Warped AdS_3 Black Holes: Are They Classically Stable?

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Outline

Introduction

AIM: study QFT in black hole spacetimes

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Have been mostly restricted to:

- asymptotically flat spacetimes (astrophysics)
- asymptotically AdS spacetimes (AdS/CFT correspondence)
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Rotating spacetimes significantly more difficult!

Frolov and Thorne (1989)
Kay and Wald (1991)
Ottewill and Winstanley (2000)
Ottewill and Duffy (2008)

Topologically Massive Gravity

Simplification: consider (2+1)-dimensional spacetimes

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$$S = S_{\text{E-H}} + S_{\text{C-S}}\,,$$

with:

$$S_{\text{E-H}} = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left(R + \frac{2}{\ell^2}\right)\,,$$
$$S_{\text{C-S}} = \frac{\ell}{96\pi G\nu}\int d^3x\sqrt{-g}\,\epsilon^{\lambda\mu\nu}\,\Gamma^{\rho}_{\lambda\sigma}\left(\partial_\mu\Gamma^{\sigma}_{\rho\nu} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\rho}\right)\,.$$

Deser, Jackiw, Templeton (1982)

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- **Massive** propagating degree of freedom.
- **Third-order** derivative theory.
- GR solutions $\subset$ TMG solutions (eg AdS, BTZ black hole)

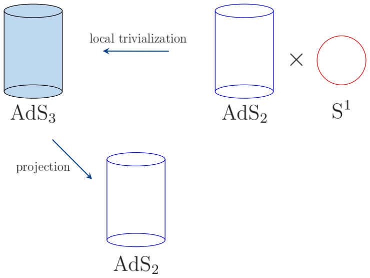
Outline

Hopf fibration of AdS_3

Hopf fibration of AdS_3:

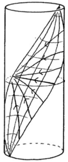
S^1 (fibre) $\hookrightarrow AdS_3 \to AdS_2$ (base)

$$ds^2 = \frac{1}{4}\Big[-\cosh^2\sigma\, d\tau^2 + d\sigma^2 + (du + \sinh\rho\, d\tau)^2\Big]$$



The Hopf fibres are the flow lines of a spacelike congruence of a Killing vector field of AdS_3.

Bengtsson, Sandin (2005)



Hopf fibration of $WAdS_3$

- **Spacelike warped AdS_3**: spacelike fibres are scaled relative to the base AdS_2:

$$ds^2 = \frac{1}{4}\Big[-\cosh^2\sigma\, d\tau^2 + d\sigma^2 + \lambda^2(du + \sinh\sigma\, d\tau)^2\Big]$$

If $\lambda^2 > 1$ this is the **spacelike stretched AdS_3** metric.

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- In the context of 2+1 gravity:

$$ds^2 = \frac{\ell^2}{\nu^2+3}\left[-\cosh^2\sigma\, d\tau^2 + d\sigma^2 + \frac{4\nu^2}{\nu^2+3}(du + \sinh\sigma\, d\tau)^2\right]$$

  is a new solution of TMG but *not* of Einstein gravity.

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- They are thought to be **perturbatively stable** vacua of TMG (with suitable boundary conditions) in the range $\nu > 1$.

  Anninos, Esole, Guica (2009)

Why study warped AdS_3 solutions?

- It is conjectured that TMG with suitable warped AdS_3 boundary conditions is dual to a 2D CFT with central charges:

$$c_L = \frac{4\nu\ell}{G(\nu^2+3)}\,, \qquad c_R = \frac{(5\nu^2+3)\ell}{G\nu(\nu^2+3)}\,.$$

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- Causal structure of warped AdS_3 black holes resembles that of asymptotically flat black holes in 3+1 dimensions.

From AdS_3 to BTZ

- Any spacetime with constant $\Lambda < 0$ must arise from **identifications of points** in AdS, through a **discrete subgroup** of its symmetry group O(2,2).

