Three-form Cosmology

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Koivisto & Nunes, PLB, arXiv:0907.3883
Koivisto & Nunes, PRD, arXiv:0908.0920
Mulryne, Noller & Nunes, JCAP, arXiv:1209.2156
Koivisto & Nunes, arXiv:1212.2541

What is a three-form?

Totally antisymmetric tensor with three indeces

$$A_{ijk} = -A_{jik}$$

For example, a three-form defines the cross product

$$(\vec{a} \times \vec{b})_i = \epsilon_{ijk}\, a_j b_k$$

where ϵ_{ijk} is the Levi-Civita symbol.

Why forms?

1. To test the robustness of scalars.
   Is it the only natural possibility?
   Scalars have not been detected yet.

2. Vectors and two-forms,
   Support anisotropy;
   Require non-minimal couplings $\longrightarrow$ possible instability.

   Three-forms,
   Viable isotropic inflation.

3. They exist in fundamental theories
   String theory.

Three-form action

Action for the three-form $A_{\mu\nu\rho}$

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} - \frac{1}{48} F^2(A) - V(A^2) \right)$$

where

$$F_{\mu\nu\rho\sigma} = 4\nabla_{[\mu} A_{\nu\rho\sigma]} = \nabla_\mu A_{\nu\rho\sigma} - \nabla_\sigma A_{\mu\nu\rho} + \nabla_\rho A_{\sigma\mu\nu} - \nabla_\nu A_{\rho\sigma\mu}$$

We have the equations of motion:

$$\nabla \cdot F = 12 V'(A^2) A$$

and due to antisymmetry we have the additional constraints:

$$\nabla \cdot V'(A^2) A = 0$$

Equations of motion

Consider flat FRW cosmology:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Most general three-form compatible with FRW:

$$A_{ijk} = a^3(t)\epsilon_{ijk}\chi(t)$$

Equations of motion of the field X:

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} + 3\dot{H}\chi = 0$$

Equation of motion of background fluid:

$$\dot{\rho}_B = -3\gamma H\rho_B$$

Equations of motion

Friedmann equation

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}(\dot\chi + 3H\chi)^2 + V(\chi) + \rho_B\right)$$

can also write:

$$H^2 = \frac{\kappa^2}{3}\frac{V + \rho_B}{1 - \kappa^2(\chi' + 3\chi)^2/6}$$

$$\Rightarrow \qquad \kappa|\chi' + 3\chi| < \sqrt{6}. \qquad\qquad {}' = d/d\ln a.$$

Evolution of the Hubble rate:

$$\dot H = -\frac{\kappa^2}{2}\left(V_{,\chi}\chi + \gamma\rho_B\right)$$

Equation of state parameter of X:

$$w_\chi = -1 + \frac{V_{,\chi}\chi}{\rho_\chi}$$

Universe de Sitter with $V = 0$; Superinflation when $V_{,\chi}\chi < 0$.

Critical points

Rewriting the equations of motion in the form of system of first order differential equations:

$$
\begin{aligned}
x' &= 3\left(\sqrt{\frac{2}{3}}y - x\right) \\
y' &= -\frac{3}{2}\lambda(x)\left(1 - y^2 - w^2\right)\left[xy - \sqrt{\frac{2}{3}}\right] + \frac{3}{2}\gamma w^2 y \\
w' &= -\frac{3}{2}w\left(\gamma + \lambda(x)\left(1 - y^2 - w^2\right)x - \gamma w^2\right)
\end{aligned}
$$

where

$$
x \equiv \kappa\chi\,, \qquad y \equiv \frac{\kappa}{\sqrt{6}}(\chi' + 3\chi)\,, \qquad z^2 = \frac{\kappa^2 V}{3H^2}
$$

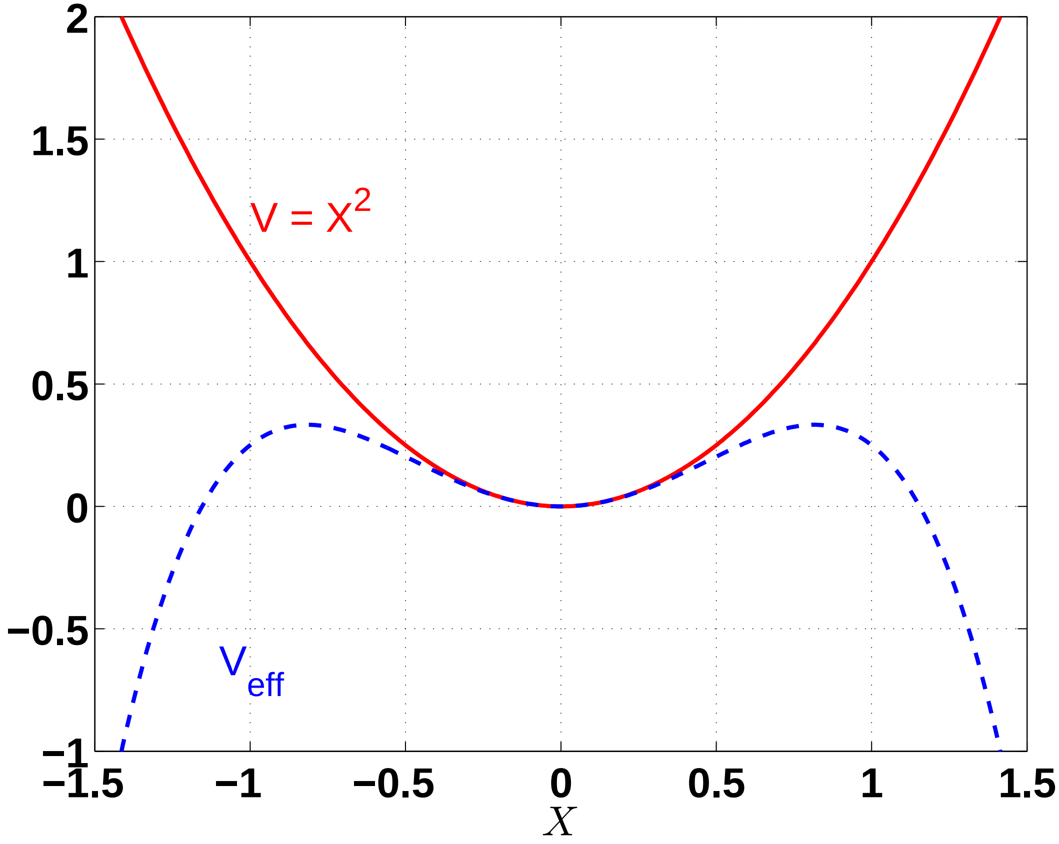
$$
w^2 \equiv \frac{\kappa^2\rho_B}{3H^2}\,, \qquad \lambda(x) \equiv -\frac{1}{\kappa}\frac{V_{,\chi}}{V}
$$

The Friedmann equation $\Rightarrow$ $y^2 + z^2 + w^2 = 1$

Critical points

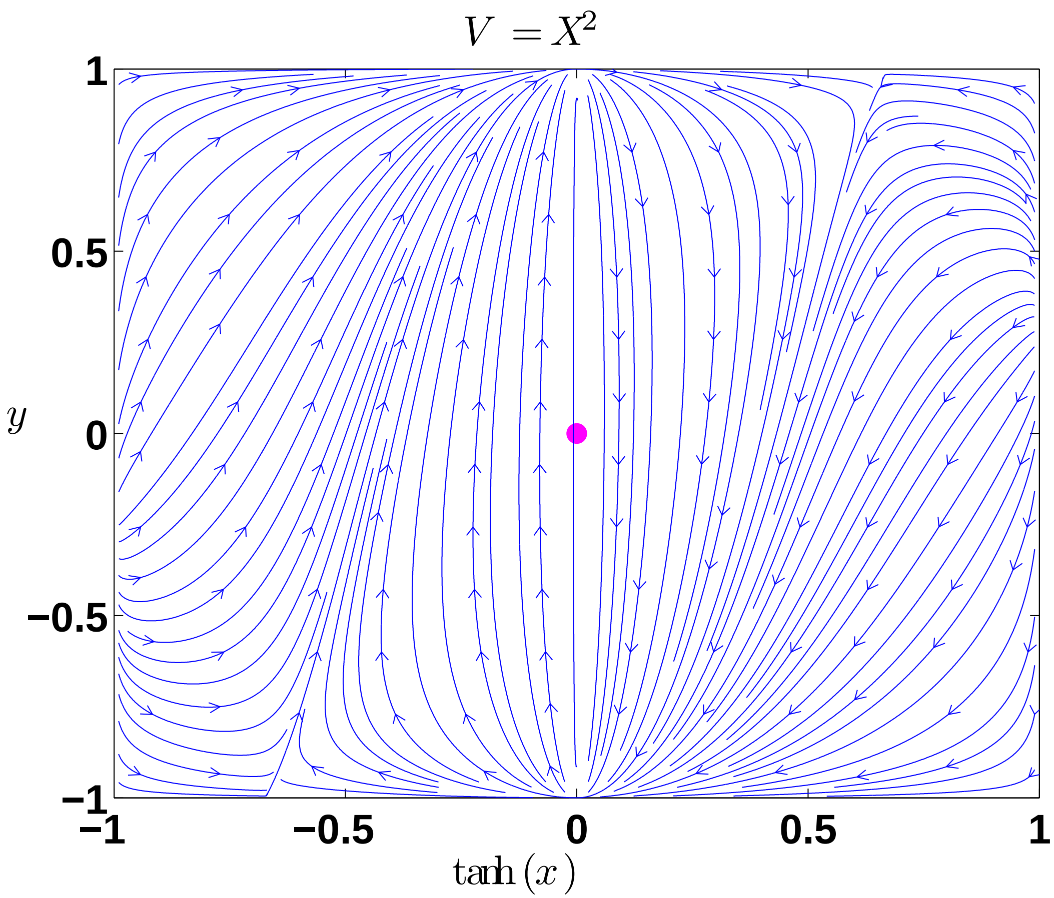
	x	y	w	$\dot{H}/H^2$	λ	description
A	0	0	± 1	$-3\gamma/2$	any	matter domination
$\mathrm{B}_\pm$	$\pm\sqrt{2/3}$	± 1	0	0	any	kinetic domination
C	x_{ext}	$\sqrt{3/2}\,x_{\mathrm{ext}}$	0	0	0	potential extrema

