\omega(2-\gamma)+2}{3\omega\gamma(2-\gamma)+4}} \equiv a_* t^{\beta}$$

$$\phi(t,\varrho) = \phi_0\left(\frac{t}{t_0}\right)^{\frac{2(4-3\gamma)}{3\omega\gamma(2-\gamma)+4}} A^{-\frac{2}{\alpha}(\alpha^2-1)}$$

$$\alpha = \sqrt{\frac{2(\omega+2)}{2\omega+3}}$$

$$\rho^{(m)}(t,\varrho) = \rho_0^{(m)} \left(\frac{a_0}{a(t)} \right)^{3\gamma} A^{-2\alpha}$$

Matter source is a perfect fluid with $P^{(m)} = (\gamma - 1)\,\rho^{(m)}$ with $\gamma =$const. $m, \alpha, \phi_0, a_0, \rho_0^{(m)}, t_0$ are > 0. Areal radius is

$$r = a(t)\varrho \left(1 + \frac{m}{2\alpha\varrho} \right)^2 A^{\frac{1}{\alpha}(\alpha-1)(\alpha+2)} = a(t)\tilde{r} A^{\frac{1}{\alpha}(\alpha-1)(\alpha+2)}$$

Require that $\omega_0 > -3/2$ and $\beta \geq 0$. Interpreted in [1]F, Vitagliano, Sotiriou, Liberati '12, solve $g^{RR} = 0$ numerically. According to the parameter values, several behaviours are possible. The "S-curve" familiar from the HMN solution is reproduced in a certain region of the parameter space, but different behaviours appear for other combinations of the parameters. In certain regions of the parameter space, CMB contains a naked singularity created with the universe. In other regions of the parameter space, pairs of black hole and cosmological apparent horizons appear and bifurcate, or merge and disappear. Larger parameter space involved, CMB class exhibits most varied and richer phenomenology of AHs seen (some new one).

Clifton's solution of $f\left(R^c{}_c\right)$ gravity

Metric $f\left(R^c{}_c\right)$ gravity described by

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g}\, f(R^c{}_c) + S^{(matter)} \,,$$

Jebsen-Birkhoff theorem of GR fails. A solution of vacuum $f(R^c{}_c) = (R^c{}_c)^{1+\delta}$ gravity was found by Clifton '06

$$ds^2 = -A_2(r)dt^2 + a^2(t)B_2(r)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$$

$$
\begin{aligned}
A_2(r) &= \left(\frac{1 - C_2/r}{1 + C_2/r}\right)^{2/q} , \\
B_2(r) &= \left(1 + \frac{C_2}{r}\right)^4 A_2(r)^{\,q+2\delta-1} , \\
a(t) &= t^{\frac{\delta(1+2\delta)}{1-\delta}} , \\
q^2 &= 1 - 2\delta + 4\delta^2 .
\end{aligned}
$$

Solar System tests require $\delta = (-1.1 \pm 1.2) \cdot 10^{-5}$, and $f''\left(R^c{}_c\right) \geq 0$ for local stability: assume $0 < \delta < 10^{-5}$. Clifton solution is conformal to the Fonarev and conformally static. AHs studied in VF '10, using areal radius and new time, metric becomes

$$
ds^2 = -A_2 D F^2 d\bar{t}^2 + \frac{dR^2}{A_2^q C^2 D} + R^2 d\Omega_{(2)}^2 ,
$$

$$D \equiv 1 - \frac{A_2^{2(\delta-1)}}{C^2}\,\dot{a}^2 \tilde{r}^2 = 1 - \frac{A_2^{-q-1}}{C^2}\, H^2 R^2 \,.$$

AHs located by parametric representation

$$R(x) = t(x)^{\frac{\delta(1+2\delta)}{1-\delta}} \frac{C_2}{x} \,(1-x)^{\frac{q+2\delta-1}{q}} \,(1+x)^{\frac{q-2\delta+1}{q}} \;,$$

$$t(x) = \left\{ \frac{(1-\delta)}{\delta\,(1+2\delta)\,C_2} \, \frac{x\,(1+x)^{\frac{2(-q+\delta-1)}{q}}}{(1-x)^{\frac{2(\delta-1)}{q}}} \left[1 + \frac{2\,(q+2\delta-1)\,x}{q(1-x)^2} \right] \right\}^{\frac{\;}{2}}$$

where $x \equiv C_2/r$: same “S-curve” phenomenology of the HMN solution.