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- To construct BTZ, the discrete subgroup G is generated by a **Killing vector** ξ:

$$G = \{\exp(t\xi) : t \in 2\pi\mathbb{Z}\}$$

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- ξ can be timelike, null and or spacelike.

$$\mathrm{BTZ} := \{x \in \mathrm{AdS}_3/\!\sim\; : \xi^2(x) > 0\}$$

  BTZ is **geodesically incomplete**:

$$\partial\mathrm{BTZ} = \{x \in \mathrm{AdS}_3/\!\sim\; : \xi^2(x) = 0\}$$

  corresponds to a **singularity** in the causal structure.

  Regions where $\xi^2(x) < 0$ would have **closed timelike curves**.

Banados, Teitelboim, Zanelli (1993)

Warped AdS_3 black hole

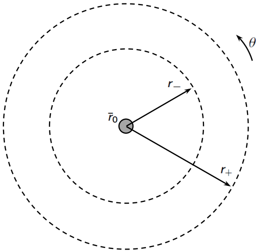
Spacelike stretched black hole:

$$ds^2 = dt^2 + \frac{\ell^2 dr^2}{4R^2(r)N^2(r)} + 2R^2(r)N^\theta(r)dtd\theta + R^2(r)d\theta^2$$

$$R^2(r) = \frac{r}{4}\Big[3(\nu^2-1)r + (\nu^2+3)(r_+ + r_-) - 4\nu\sqrt{r_+ r_-(\nu^2+3)}\Big]$$

$$N^2(r) = \frac{(\nu^2+3)(r-r_+)(r-r_-)}{4R(r)^2}$$

$$N^\theta(r) = \frac{2\nu r - \sqrt{r_+ r_-(\nu^2+3)}}{2R(r)^2}$$



Anninos, Li, Padi, Song, Strominger (2008)

$\nu > 1$ is the **warp factor** of the spacetime.

$\nu \to 1$ recovers the **BTZ black hole**.

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  $\implies$ no **stationary limit surface**

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- Consider $\xi(r) = \partial_t + \Omega(r)\partial_\theta$, with $\Omega_-(r) < \Omega(r) < \Omega_+(r)$ and

$$\Omega_\pm(r) = -\frac{2}{2\nu r - \sqrt{r_+ r_-(\nu^2+3)} \pm \sqrt{(r-r_+)(r-r_-)(\nu^2+3)}}$$

  $\exists$ "**stationary** observers" (following orbits of $\xi(r)$)

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- $\Omega(r) \to \Omega_\mathcal{H}$ when $r \to r_+$, with

$$\Omega_\mathcal{H} = -\frac{2}{2\nu r_+ - \sqrt{r_+ r_-(\nu^2+3)}}$$

  $\Omega_\mathcal{H}$ is the angular velocity of the event horizon

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- $\chi = \partial_t + \Omega_{\mathcal{H}}\partial_\theta$ is the Killing vector field that generates the horizon

  It is null at

  $$r = r_+ \qquad \text{and} \qquad r = r_{\mathcal{C}} = \frac{4\nu^2 r_+ - (\nu^2+3)r_-}{3(\nu^2-1)}$$

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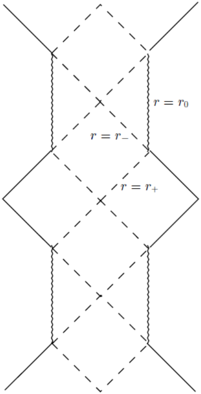
- Asymptotic form of the metric at $r \to \infty$:

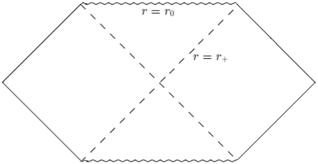
  $$ds^2 = dt^2 + \frac{\ell^2 dr^2}{(\nu^2+3)r^2} + 2\nu r\, dtd\theta + \frac{3(\nu^2-1)r^2}{4}d\theta^2$$

  It is **locally** the metric of the spacelike stretched AdS_3

Causal structure of warped AdS_3 black holes

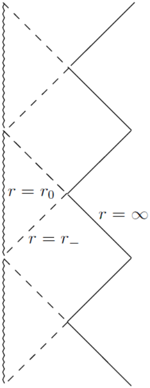


$r_0 < r_- < r_+$



$r_0 = r_- < r_+$

- **Not** asymptotically AdS_3!
- Similar to asymptotically flat black holes!
- Arena to obtain valuable insights for difficult problems with the Kerr black hole!