Effective potentials

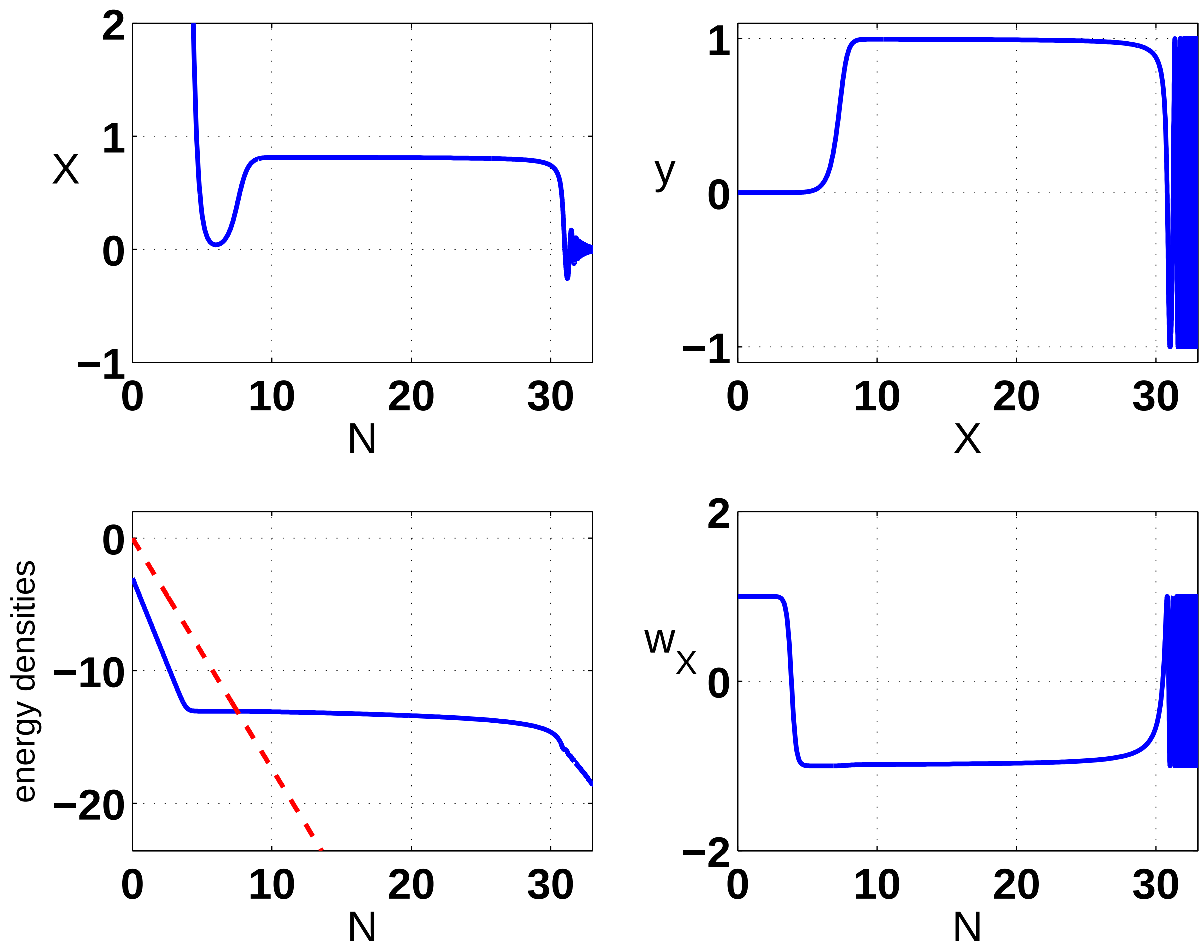


$$V_{\mathrm{eff},\chi} = V_{,\chi} + 3\dot{H}\chi$$
$$= V_{,\chi}\left(1 - \frac{3}{2}(\kappa\chi)^2\right) - \frac{3}{2}\gamma\kappa^2\rho_B\chi$$

Phase space



Example evolutions: $V = \chi^2$



Example evolutions: $V = \chi^n$

Epoch of tracking:

$$N_s = \frac{1}{3n} \ln \left(\frac{V_i}{3H_i^2 y_i^2} \right)$$

Point of turn around:

$$N_t = \frac{1}{3\,(1+\gamma/2)} \ln \left(\frac{2}{\gamma} \frac{B}{A} \right)$$

where $A = \sqrt{2/3}\, y_i/(1+\gamma/2)$ and $B = \chi_i - A$.

Epoch of inflation:

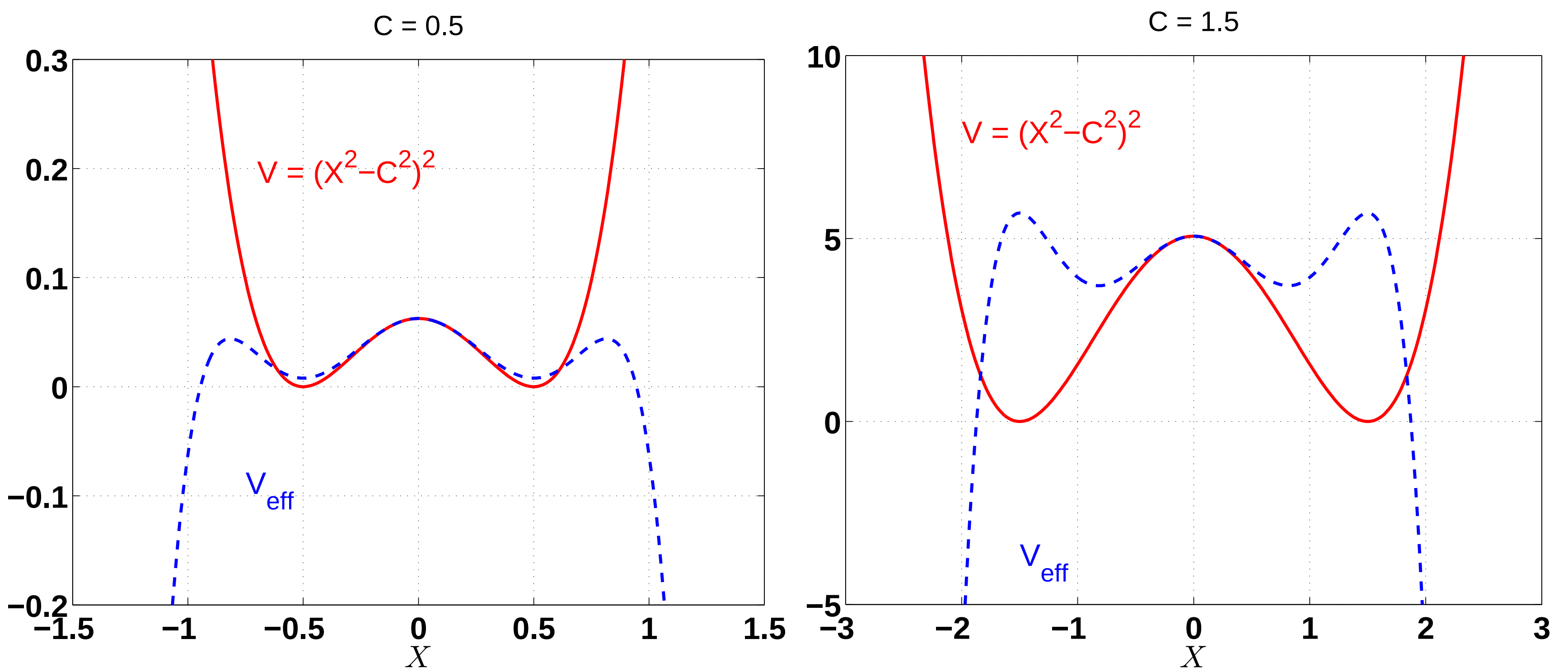
$$\Delta N = \frac{2}{9n} \frac{1}{2/3 - (\kappa \chi_{\rm init})^2} - \frac{1}{2}$$

Oscillations:

$$\langle w_\chi \rangle = \frac{n-2}{n+2}$$

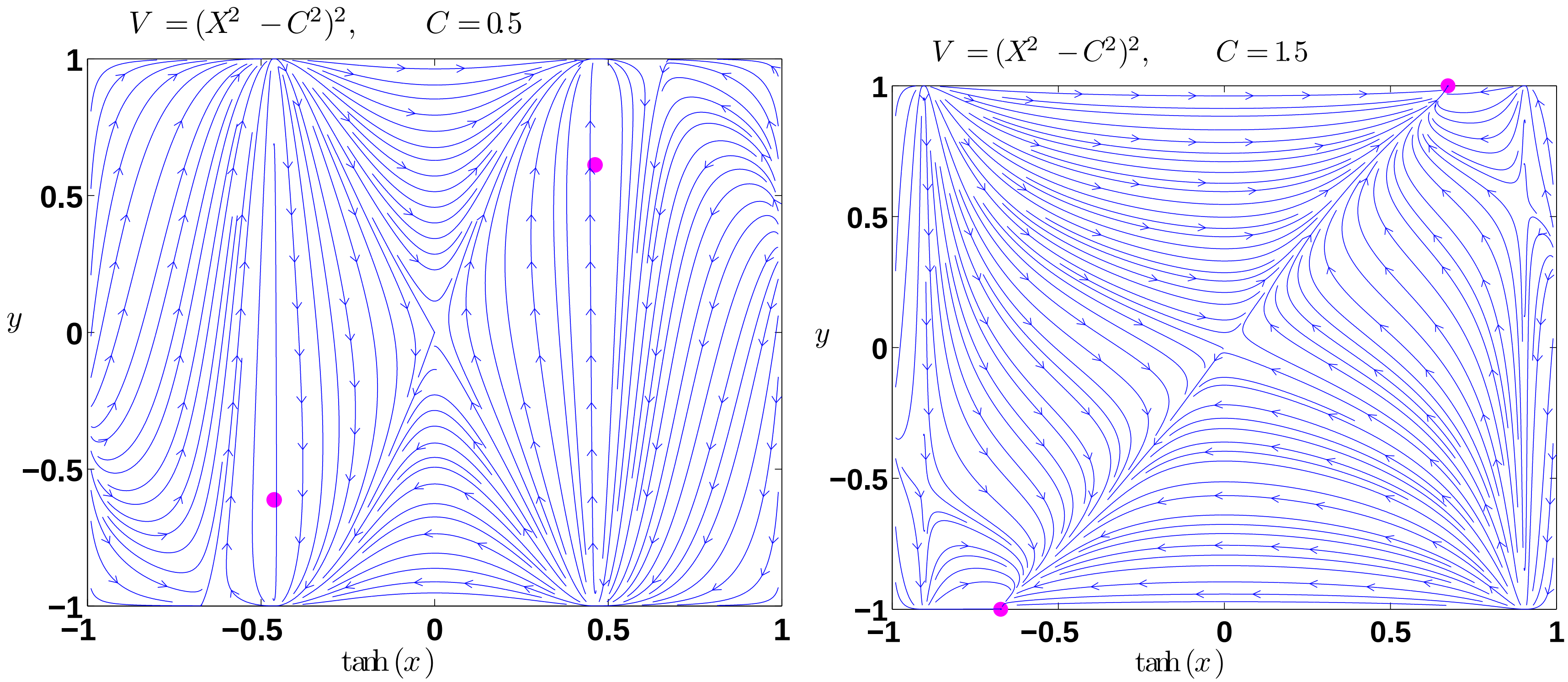
Thus for $n = 2$ the field behaves as dust, $\langle w_\chi \rangle = 0$ and for $n = 4$ it mimics radiation, $\langle w_\chi \rangle = 1/3$.