Other solutions

Few other solutions known in BD theory Sakai & Barrow, Einstein-Gauss-Bonnet gravity Nozawa & Maeda '08, higher order gravity Charmousis, Lovelock gravity Maeda, Willison, Ray '11 (Einstein-Gauss-Bonnet and Lovelock appropriate in $D > 4$, bestiary then includes Myers-Perry BHs, black strings, black rings, black Saturns, *etc.*). Add stringy/supergravity BHs. Misner-Sharp-Hernandez mass and Kodama vector defined in GR and Einstein-Gauss-Bonnet gravity (perhaps in FLRW in $f(R)$ gravity)

ISOLATED SCALAR-TENSOR BHs

or ISOLATED BEASTS ARE TAMED

[T.P. Sotiriou & VF PRL 2012]

- In GR spacetime singularities are generic (Hawking & Penrose) and they are usually cloacked by horizons (Cosmic Censorship).
- GR: **stationary** black holes (endpoint of grav. collapse) must be **axisymmetric** (Hawking '72). Asympt. flat black holes in GR are simple.
- Non-asympt. flat black holes can be very complicated: "cosmological" black holes have appearing/disappearing apparent horizons (McVittie, generalized McVittie, LTB, Husain-Martinez-Nuñez, Fonarev, ...). Interaction between black hole and cosmic "background".
- Scalar-tensor, $f(R)$ gravity, higher order gravity, low-energy effective actions for quantum gravity, etc.: Birkhoff's theorem is lost.

Prototype: Brans-Dicke theory (Jordan frame)

$$S_{BD} = \int d^4x \sqrt{-\hat{g}} \left[\varphi \hat{R} - \frac{\omega_0}{\varphi} \hat{\nabla}^\mu \varphi \hat{\nabla}_\mu \varphi + L_m(\hat{g}_{\mu\nu}, \psi) \right]$$

- Hawking '72: endpoint of axisymmetric collapse in this theory must be GR black holes. Result generalized *for spherical symmetry only* by Bekenstein + Mayo '96, Bekenstein '96, + bits and pieces of proofs.
- What about more general theories?

$$S_{ST} = \int d^4x \sqrt{-\hat{g}} \left[\varphi \hat{R} - \frac{\omega(\varphi)}{\varphi} \hat{\nabla}^\mu \varphi \hat{\nabla}_\mu \varphi - V(\varphi) + L_m(\hat{g}_{\mu\nu}, \psi) \right]$$

This action includes metric and Palatini $f(R)$ gravity inportant for cosmology.

A SIMPLE PROOF

This work (T.P. Sotiriou & VF 2012, *Phys. Rev. Lett.* 108, 081103): extend result to *general* scalar-tensor theory

$$S_{ST} = \int d^4x \sqrt{-\hat{g}} \left[\varphi \hat{R} - \frac{\omega(\varphi)}{\varphi} \hat{\nabla}^{\mu}\varphi \hat{\nabla}_{\mu}\varphi - V(\varphi) + L_m(\hat{g}_{\mu\nu}, \psi) \right]$$

we require

- **asymptotic flatness** (collapse on scales $\ll H_0^{-1}$): $\varphi \to \varphi_0$ as $r \to +\infty$, $V(\varphi_0) = 0$, $\varphi_0 \, V'(\varphi_0) = 2\,V(\varphi_0)$
- **stationarity** (endpoint of collapse).

Use Einstein frame $\hat{g}_{\mu\nu} \to g_{\mu\nu} = \varphi \, \hat{g}_{\mu\nu}$, $\varphi \to \phi$ with

$$d\phi = \sqrt{\frac{2\omega(\varphi) + 3}{16\pi}} \, \frac{d\varphi}{\varphi} \qquad (\omega \neq -3/2)$$

brings the action to

$$S_{ST} = \int d^4x\sqrt{-g}\Big[\frac{R}{16\pi} - \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - U(\phi) + L_m(\hat{g}_{\mu\nu},\psi)\Big]$$

where $U(\phi) = V(\varphi)/\varphi^2$. Field eqs. are

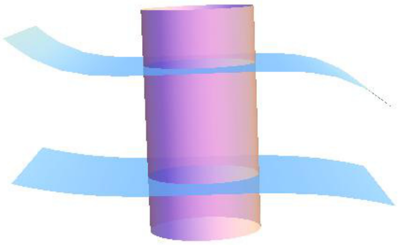
$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}\hat{g}_{\mu\nu} = \frac{\omega(\varphi)}{\varphi^2}\left(\hat{\nabla}_\mu\varphi\hat{\nabla}_\nu\varphi - \frac{1}{2}\hat{g}_{\mu\nu}\,\hat{\nabla}^\lambda\varphi\hat{\nabla}_\lambda\varphi\right) + \frac{1}{\varphi}\left(\hat{\nabla}_\mu\hat{\nabla}_\nu\varphi - \hat{g}_{\mu\nu}\hat{\Box}\varphi\right) - \frac{V(\varphi)}{2\varphi}\hat{g}_{\mu\nu}\,,$$

$$(2\omega+3)\,\hat{\Box}\varphi = -\omega'\,\hat{\nabla}^\lambda\varphi\hat{\nabla}_\lambda\varphi + \varphi\,V' - 2V\,,$$