$r_0 < r_- = r_+$

Jugeau, Moutsopoulos, Ritter (2010)

Outline

Scalar field on the background of a warped AdS_3 black hole

- **Scalar field** Φ on the background of a spacelike stretched black hole:

$$\left(\nabla^2 - m^2\right)\Phi(t, r, \theta) = 0$$

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- **Exact** mode solutions:

$$\Phi_{\omega k}(t, r, \theta) \sim e^{-i\omega t + ik\theta}\, z^{\alpha}(1 - z)^{\beta} F(a, b, c; z)$$

$$z = \frac{r - r_+}{r - r_-}$$

α, β, a, b, c functions of ω and k

$F(a, b, c; z)$ hypergeometric function

Effective potential

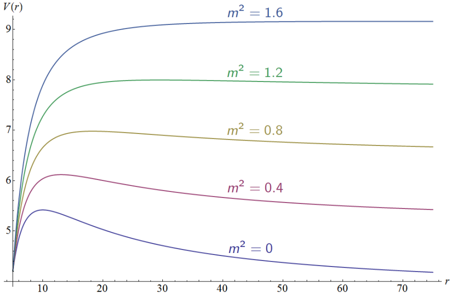
- Let

$$\Phi(t, r, \theta) = e^{-i\omega t + ik\theta}\phi_{\omega k}(r)\,, \qquad \phi_{\omega k}(r) \equiv R(r)^{-1/2}\,\varphi_{\omega k}(r)$$

- Radial equation:

$$\left(\frac{d^2}{dr_*^2} + (\omega^2 - V_{\omega k}(r))\right)\varphi_{\omega k}(r) = 0\,,$$

$V_{\omega k}(r)$ is the **effective potential** felt by the scalar field



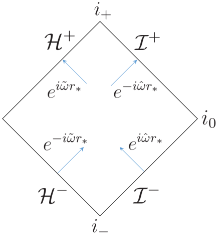
Asymptotic solutions

- **Near-horizon** limit ($r_* \to -\infty$):

$$\varphi_{\omega k}(r_*) = A_{\omega k}\, e^{i\tilde{\omega} r_*} + B_{\omega k}\, e^{-i\tilde{\omega} r_*}$$

where

$$\tilde{\omega} \equiv \omega - k\Omega_{\mathcal{H}}$$



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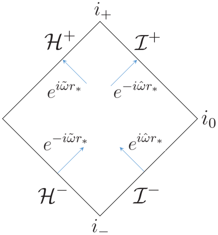


- **Near-infinity** limit ($r_* \to +\infty$):

$$\varphi_{\omega k}(r_*) = C_{\omega k}\, e^{i\hat{\omega} r_*} + D_{\omega k}\, e^{-i\hat{\omega} r_*}$$

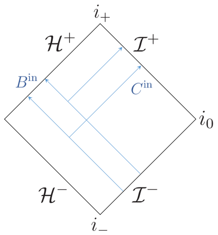
where

$$\hat{\omega} \equiv \begin{cases} \sqrt{\omega^2 - \omega_m^2}\,, & \omega > \omega_m \geq 0 \\ -\sqrt{\omega^2 - \omega_m^2}\,, & \omega < -\omega_m \leq 0 \end{cases}$$

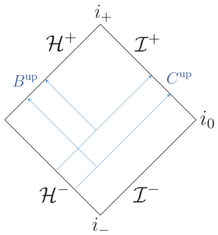
$$\omega_m \equiv \frac{1}{2}\frac{\nu^2 + 3}{\sqrt{3(\nu^2 - 1)}}\sqrt{1 + \frac{4m^2}{\nu^2 + 3}}$$

Modified "**Breitenlohner-Freedman bound**": $m^2 \geq -\dfrac{\nu^2 + 3}{4}$

Basis modes and superradiance



in modes



up modes

- "**in**" modes:

$$\varphi^{\rm in}_{\omega k}(r_*) = \begin{cases} B^{\rm in}_{\omega k}\, e^{-i\tilde{\omega} r_*}\,, & r_* \to -\infty \\ e^{-i\hat{\omega} r_*} + C^{\rm in}_{\omega k}\, e^{i\hat{\omega} r_*}\,, & r_* \to +\infty \end{cases}$$