Effective potentials



$$
\begin{aligned}
V_{\text{eff},\chi} &= V_{,\chi} + 3\dot{H}\chi \\
&= V_{,\chi}\left(1 - \frac{3}{2}(\kappa\chi)^2\right) - \frac{3}{2}\gamma\kappa^2\rho_B\chi
\end{aligned}
$$

Phase space


$V = (X^2 - C^2)^2, \quad C = 0.5$
1
0.5
0
−0.5
−1
y
−1
−0.5
0
0.5
1
$\tanh(x)$
$V = (X^2 - C^2)^2, \quad C = 1.5$
1
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y
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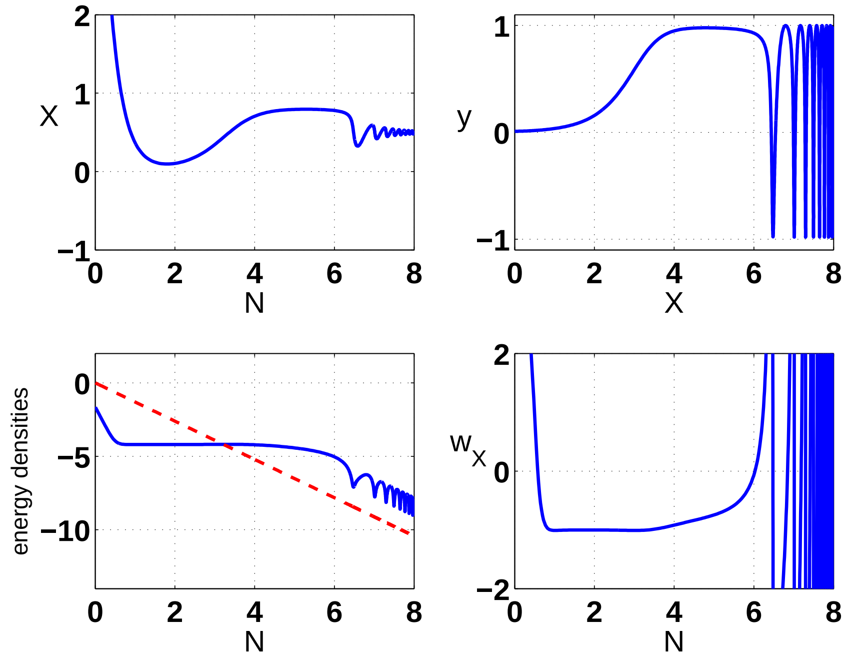
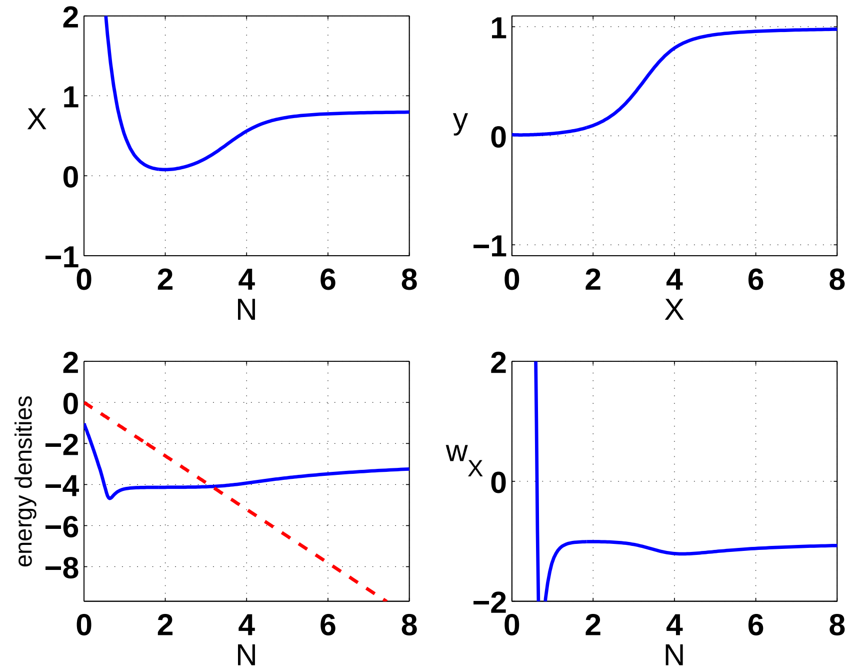
$\tanh(x)$

Example evolutions: $V = (\chi^2 - C^2)^2$

$C = 0.5$



$C = 1.5$



Stability analysis

$V(\chi)$	A	B	C
$\exp(-\beta\chi)$	U	B_+ stable for $\beta > 0$ B_- stable for $\beta < 0$	S
$\exp(-\beta\chi^2)$	U	stable for $\beta > 0$	stable for $\beta < 0$
χ^2	U	U	S
χ^4	U	U	S
$\left(\chi^2 - C^2\right)^2$	U	stable for $C > \sqrt{2/3}$	C_1 stable, C_2 unstable

C_1 are the minima
C_2 is the local maximum

Gauge invariance and stability

$$\mathcal{L} = -\frac{1}{48}F^2(A) - V(A^2)$$

F^2 is invariant under $A \to A + \nabla C$.
$V(A^2)$ breaks this symmetry resulting in extra degrees of freedom.
To see this we can make an expansion in Stückelberg fields s.t.
$A = \tilde{A} + 4[\nabla\Sigma]$

$$\mathcal{L}' = -\frac{1}{48}F^2(\tilde{A}) - V((\tilde{A} + F(\Sigma))^2)$$

is now invariant under $\tilde{A} \to \tilde{A} + [\nabla C]$; $\quad \Sigma \to \Sigma - C/4$.

Expanding the potential around $\tilde{A}$

$$\mathcal{L}' = \mathcal{L} - V'(\tilde{A}^2)F^2(\Sigma)$$

Presence of ghost field for

$$V'(\tilde{A}^2) < 0 \quad \Leftrightarrow \quad V_{,\chi}\chi < 0$$

i.e. when there is phantom behaviour ($w_\chi < -1$).

Part I:

Inflation

Cosmological perturbations

General perturbations about FRW background:

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right)$$

$$h_{ij} = a^2 e^{2\zeta}\delta_{ij}$$

ζ is the curvature perturbation, and we expand N_i and N as:

$$N_i = \psi_{,i} + \tilde{N}_i \qquad\qquad N = 1 + \tilde{\alpha}$$

Perturbations of the three-form:

$$A_{0ij} = a(t)\epsilon_{ijk}\,\alpha_{,k}$$
$$A_{ijk} = a^3(t)\epsilon_{ijk}(\chi(t) + \alpha_0)$$

Vector perturbations are decaying and can be ignored.

The second order action

The second order action in scalar perturbations:

$$S_2 = \int \mathrm{d}t\mathrm{d}^3x \left[a^3 \frac{\Sigma}{H^2} \dot{\zeta}^2 - a\epsilon(\partial\zeta)^2 \right]$$

where the speed of sound

$$c_s^2 = \frac{V_{,\chi\chi}\chi}{V_{,\chi}}$$

and

$$\epsilon = -\frac{\dot{H}}{H^2} \qquad\qquad \Sigma = \frac{H^2\epsilon}{c_s^2}$$

ζ is conserved on the large scales.

$$\dot{\zeta} = \frac{c_s^2}{\epsilon} \frac{\nabla^2}{a^2} \left(\psi + \frac{\zeta}{H} \right)$$

Power spectrum of scalar perturbations

The 2-point correlation function

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = (2\pi)^5\delta^3(\mathbf{k}_1+\mathbf{k}_2)\frac{P_\zeta(k_1)}{2k_1^3},$$

The power spectrum

$$P_\zeta \equiv \frac{1}{2\pi^2}k^3\left|\zeta_k\right|^2 = \frac{1}{2(2\pi)^2\epsilon c_s}\left.\frac{H^2}{M_{\rm Pl}^2}\right|_*$$

$*$ indicates horizon crossing $c_s k = aH$.

The spectral index n_s is

$$1-n_s = 2\epsilon+\frac{\dot\epsilon}{\epsilon H}+\frac{\dot c_s}{c_s H}$$

Power spectrum of tensor perturbations

Since the three-form does not generate tensor perturbations, their evolution equation is as usual,

$$\ddot{h} + 3H\dot{h} - \frac{\nabla^2}{a^2}h = 0$$

Tensor power spectrum:

$$\mathcal{P}_T = \frac{2}{\pi^2}\frac{H^2}{M_{\rm Pl}^2}\bigg|_*$$

The tensor spectral index is then

$$n_T = -2\epsilon$$

Consistency relation

Ratio of tensor to scalar perturbations

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta} = 16\, c_S \, |\epsilon|$$

Thus it is in principle possible to distinguish the three-form inflation from scalar field already from the spectra of linear perturbations.

The third order action

The third order action of scalar perturbations:

$$
\begin{aligned}
S_3 &= \int \Bigg\{ dt d^3x [-\epsilon a \zeta (\partial \zeta)^2 - a^3 (\Sigma + 2\lambda) \frac{\dot{\zeta}^3}{H^3} + \frac{3a^3\epsilon}{c_s^2} \zeta \dot{\zeta}^2 \\
&+ \frac{1}{2a} \left(3\zeta - \frac{\dot{\zeta}}{H} \right) \left(\partial_i \partial_j \psi \partial_i \partial_j \psi - \partial^2 \psi \partial^2 \psi \right) - 2a^{-1} \partial_i \psi \partial_i \zeta \partial^2 \psi \Bigg\}
\end{aligned}
$$

where λ is

$$
\lambda = -\frac{1}{12} \frac{V_{,\chi}^3 V_{,\chi\chi\chi}}{V_{,\chi\chi}^3}
$$

At tree level in quantum field theory, and in the interaction picture, the In-In (equal time) three-point correlation function is given by the expression

$$
\langle \zeta(t, \mathbf{k}_1) \zeta(t, \mathbf{k}_2) \zeta(t, \mathbf{k}_3) \rangle = -i \int_{t_0}^{t} \mathrm{d}t' \langle [\zeta(t, \mathbf{k}_1) \zeta(t, \mathbf{k}_2) \zeta(t, \mathbf{k}_3), H_{\mathrm{int}}(t')] \rangle
$$

The bispectrum and non-Gaussianity

The non-Gaussianity of the CMB in the WMAP observations is analyzed by assuming

$$\zeta = \zeta_L - \frac{3}{5} f_{\rm NL} \zeta_L^2$$

where ζ_L is the linear Gaussian part the perturbations, and $f_{\rm NL}$ is an estimator parameterizing the size of the non-Gaussianity.