$\Omega = \Omega(\varphi) \longrightarrow$ same symmetries as in the J. frame:

- ξ^μ timelike Killing vector (stationarity)
- ζ^μ spacelike at spatial infinity (axial symmetry).

Consider, in vacuo, a 4-volume $\mathcal{V}$ bounded by the horizon H, two Cauchy hypersurfaces $\mathcal{S}_1$, $\mathcal{S}_2$, and a timelike 3-surface at infinity



multiply $\Box\phi = U'(\phi)$ by U', integrate over $\mathcal{V} \longrightarrow$

$$\int_{\mathcal{V}} d^4x\sqrt{-g}\, U'(\phi)\Box\phi = \int_{\mathcal{V}} d^4x\sqrt{-g}\, U'^2(\phi)$$

rewrite as

$$\int_{\mathcal{V}} d^4x\sqrt{-g}\left[U''(\phi)\nabla^\mu\phi\nabla_\mu\phi + U'^2(\phi)\right]$$
$$= \int_{\partial\mathcal{V}} d^3x\sqrt{|h|}\, U'(\phi)n^\mu\nabla_\mu\phi$$

where n^μ =normal to the boundary, h =determinant of the induced metric $h_{\mu\nu}$ on this boundary. Split the boundary into its constituents $\int_{\mathcal{V}} = \int_{\mathcal{S}_1} + \int_{\mathcal{S}_2} + \int_{horizon} + \int_{r=\infty}$ Now, $\int_{\mathcal{S}_1} = -\int_{\mathcal{S}_2}$, $\int_{r=\infty} = 0$, $\int_{horizon} d^3x\sqrt{|h|}\, U'(\phi)n^\mu\nabla_\mu\phi = 0$ because of the symmetries.

$$\longrightarrow \int_{\mathcal{V}} d^4x\sqrt{-g}\left[U''(\phi)\nabla^\mu\phi\nabla_\mu\phi + U'^2(\phi)\right] = 0\,.$$

Since $U'^2 \geq 0$, $\nabla^\mu \phi$ (orthogonal to both ξ^μ, ζ^μ on H) is spacelike or zero, and $U''(\phi) \geq 0$ for stability (black hole is the endpoint of collapse!), it must be $\nabla_\mu \phi \equiv 0$ in $\mathcal{V}$ and $U'(\phi_0) = 0$. For $\phi =$const., **theory reduces to GR, black holes must be Kerr**.

- Metric $f(R)$ gravity is a special case of BD theory with $\omega = 0$ and $V \neq 0$.
- for $\omega = -3/2$, vacuum theory reduces to GR, Hawking's theorem applies (Palatini $f(R)$ gravity is a special BD theory with $\omega = -3/2$ and $V \neq 0$).

Exceptions not covered by our proof:

- theories in which $\omega \to \infty$ somewhere
- theories in which φ diverges (at ∞ or on the horizon) ex: maverick solution of Bocharova *et al.* '80 (unstable).
- Proof extends immediately to electrovacuum/conformal matter ($T = 0$).

CONCLUSIONS AND OPEN PROBLEMS

- Rich bestiary (phenomenology and dynamics) of evolving horizons.
- Are AHs/trapping horizons the “right’ quantities for thermodynamics? Is their thermodynamics meaningful? Is the Kodama prescription correct? (conflicting views)
- An adiabatic approximation should be meaningful. Do fast-evolving horizons require non-equilibrium thermodynamics?
- Even though Birkhoff’s theorem is lost, black holes which are the endpoint of axisymmetric gravitational collapse (and asympt. flat) in *general* scalar-tensor gravity are the same as in GR (*i.e.*, Kerr-Newman). Proof extends to electrovacuum.
- Exceptions (exact solutions) are unphysical or unstable solutions which cannot be the endpoint of collapse, or do not satisfy the Weak/Null Energy Condition.

- Asymptotic flatness is a technical assumption, but can't eliminate it at the moment. Excludes "large" primordial black holes in a "small" universe.
- What about more general theories with other degrees of freedom?

OBRIGADO!