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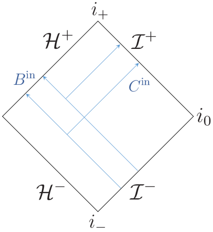

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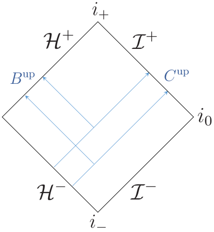

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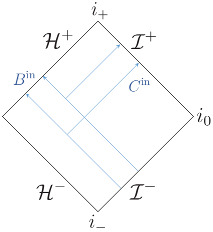
- "**up**" modes

$$\varphi^{\rm up}_{\omega k}(r_*) = \begin{cases} e^{i\tilde{\omega} r_*} + B^{\rm up}_{\omega k}\, e^{-i\tilde{\omega} r_*}\,, & r_* \to -\infty \\ C^{\rm up}_{\omega k}\, e^{i\hat{\omega} r_*}\,, & r_* \to +\infty \end{cases}$$

Basis modes and superradiance



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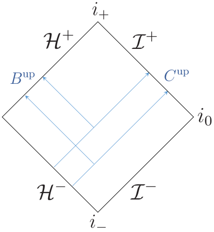
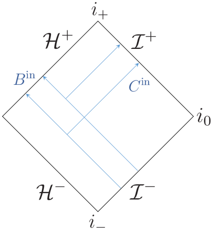


up modes

- From Wronskian relations:

$$\tilde{\omega}\,|B^{\rm in}_{\omega k}|^2 = \hat{\omega}\left(1 - |C^{\rm in}_{\omega k}|^2\right), \qquad \tilde{\omega}\left(1 - |B^{\rm up}_{\omega k}|^2\right) = \hat{\omega}\,|C^{\rm up}_{\omega k}|^2$$

Basis modes and superradiance



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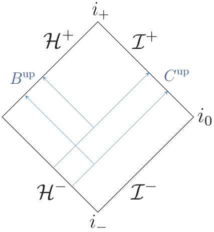

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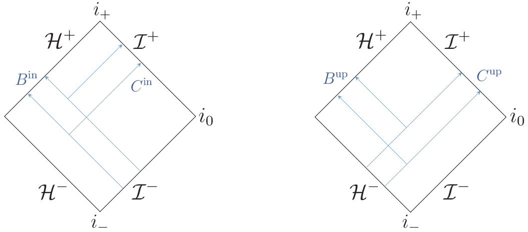
- **Superradiance** when:

$$|C^{\text{in}}_{\omega k}|^2 > 1 \iff \tilde{\omega}\hat{\omega} < 0\,, \qquad |B^{\text{up}}_{\omega k}|^2 > 1 \iff \tilde{\omega}\hat{\omega} < 0$$

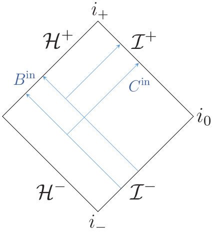
Basis modes and superradiance



in modes   up modes

- Definition of **positive frequency** $\Longleftrightarrow$ location of a **locally nonrotating observer**
  - **in** modes: LNRO at infinity $\Longrightarrow \omega > 0$ (i.e. $\hat{\omega} > 0$)
    $\exists$ superradiance iff $\tilde{\omega} < 0$, i.e. $\omega_{\boldsymbol{m}} < \omega < k\Omega_{\mathcal{H}}$
  - **up** modes: LNRO at horizon $\Longrightarrow \tilde{\omega} > 0$
    $\exists$ superradiance iff $\hat{\omega} < 0$, i.e. $\omega < -\omega_{\boldsymbol{m}}$

Basis modes and superradiance



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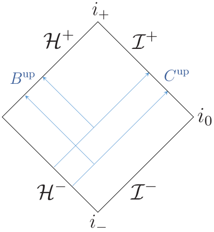
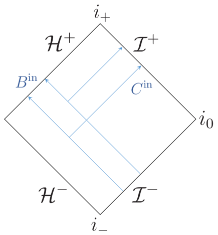


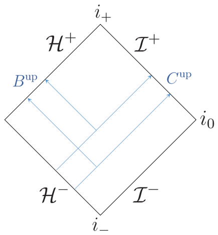
up modes

- Whatever the choice of positive frequency classical superradiance is **always** present (when physically motivated boundary conditions are imposed)!