The three-point correlation function:

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \frac{1}{k_1^3 k_2^3 k_3^3} {P_\zeta}^2 \mathcal{A}(k_1, k_2, k_3)$$

$$f_{\rm NL} = \frac{10}{3} \frac{1}{k_1^3 + k_2^3 + k_3^3} \mathcal{A} \qquad\qquad f_{\rm NL}^{\rm equil} = 30 \frac{1}{K^3} \mathcal{A}$$

where $K/3 = k_1 = k_2 = k_3$.

$$f_{\rm NL}^{\rm equil} \approx \frac{5}{81} \left(\frac{1}{c_s^2} - 1 - 2\frac{\lambda}{\Sigma} \right) - \frac{35}{108} \left(\frac{1}{c_s^2} - 1 \right) + \ldots$$

The dual theory

We define duals as:

$$(\star F) = \frac{1}{4!}\epsilon_{\alpha\beta\gamma\delta}F^{\alpha\beta\gamma\delta} \equiv \Phi \qquad F_{\alpha\beta\gamma\delta} = -\epsilon_{\alpha\beta\gamma\delta}\Phi$$

$$(\star A)_\alpha = \frac{1}{3!}\epsilon_{\alpha\beta\gamma\delta}A^{\beta\gamma\delta} \equiv B_\alpha \qquad A_{\beta\gamma\delta} = -\epsilon_{\alpha\beta\gamma\delta}B^\alpha$$

which allows us to write the equivalent formulations of the Lagrangians:

$$\mathcal{L}_{IV}(F, \nabla\cdot F) = -\frac{1}{48}F^2 + 2A^2(\nabla\cdot F)V'\left(A^2(\nabla\cdot F)\right) - V(A^2(\nabla\cdot F))$$

$$\mathcal{L}_{III}(A, \nabla A)) = -\frac{1}{3}\left[\nabla A\right]^2 - V(A^2)$$

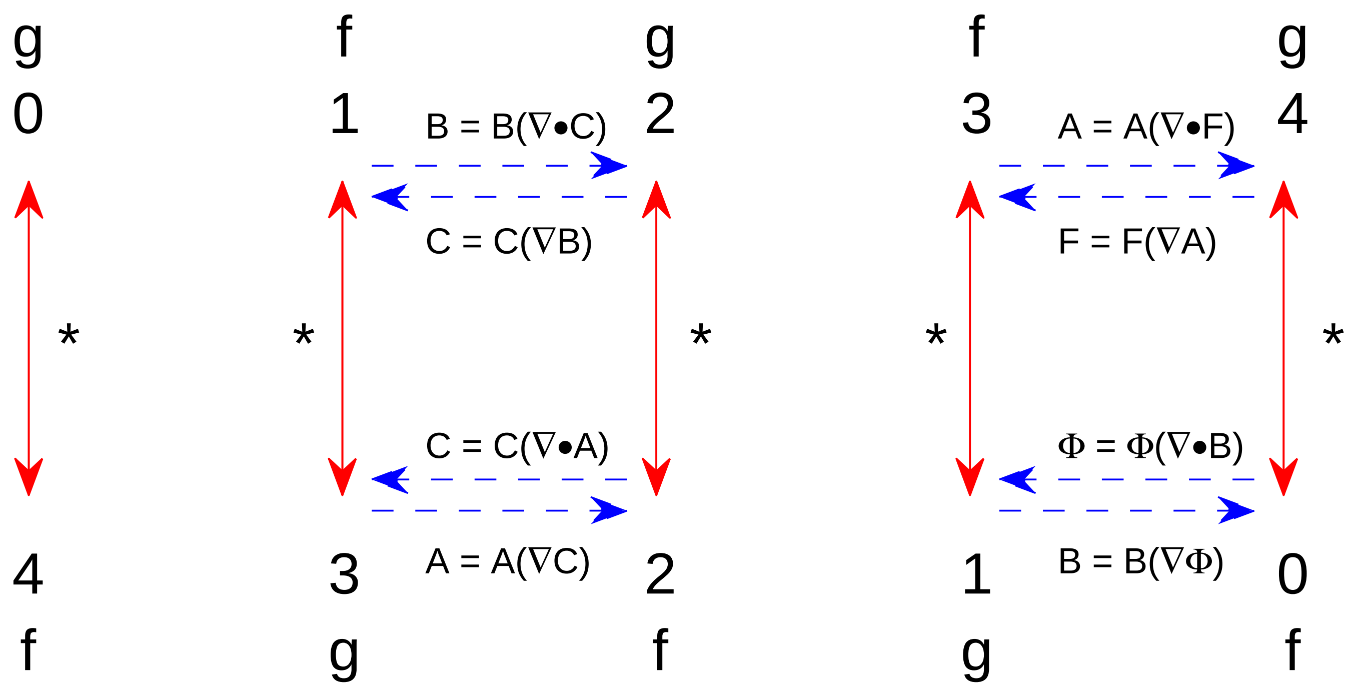
$$\mathcal{L}_{I}(B, \nabla\cdot B) = \frac{1}{2}(\nabla\cdot B)^2 - V\left(-6B^2\right)$$

$$\mathcal{L}_0(\Phi, \nabla\Phi) = -\frac{1}{2}\Phi^2 - 12B^2(\nabla\Phi)V'\left(-6B^2(\nabla\Phi)\right) - V\left(-6B^2(\nabla\Phi)\right)$$

We recognise $\mathcal{L}_0$ as a $p(X,\Phi)$ theory or k-inflation/essence. $X = -\nabla^\mu\Phi\nabla_\mu\Phi$.

Equivalent formulations

Equivalence between Lagrangian descriptions:



Faraday formulation

Gauge fixing formulation

$$\mathcal{L}_f = f\left(F^2(x)\right) - V(x^2)\,,$$

$$\mathcal{L}_g = g\left((\nabla \cdot x)^2\right) - U(x^2)$$

Example I: Power law potential

In the original three-form theory

$$\mathcal{L} = -\frac{1}{48}F^2 - V_0 A^{2p}$$

and in the $p(X,\phi)$ theory

$$\mathcal{L}_\phi = (2p-1)\left(\frac{1}{V_0}\right)^{1/(2p-1)}\left(\frac{X}{24p^2}\right)^{\frac{p}{2p-1}} - \frac{1}{2}\phi^2$$

In either approach we obtain: $c_s^2 = 2p-1$
N e-folds before the end of inflation:

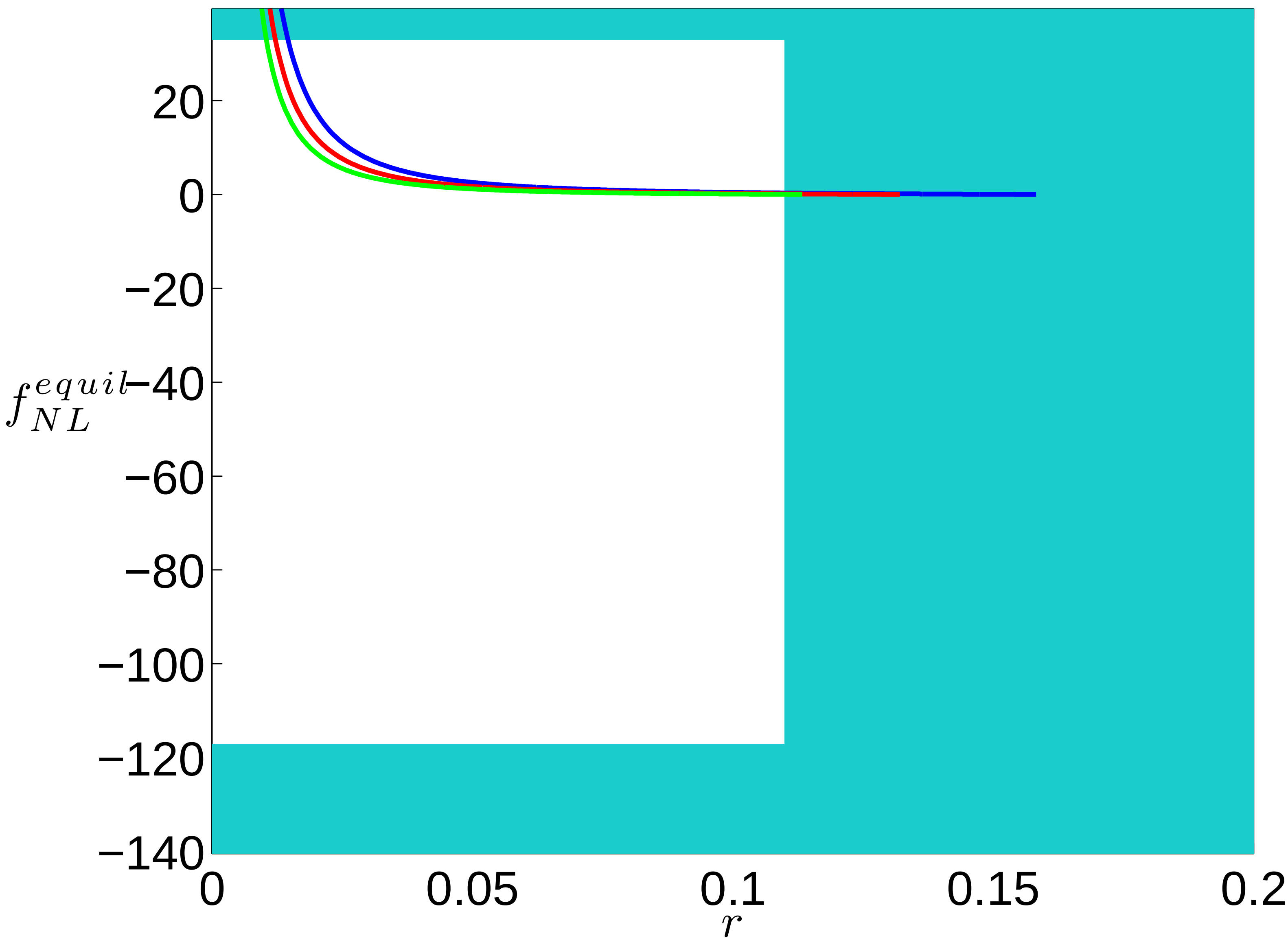
$$\chi_N^2 = \frac{2}{3} - \frac{4}{18p}\frac{1}{1+2N} \qquad \epsilon_N \approx \frac{1}{1+2N}$$

The spectral index for $N = 60$ gives

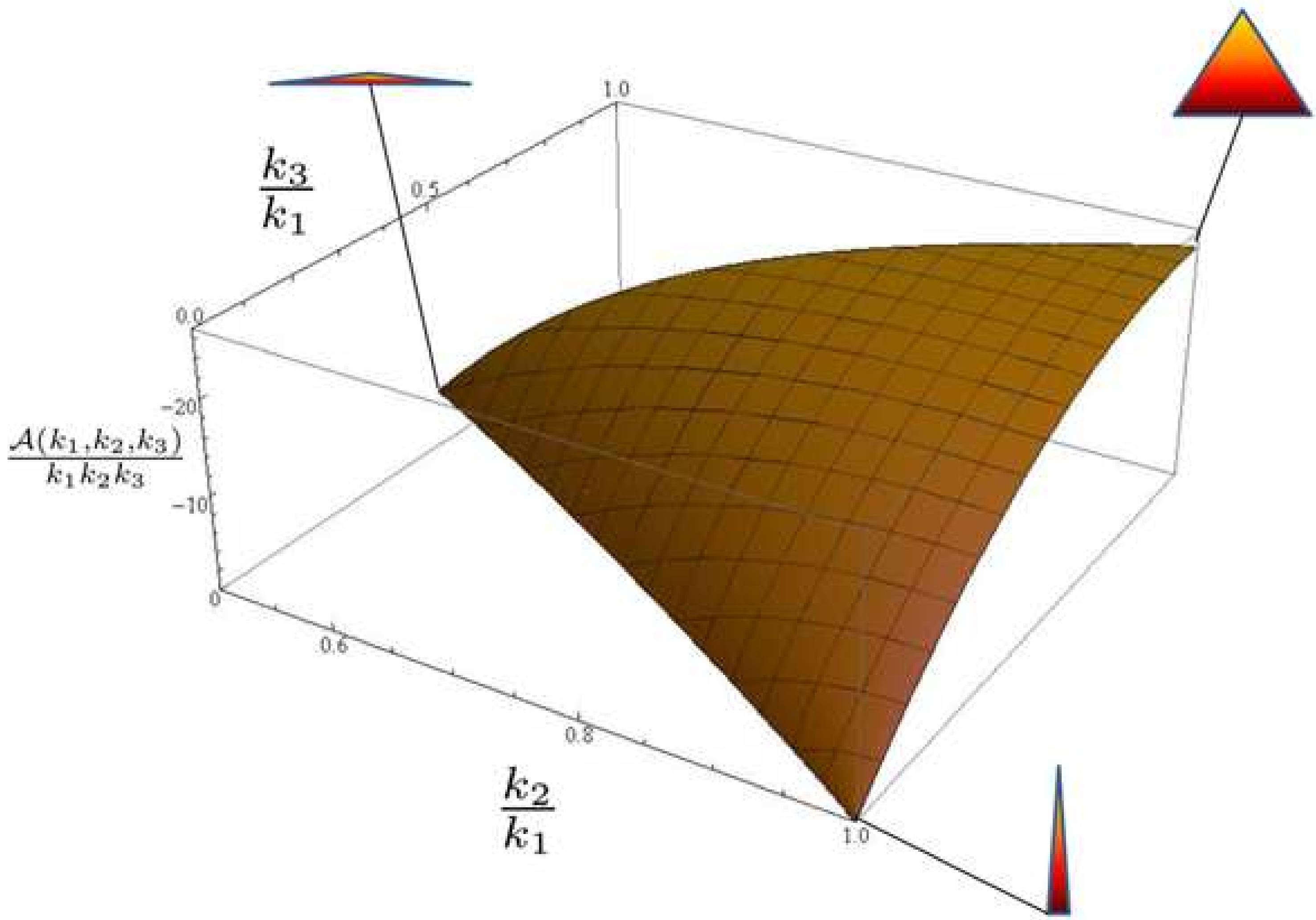
$$n_s \approx -4\epsilon \approx 0.97$$

Power law potential

Bounds from Planck in blue. Lines are for N = 50, 60, 70.



Power law potential



Amplitude dominant in the equilateral shape.

Example II: Exponential potential

In the original three-form theory

$$\mathcal{L} = -\frac{1}{48}F^2 - V_0 \exp(\beta A^2)$$

and in the $p(X, \phi)$ theory:

$$\mathcal{L}_0 = (W(x) - 1)\, V_0 \exp\left(\frac{1}{2}W(x)\right) - \frac{1}{2}\phi^2$$

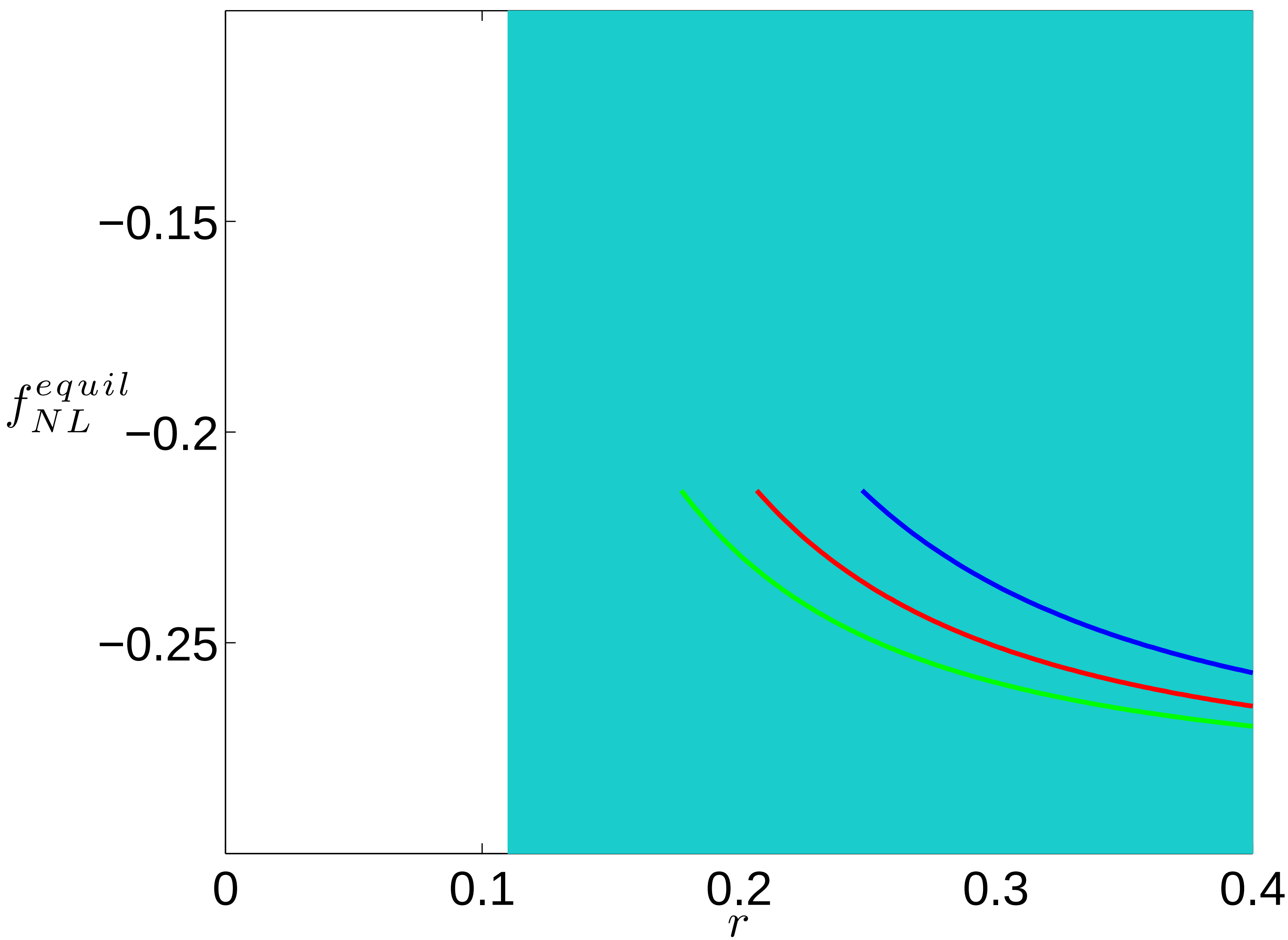
where $W(x)$ is the Lambert-W function and $x = X/12\beta V_0^2$.
N e-folds before the end of inflation

$$\chi_N^2 = \frac{2}{3} - \frac{1}{18\beta}\frac{1}{1+\sqrt{6}N}$$

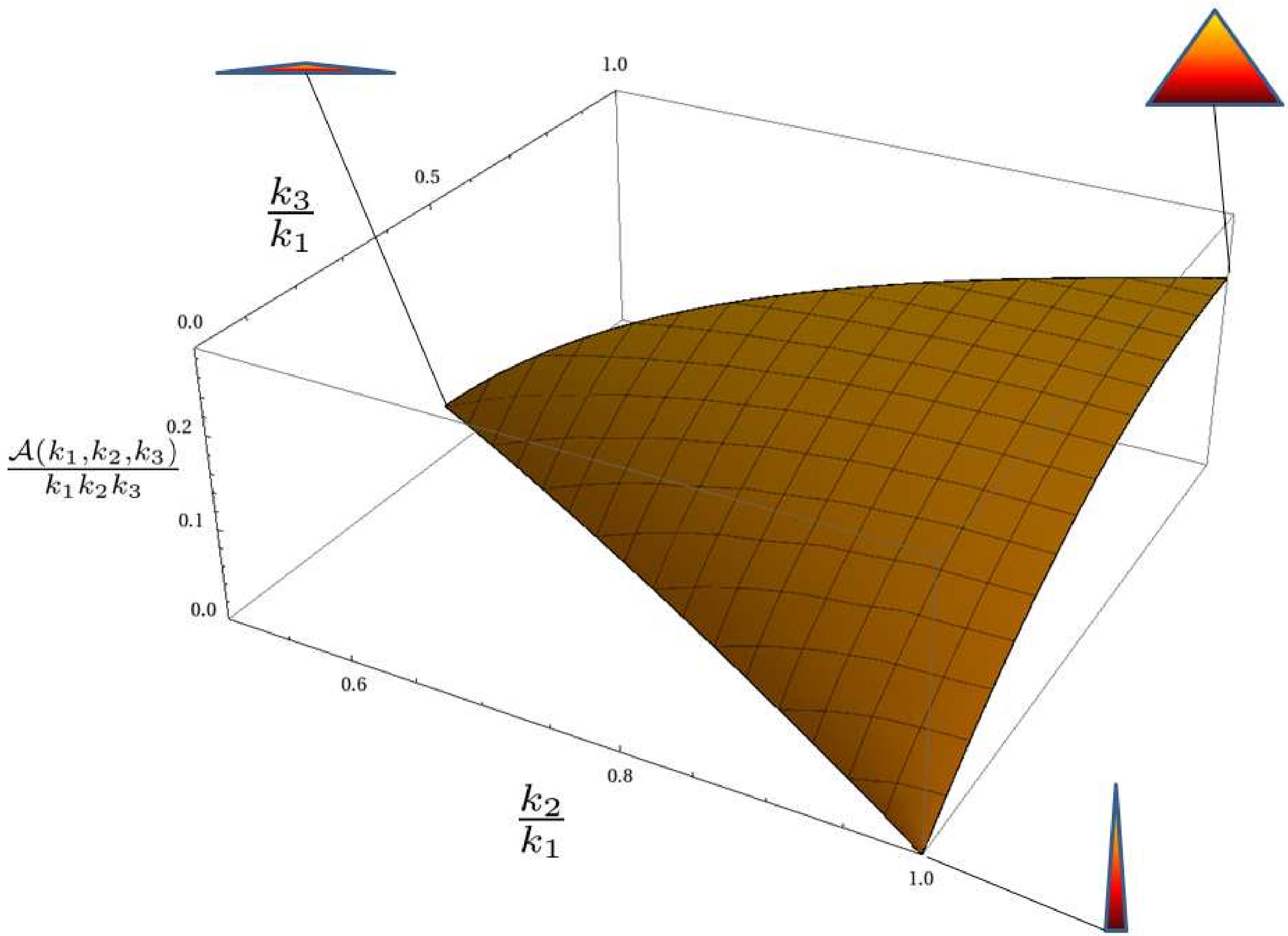
and the spectral index gives for $N = 60$

$$n_s \approx 0.97$$

Exponential potential



Exponential potential



Amplitude dominant in the equilateral shape.