Basis modes and superradiance



in modes

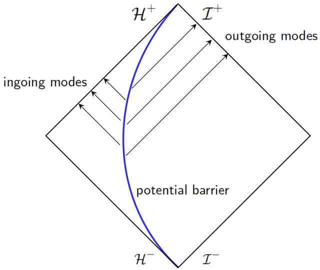


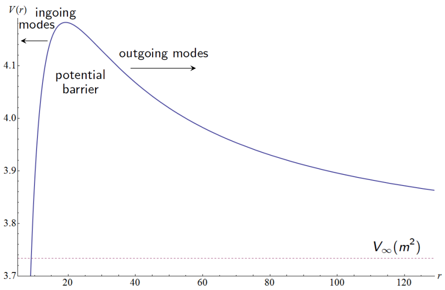
up modes

- Whatever the choice of positive frequency classical superradiance is **always** present (when physically motivated boundary conditions are imposed)!
- This is:
  - **similar** to the Kerr spacetime;
  - **in contrast** with the BTZ and Kerr-AdS spacetimes.

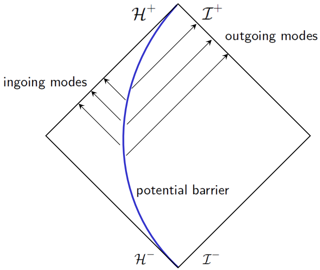
Outline

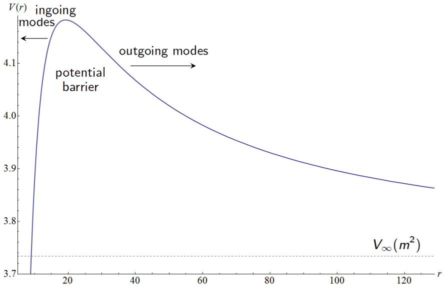
Quasinormal modes





Quasinormal modes





Boundary conditions:

- **Ingoing** modes at the event horizon;
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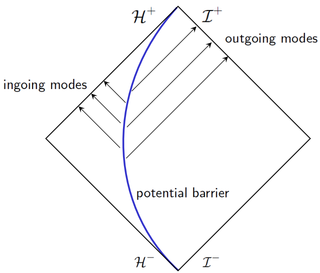

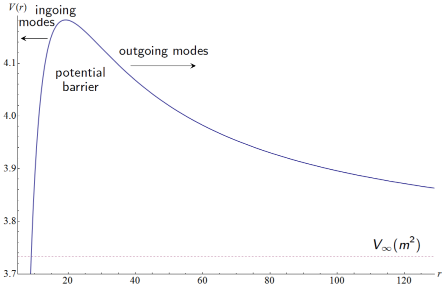

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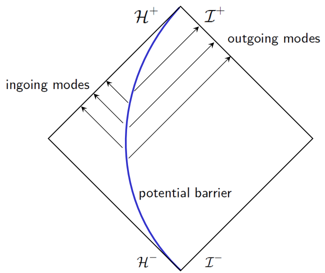
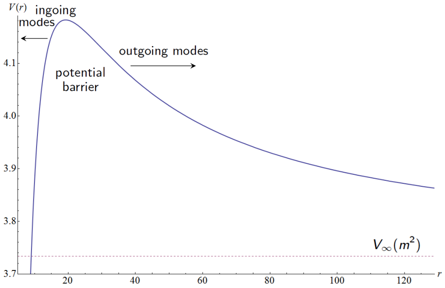
Quasinormal modes





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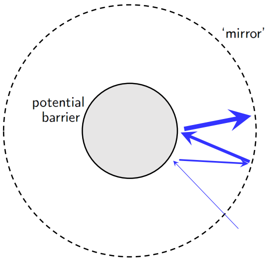
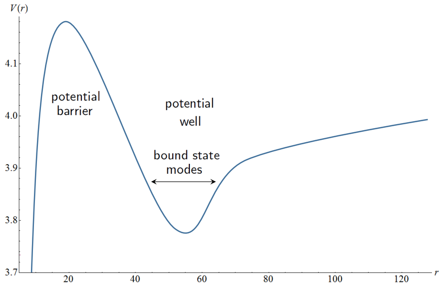
If $\mathrm{Im}(\omega_n) > 0$ for some n: mode is **unstable**!

Superradiant and bound state modes





Bound state modes: localised in the potential well

Boundary conditions:

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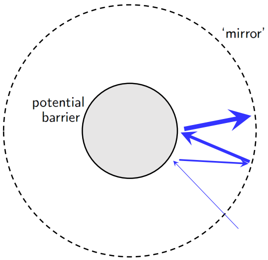
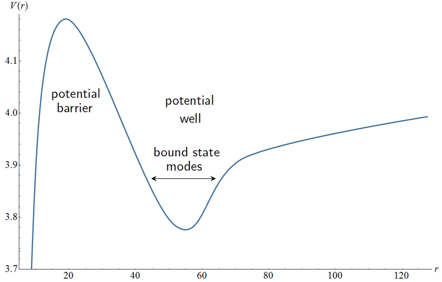
Superradiant and bound state modes





Bound state modes: localised in the potential well

Boundary conditions:

- **Ingoing** modes at the event horizon;
- **Exponentially decreasing** modes at infinity.

$\text{Im}(\omega_n) > 0 \implies$ **superradiant bound state mode** $\implies$ instability!

Eigenfrequencies for quasinormal and bound state modes

Eigenfrequencies can be obtained in closed form!

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$(+)$ quasinormal frequencies $\qquad$ $(-)$ bound state frequencies

- $$(\omega_\pm)_n^{(R)} = \frac{\nu^2+3}{d^2\delta^2 - 3(\nu^2-1)}\left\{-d\delta\left(\frac{4kd}{\nu^2+3} + i\left(n+\frac{1}{2}\right)\right) \pm i(e - i\,\mathrm{sgn}(k) f)\right\}$$

$$d = \frac{1}{r_+ - r_-}, \qquad \delta = 2\nu(r_+ + r_-) - 2\sqrt{(\nu^2+3)r_+ r_-},$$

$$e = \sqrt{\frac{\sqrt{E^2+F^2}+E}{2}}, \qquad f = \sqrt{\frac{\sqrt{E^2+F^2}-E}{2}},$$

$$E = \frac{1}{4}\left(1+\frac{4m^2}{\nu^2+3}\right)d^2\delta^2 - 3(\nu^2-1)\left[\frac{1}{4}\left(1+\frac{4m^2}{\nu^2+3}\right) + \left(\frac{4kd}{\nu^2+3}\right)^2 - \left(n+\frac{1}{2}\right)^2\right],$$

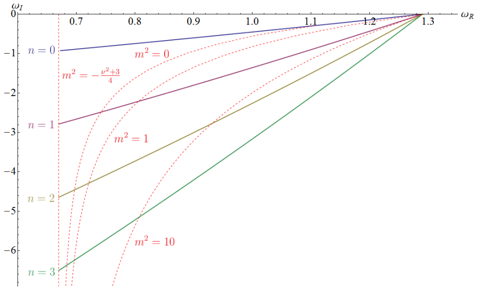
$$F = -3(\nu^2-1)\left(n+\frac{1}{2}\right)\frac{8kd}{\nu^2+3}.$$

- $$(\omega_\pm)_n^{(L)} = -i\left[(2n+1)\nu \mp \sqrt{3(\nu^2-1)\left(n+\frac{1}{2}\right)^2 + \frac{\nu^2+3}{4}\left(1+\frac{4m^2}{\nu^2+3}\right)}\right]$$

Eigenfrequencies for quasinormal and bound state modes

- **Quasinormal** eigenfrequencies

  $\mathrm{Im}(\omega_n) < 0$

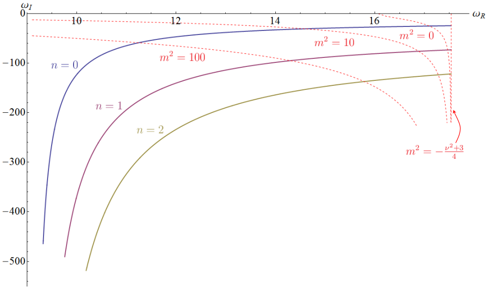


- **Bound state** eigenfrequencies

  $\mathrm{Re}(\omega_n) > k\Omega_{\mathcal{H}}$

  $\mathrm{Im}(\omega_n) < 0$

  No superradiant instabilities, *in contrast with Kerr!*



Effective potential

Why are none of the exponentially decreasing modes superradiant?