Part II:

Dark Energy

Three-form action

Action for the three-form $A_{\mu\nu\rho}$

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} - \frac{1}{48}F^2(A) - V(A^2) - \sum_a m_a(A^2)\delta(x - x(\lambda))\sqrt{\frac{\dot{x}^2}{-g}}\right)$$

where

$$F_{\mu\nu\rho\sigma} = 4\nabla_{[\mu}A_{\nu\rho\sigma]} = \nabla_\mu A_{\nu\rho\sigma} - \nabla_\sigma A_{\mu\nu\rho} + \nabla_\rho A_{\sigma\mu\nu} - \nabla_\nu A_{\rho\sigma\mu}$$

We have the equations of motion:

$$\nabla \cdot F = 12\left(V'(A^2) + 2\rho\frac{m'(A^2)}{m(A^2)}\right)A$$

and due to antisymmetry we have the additional constraints:

$$\nabla \cdot \left(V'(A^2) + 2\rho\frac{m'(A^2)}{m(A^2)}\right)A = 0$$

Equations of motion

Consider flat FRW cosmology:

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Most general three-form compatible with FRW:

$$A_{ijk} = a^3(t)\epsilon_{ijk}\chi(t)$$

Equations of motion of the field χ with $f \equiv 2m_{,\chi}/m$

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} + 3\dot{H}\chi = -\kappa\rho_m f$$

Equation of motion of dark matter fluid:

$$\dot{\rho}_m + 3H\rho_m = \kappa\rho_m f$$

Equations of motion

Friedmann equation

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}(\dot{\chi} + 3H\chi)^2 + V(\chi) + \rho_m\right)$$

can also write:

$$H^2 = \frac{\kappa^2}{3} \frac{V + \rho_m}{1 - \kappa^2(\chi' + 3\chi)^2/6}$$

$\Rightarrow \qquad \kappa|\chi' + 3\chi| < \sqrt{6}. \qquad\qquad {}' = d/d\ln a.$

Evolution of the Hubble rate:

$$\dot{H} = -\frac{\kappa^2}{2}\left(V_{,\chi}\chi + (1 + \kappa f \chi)\rho_m\right)$$

Equation of state parameter of χ:

$$w_\chi = -1 + \frac{V_{,\chi} + \kappa f \rho_m}{\rho_\chi}\chi$$

Effective potential

$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} + 3\dot{H}\chi = -\kappa\rho_m f$$

$$V_{\mathrm{eff},\chi} = V_{,\chi}\left(1 - \frac{3}{2}(\kappa\chi)^2\right) - \frac{3}{2}\kappa^2\rho_m\chi + \kappa f\rho_m\left(1 - \frac{3}{2}(\kappa\chi)^2\right)$$

We are going to study 4 cases:

(i) $V = 0$, $f = 0$;

(ii) $V = 0$, $f \neq 0$;

(iii) $V \neq 0$, $f = 0$;

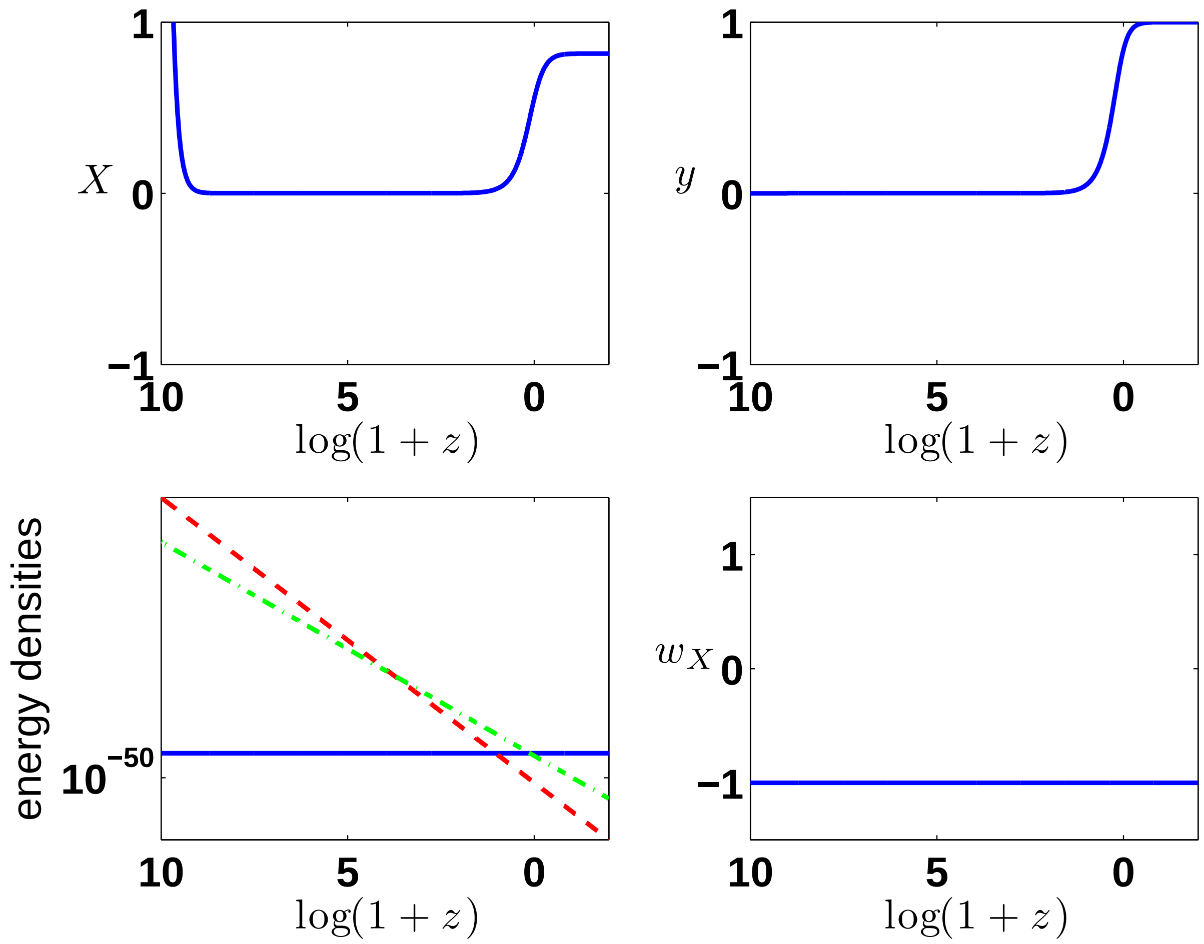
(iv) $V \neq 0$, $f \neq 0$.

Case: $V = 0$, $f = 0$;

$$y_i \equiv \chi_i' + 3\chi_i \neq 0 \qquad \text{otherwise} \qquad \rho_\chi \equiv 0.$$

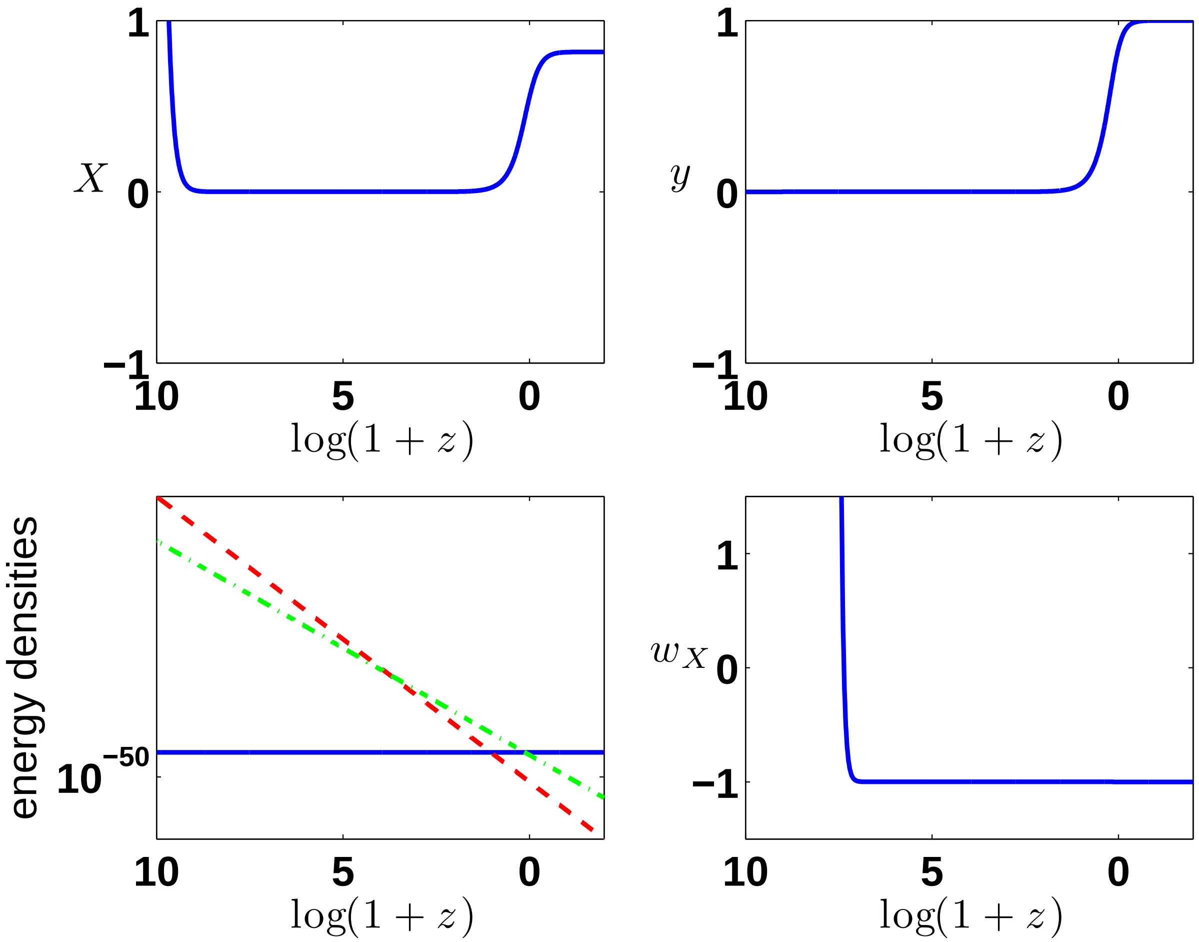
$$w_\chi \equiv -1$$

It is a cosmological constant!