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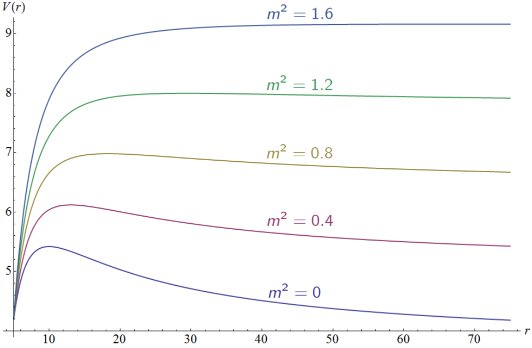
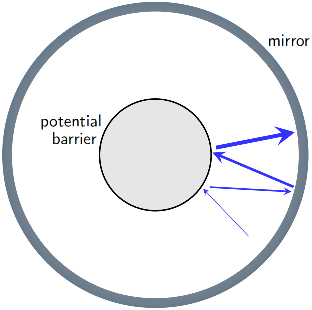
The effective potential never forms a potential well



What happens if we add an actual mirror?



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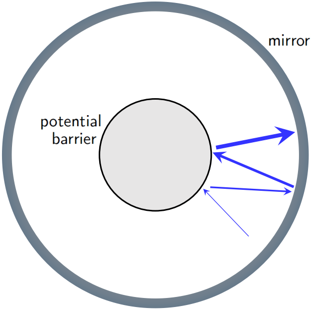

Boundary conditions (for bound state modes):

- **Ingoing** modes at the event horizon;
- **Vanishing** modes at the mirror (Dirichlet boundary condition).

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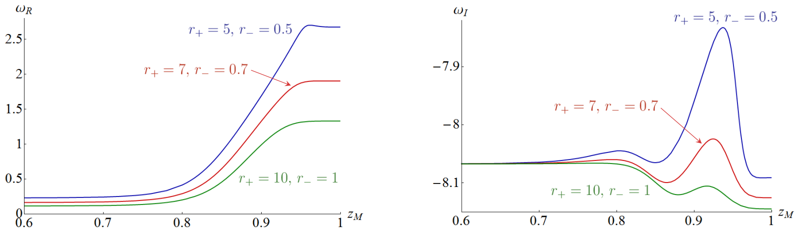
Right eigenfrequency $\omega_{\mathcal{M}}^{(R)}$ vs position of the mirror



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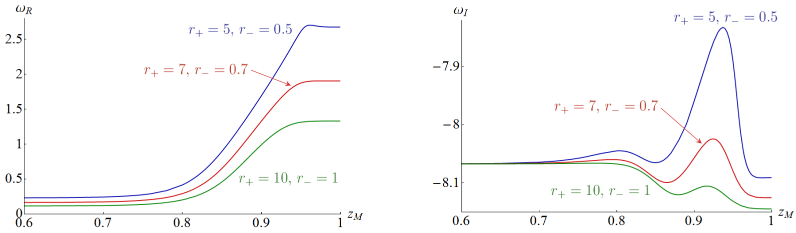

- $\mathrm{Re}(\omega_{\mathcal{M}}^{(R)}) > k\Omega_{\mathcal{H}}$
- $\mathrm{Re}(\omega_{\mathcal{M}}^{(R)}) \to k\Omega_{\mathcal{H}}$ as $r_{\mathcal{M}} \to r_+$
- $\mathrm{Re}(\omega_{\mathcal{M}}^{(R)}) \to \mathrm{Re}(\omega^{(R)})$ as $r_{\mathcal{M}} \to \infty$