Case: $V = 0$, $f \neq 0$;

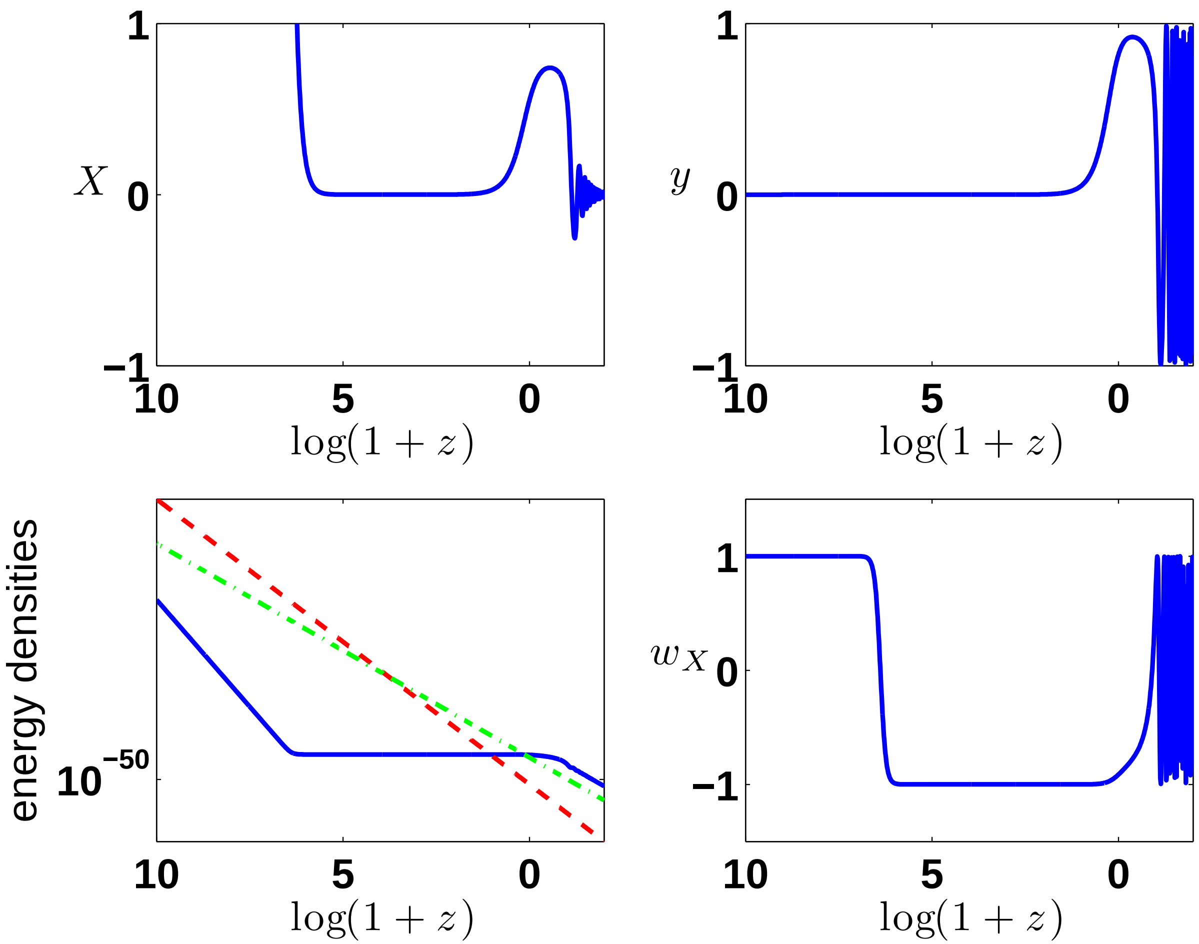
$$w_\chi = -1 + \frac{\kappa f \rho_m}{\rho_\chi}$$



Case: $V \neq 0$, $f = 0$;

With $V = V_0\chi^2$;

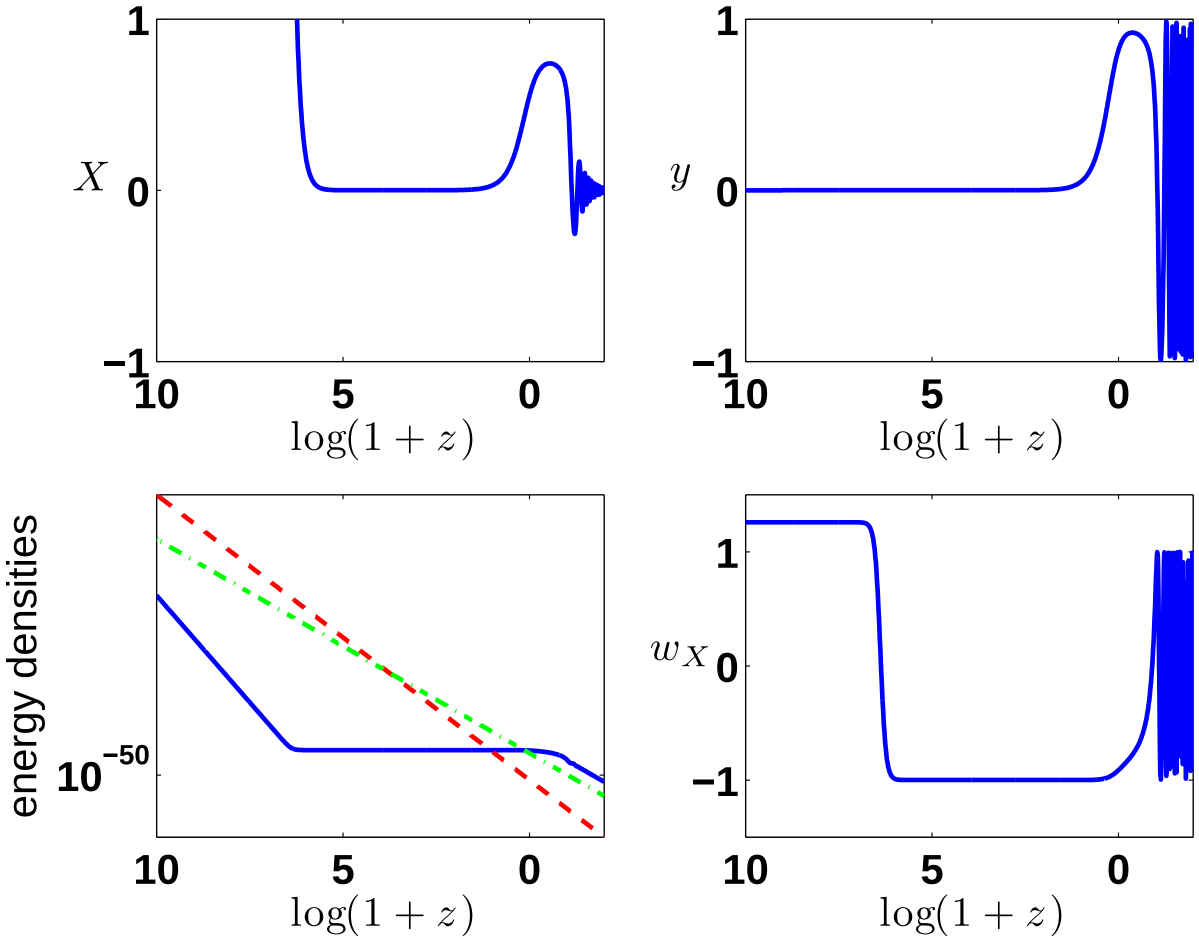
$$w_\chi = -1 + \frac{V_{,\chi}}{\rho_\chi}\chi$$



Case: $V \neq 0$, $f \neq 0$;

With $V = V_0\chi^2$;

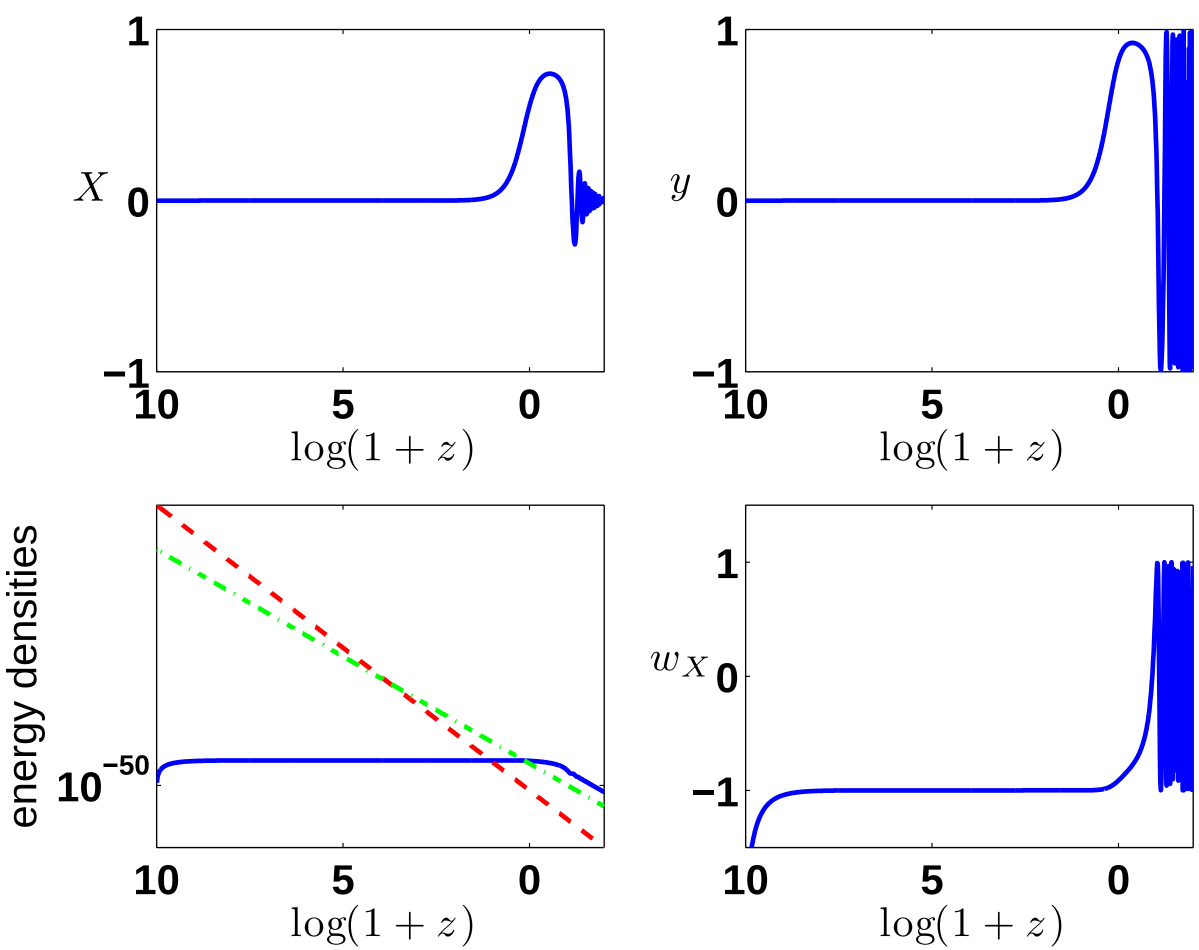
$$w_\chi = -1 + \frac{V_{,\chi} + \kappa f \rho_m}{\rho_\chi}\chi$$



Case: $V \neq 0$, $f \neq 0$;

With $V = V_0\chi^2$ and $\chi_i = \chi_i' = 0$

$$w_\chi = -1 + \frac{V_{,\chi} + \kappa f \rho_m}{\rho_\chi}\chi$$



Newtonian limit of linear perturbations

Linear evolution of matter density perturbations

$$\ddot{\delta}_m + \left(2H + \kappa f \dot{\chi} - \frac{2\dot{F}}{1-F}\right)\dot{\delta}_m = \left(\frac{\kappa_{\rm eff}^2}{2}\rho_m - \frac{k^2}{a^2}c_{\rm eff}^2\right)\delta_m$$

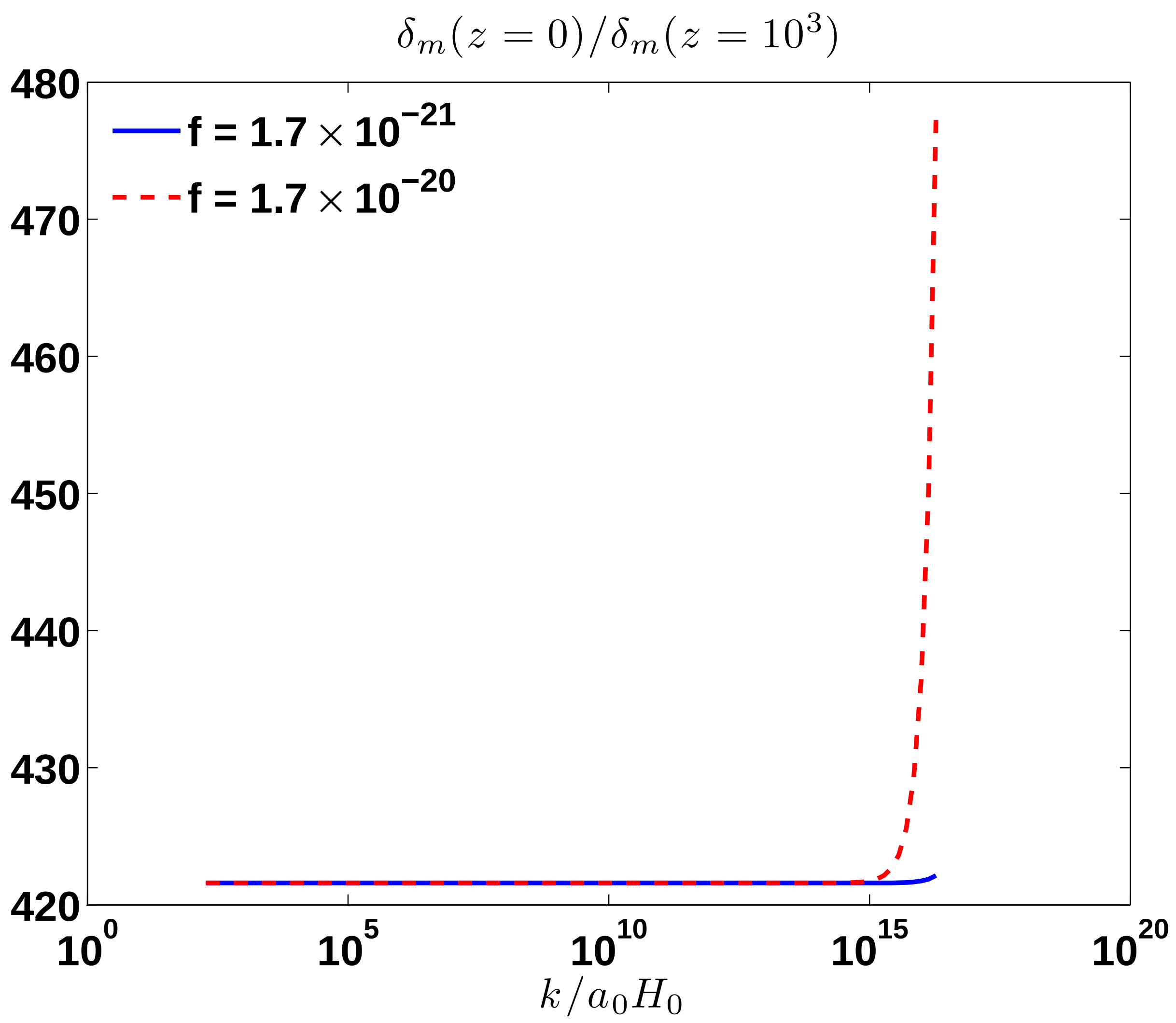
$$\kappa_{\rm eff}^2 = \frac{\kappa^2}{1-F}\left[1 + \frac{2}{\kappa^2\rho_m}\left(\ddot{F} + (2H + \kappa f\dot{\chi})\dot{F} - \frac{\kappa^2}{2}\frac{V_{,\chi}\kappa f \rho_m}{V_{,\chi\chi} + \kappa f_{,\chi}\rho_m}\right)\right]$$

$$c_{\rm eff}^2 = \frac{F}{1-F}\,, \qquad\qquad F \equiv -\frac{\kappa^2 f^2 \rho_m}{V_{,\chi\chi} + \kappa f_{,\chi}\rho_m}$$

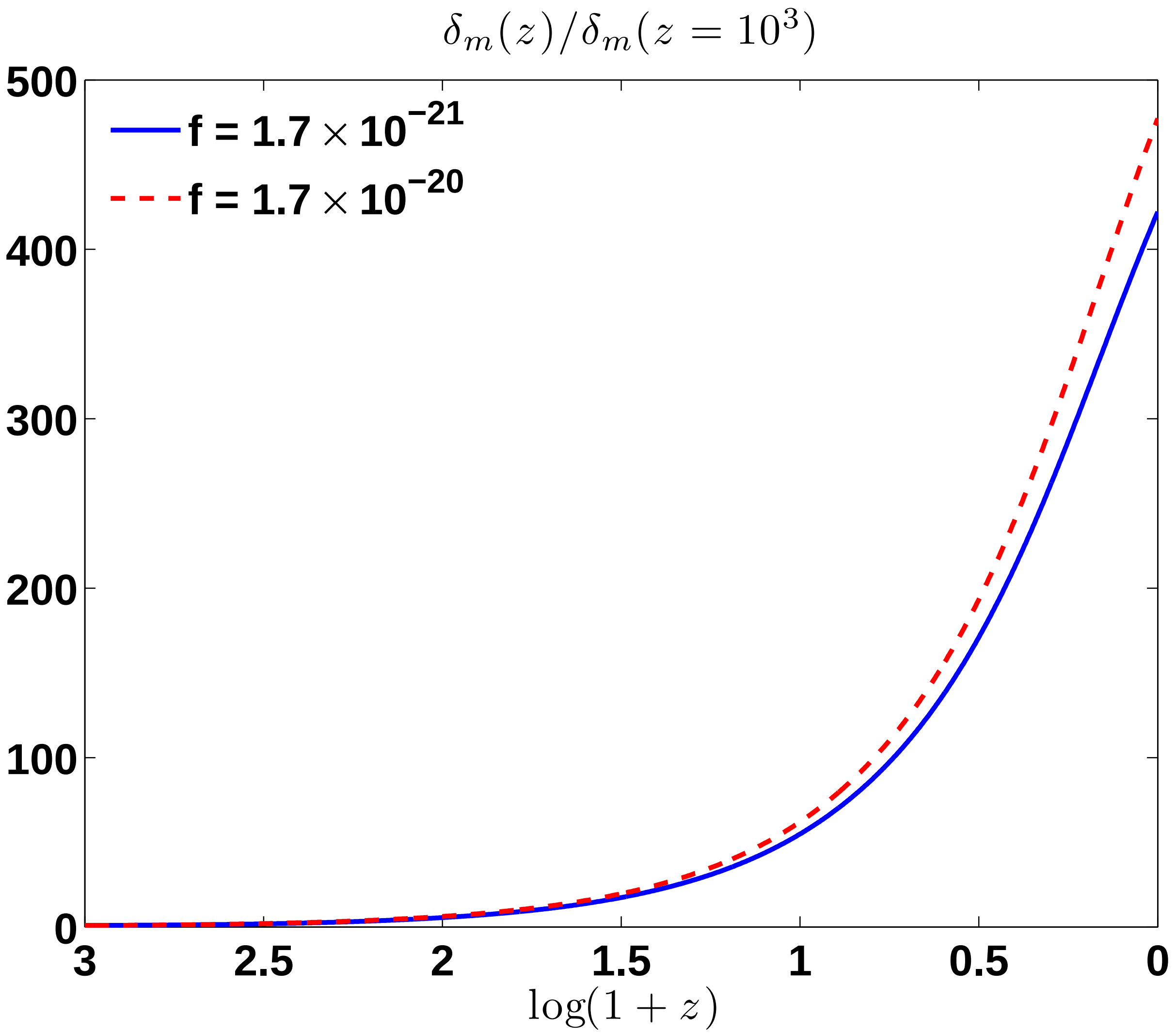
$F < 0 \quad \Rightarrow \quad c_{\rm eff}^2 < 0 \quad \Rightarrow$

for sufficiently large modes there is extra source term for growth of perturbations!

Growth at a given time



Growth for a given mode



Summary

- Three-forms possess accelerating attractors and saddle points which can describe three-form driven inflation or dark energy;
- Three-forms can give rise to viable cosmological scenarios with potentially observable signatures distinct from standard single scalar field models;
- Equivalence between models with alternative Lagrangian descriptions.
- Scalar spectral index predicted to be $n_s \approx 0.97$ for $N = 60$.
- Models have typically either large non-Gaussianity, e.g. power law potential or large ratio of tensor to scalar perturbations.
- In the presence of a coupling to dark matter, growth of structure is enhanced for small scales.

Further afield

- Reheating/preheating [De Felice et al.];
- Multiple 3-forms (assisted inflation/quintessence);
- Quintessential inflation;
- Formation of structure (analytical and numerical methods);
- Screening mechanisms.