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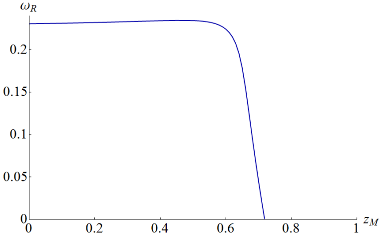
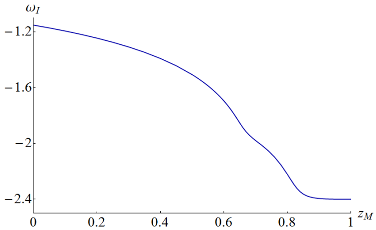
Left eigenfrequency $\omega_{\mathcal{M}}^{(L)}$ vs position of the mirror





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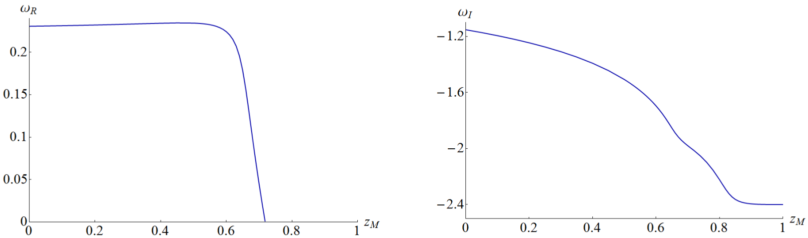

- $\mathrm{Re}(\omega_{\mathcal{M}}^{(L)}) \to k\Omega_{\mathcal{H}}$ as $r_{\mathcal{M}} \to r_+$
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Outline

QFT on warped AdS_3 black holes

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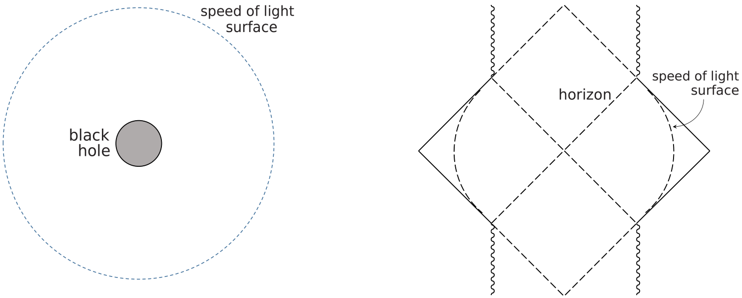
- QFT on **rotating** black holes is a challenging problem:
  - Superradiant modes require care.
  - The **Hartle-Hawking vacuum state** is **not** well defined!

Frolov and Thorne (1989)
Kay and Wald (1991)
Ottewill and Winstanley (2000)
Ottewill and Duffy (2008)

Hartle-Hawking vacuum on a warped AdS_3 black hole



Beyond the speed of light surface, the Hartle-Hawking vacuum would have to rotate with a speed **greater** than the speed of light.

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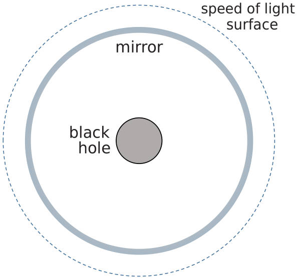

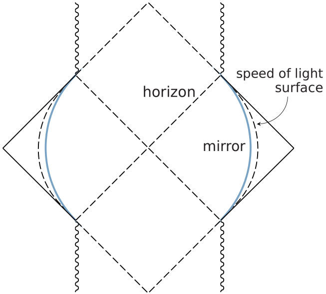

Beyond the speed of light surface, the Hartle-Hawking vacuum would have to rotate with a speed **greater** than the speed of light.

If a **mirror** is put between the horizon and the speed of light surface, an Hartle-Hawking vacuum is **well defined**.

Stress-energy tensor for a scalar field in the Hartle-Hawking vacuum

- **AIM:** compute the expectation value of the renormalised stress-energy tensor $\langle T_{\mu\nu}(x)\rangle_{\text{ren}}$ for a scalar field in the Hartle-Hawking vacuum

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Work in progress...

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- QFT computations on warped AdS_3 black holes may give valuable insights for the Kerr case.

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- What is the **renormalised stress-energy tensor** for a field in the Hartle-Hawking vacuum state? What information does it provide for the Kerr case?

THANK YOU FOR YOUR ATTENTION!

More information in:

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